Fuzzy Noise Maps and Fuzzy Noise Limits for Impact Assessment

(Invited Paper)

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Abstract— **Impact assessment involves estimating the value of a number of well-chosen indicators and comparing this estimate to a predefined set of limits. In this paper, it is argued that imperfection is ubiquitous in this process and therefore fuzzy set theory is a suitable framework for it. Imperfection and uncertainty emerge indeed naturally in a noise mapping process where large areas need to be investigated and not all input data is available or of sufficient quality. At the other side of the comparison, impact limits are often vaguely formulated as e.g. "unacceptable level" or "no adverse health effect". Additional uncertainty is added via the exposure-response relationships thus naturally leading to fuzzy noise limits. The proposed evaluation system involves an unacceptable level, a sustainable limit, and a crisp policy goal. A few examples are given to illustrate the use of the proposed fuzzy set based theoretical framework.**

I. Introduction

A. Terminology

This application of fuzzy systems is situated in the field of environmental noise assessment. Some of the most important terms are summarized in this paragraph:

- dB logarithmic scale on the acoustic pressure
- dBA acoustic pressure frequency weighted with 'A'-curve to mimic the sensitivity of the human hearing system to different frequencies.
- L_{eq} the energetic mean of the noise level
- $L_{A,eq}$ the energetic mean of the 'A'-weighted noise level
- SEL the integral of the noise level, where implicitly the duration of a noise event is included

B. Background

Guidelines and standards have been proposed for environmental noise impact assessment. A first set of standards is oriented towards noise exposure computation. In Europe several standards are currently in use causing differences in results even when the same input data is employed. A joint effort in unifying the standards is made in the Harmonoise project [6]. While the envisaged accuracy of the models is very high they will require a lot of input data. Data that is either not always available or is of low quality or may be too expensive to gather. The influence on the result is often hard to assess with classical techniques.

A second set of guidelines focuses on the setting of noise limits by policy. Recommended noise levels for a given situation or effect are expressed in terms of an indicator and it's limit. The World Health Organization for instance recommends L_{eq} levels below 55 dBA for "residential areas, outdoor during daytime" and an L_{eq} below 55 dBA for "fairly good speech intelligibility" [1]. When noise limits are set based on these guidelines they serve to protect the population from *adverse health effects*. Although the aforementioned indicators are crisp and can be accurately measured, vagueness in concepts like *adverse health effects* make the setting of limits an inherent vague process. This type of uncertainty is often called *hard* [11]. The setting of a noise limit is furthermore influenced by vagueness in concepts such as sharing the burden among population groups in a fair way or trading the quality of life of the individual for the quality of life of a population.

Based on these observations a formal framework is presented in following sections. Section II describes the fuzzy equivalent of calculating indicators. Section III and IV extend the setting of noise limits to the fuzzy framework. Section V discusses the fuzzy equivalent of evaluating an indicator with respect to limits while section VI treats this evaluation on a macroscopic scale.

II. Fuzzy Noise Exposure

A. Noise Level Calculation

The first step in environmental noise annoyance prediction is quantifying the sources of noise. Any imperfection encountered in this process should be carried along up to the final noise annoyance assessment. Imperfection generally denotes that data or models are not ideal and show deficiencies. Data not sampled or recorded accurately is *imprecise*. *Uncertainty* is related to the lack of knowledge of the truth value of an expression. Information is considered *vague* when it is not clearly defined or has several meanings [9]. Information can be tainted by several aspects of imperfection.

Imprecision is often modeled by probability while uncertainty and vagueness are commonly modeled by fuzzy set theory. In this paper we take the fuzzy set approach. For imprecise data a transformation is applied converting a probability distribution to a fuzzy set [8].

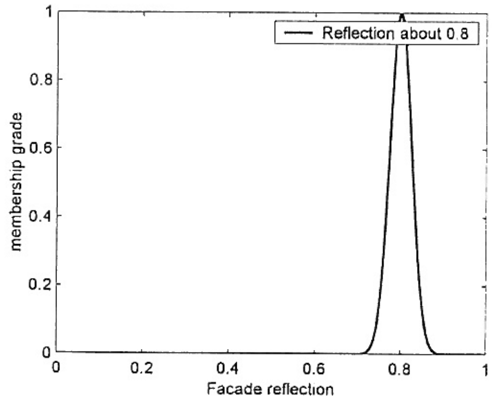

Fig. 1. Fuzzy number and interpretation

To model imperfect data the classical concept of a number is extended to a fuzzy number. The classical single value of a variable is replaced by a fuzzy set which can be seen as a possibility density function [12]. An example is the façade reflection coefficient illustrated in figure 1. The fuzzy number shown expresses that the reflection coefficient is *about 0.80*. It could be the result of an expert estimate for example. The exact reflection coefficient could be measured but is *unknown* in this case but will *likely* be around 0.80.

A fuzzy number A defined over a universe of discourse X and with possibility density function $\mu_A(x)$ can be represented by a set of α-cuts A_α defined as:

$$A_\alpha = \{x \in X : \mu_A(x) \geq \alpha\} \tag{1}$$

Huge amounts of information are needed and processed in environmental noise assessment. Therefore a suitable representation for the fuzzy numbers is essential. Criteria are memory footprint and speed of computation. Fuzzy numbers based on convex fuzzy sets are the best choice. A fuzzy set associated with the fuzzy number is convex when the N α-cuts can be written as intervals $A_{\alpha,i} = [A_{\alpha,i,inf}, A_{\alpha,i,sup}]$. Only begin and end values of the α-cuts need to be stored.

Arithmetic on fuzzy numbers is performed by using the extension principle by Zadeh [7]. Given a binary operator $\otimes$ and it's inverse $\oslash$:

$$\otimes : X \times X \rightarrow X : C = A \otimes B \tag{2}$$

$$\mu_C(x) = \sup_y \min\{\mu_A(x \oslash y), \mu_B(y \in X)\} \tag{3}$$

The computation involves a costly minimization which can be eliminated by using convex fuzzy numbers. Arithmetic on fuzzy numbers then reduces to interval arithmetic where the arithmetic is performed on the α-cuts independently of each other. The developed implementation which uses 4 α-cuts only slows down the noise mapping software by a factor 3. Considering the 8-fold amount of numbers processed perform this slowdown is acceptable. The good performance is obtained because modern processor can easily take advantage of independent and parallel computations.

Typical emission and propagation models are formulated to operate on crisp numbers. To support computation with fuzzy numbers these models have to be converted. This can be done at several meta levels. In the macro level approach the original computation models are left untouched and a fuzzy *shell* is built around them. This shell is used to let the original models run for a number of times while selecting appropriate input parameters from the input distributions. The output distribution is gradually build up while performing the simulations. A micro level approach replaces each crisp computational block by its fuzzy counter part and fuzzy information flows through the original model [3]. The latter approach is the most suitable because the monotonicity of the operations, required for convex fuzzy set calculus, can in general not be guaranteed for the complex emission and propagation models as a whole.

B. *Visualization*

One of the outputs of noise mapping are the exposure maps. A commonly used indicator is the $L_{A,eq}$. When a crisp computation is used and hence the result is crisp this single value can easily be translated into a color and used for map drawing. The result of a fuzzy noise mapping is however a possibility distribution per location. A suited measure has to be chosen to visualize the noise map and also the computed imperfection on the results.

Defuzzification by the *maximum criterion defuzzification* or *center of gravity defuzzification* can be used to pick a single crisp value out of the fuzzy set. Both are straightforward to compute when convex fuzzy sets are used.

$$\text{DF}_{\text{max}} : X \rightarrow \mathbb{R} : \text{DF}_{\text{max}}(A) = \frac{1}{2}(A_{\alpha,N,inf} + A_{\alpha,N,sup}) \tag{4}$$

$$\text{DF}_{\text{cog}} : X \rightarrow \mathbb{R} : \text{DF}_{\text{cog}}(A) \simeq \frac{\sum_i \alpha_i(A_{\alpha,i,inf} + A_{\alpha,i,sup})}{\sum_i \alpha_i} \tag{5}$$

For symmetric convex fuzzy sets these two defuzzifications are equivalent. To visualize the imperfection on the results, a wider range of measures is available. The *support* gives the total imperfection but in general overestimates it. This is a consequence of the cumulation of imperfection during the noise propagation [3]. The support represents the limits of the attainable values in the most extreme cases. The *bandwidth* or diameter of the $0.5 - \alpha$-cut is a better choice to visualize the *expected* uncertainty. A useful alternative is the entropy after Klir [7] which is a measure for information content and is defined as (possibilities need to be discretized and sorted in descending order) :

$$H_{\text{Klir}} = \sum_{i=2}^{n} \pi_i \log_2\left(\frac{i}{i-1}\right) - \sum_{i=1}^{n-1}(\pi_i - \pi_{i+1}) \log\left(1 - i \sum_{j=i+1}^{n} \frac{\pi_j}{j(j-1)}\right) \tag{6}$$

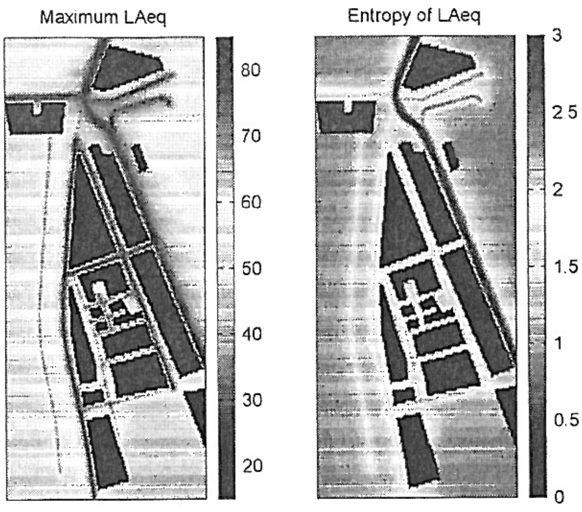

Fig. 2. Fuzzy noise maps using the maximum criterion defuzzification and entropy uncertainty measure

Although almost all noise levels are presented in the logarithmic dB scale, the computation are done using the linear physical acoustic pressures. Acoustic pressures are first transformed into the logarithmic scale before the fuzzy measures are taken. Interpretation of results is easier when the results are converted prior to the defuzzification process. Figure 2 illustrate the a defuzzified noise map using the maximum criterion and the entropy as uncertainty measure.

III. Fuzzy Noise Limits

A. Multiple Limits

Consider an indicator I, defined on a universe of discourse X_I. As example of an indicator take the façade exposure to traffic noise during the day expressed as $L_{A,eq}$. The universe of discourse can safely be set to the interval 0 to 100 dB. In setting a classical noise limit a single value I_{nl} is chosen as goal. Values of the indicator below this value suggest a *good* or *wanted* situation. Values above I_{nl} are automatically considered *bad*. This binary evaluation is however not subtle enough to represent the available scientific knowledge.

For values of an indicator there is often a difference between what science considers to possibly cause adverse health effects and what people in a society prefer. A function μ_S is used which maps values of an indicator to the set $\{0,1\}$ evaluating the indicator as *good* or *sustainable*. A similar function μ_U is defined for the *bad* or *unacceptable* situation. $\mathbb{B}$ is used as notation for $\{0,1\}$.

$$\mu_S : X \rightarrow \mathbb{B} : \mu_S(x) = (x \leq X_S) \quad (7)$$

$$\mu_U : X \rightarrow \mathbb{B} : \mu_U(x) = (x \geq X_U) \quad (8)$$

The sets S and U of *good* and *bad* values are defined as:

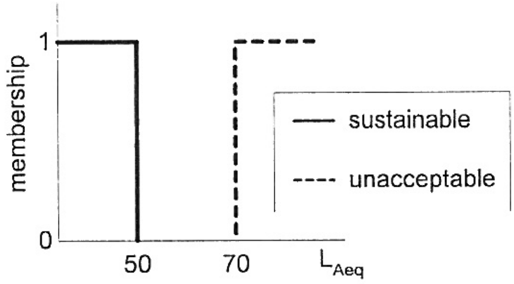

Fig. 3. Twofold crisp limit

$$S = \{x \in X : \mu_S(x) > 0\} \quad (9)$$

$$U = \{x \in X : \mu_U(x) > 0\} \quad (10)$$

This generalization does not require $X_S = X_U$. Values can be either in S or U depending on the choice of X_S and X_U. When $X_S < X_U$ and hence the intersection of S and U is empty there exist values of I that don't belong to S nor to U. These values are not *good* but not *bad* either. A less common configuration is $X_S > X_U$ and hence $S \cap U \neq \emptyset$. Values belonging to this intersection represent an unknown situation: it can be *good* or *bad*.

Turning back to the example of the $L_{A,eq}$, X_S may be set at 50 dB while 70 dB is a reasonable choice for X_U. The functions μ_S and μ_U are plotted in figure 3.

B. Fuzzy Limits

Functions μ_S and μ_U can be seen as the membership functions of the sets S and U. The image of the mapping is extended from $\mathbb{B}$ to the continuous interval $[0,1]$. This promotes the sets S and U to fuzzy sets and allows to express that some indicator values only partly belong to the sets of unacceptable or sustainable indicator values. Figure 4 shows an example of such a mapping. Already this figure gives a better picture to policy makers of what science knowledge is available about the effects of noise, including the uncertainty and vagueness.

C. Crisp Targets

Based on the observations made above one could argue that policy targets should be fuzzy as well. However since these goals are the result of debate and also try to account for technological feasibility, cost of compliance, social, economic and culture conditions they can be made crisp. The transition towards a sustainable situation can then be achieved by consequently setting new crisp targets increasingly close to the sustainable limit.

IV. Quantifying Fuzzy Noise Limits

A straightforward approach to obtain the fuzzy set membership functions is by consulting experts in the field. Each expert provides a membership function for both fuzzy sets which are then aggregated into one couple of membership function. An

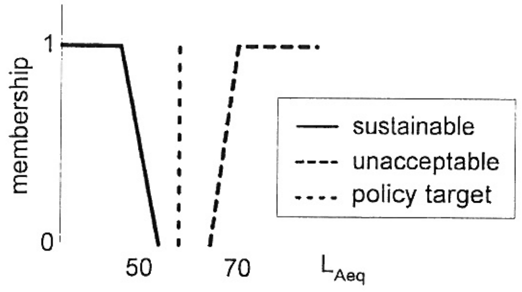

Fig. 4. Fuzzy noise limits and crisp target

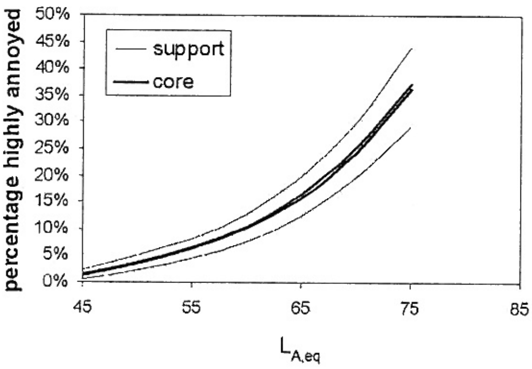

Fig. 5. Dosage-effect relationship converted to a fuzzy relation, from [2]

overview of membership function generating techniques can be found in [8].

Direct construction of fuzzy set membership functions is a rather difficult task. Consider again the example of the $L_{A,eq}$. The effect of noise on man and society has to be transformed back into limits on the $L_{A,eq}$. This must be done under conditions of vagueness and uncertainty on these effects while the $L_{A,eq}$ is sharply defined quantity.

To simplify this task, the available scientific knowledge including uncertainty on the relation between $L_{A,eq}$ and effect can be introduced explicitly. This reduces the task of the expert to drawing fuzzy limits for the effect, which are only affected by society preferences. Consider the dosage-effect relationship that relates road noise to the percentage of highly annoyed people shown in figure 5 [4]. A fuzzy extension of the dosage-effect relationship is used where the universe of discourse is X_{HA}. Based on the statistics of annoyance in surveys for each noise level, a fuzzy set is built with a small core around the mean value. The support can be for instance the 95% percentile. The result is a fuzzy relation DE between the percentage of highly annoyed people defined over X_{HA} and the $L_{A,eq}$ defined over X_L.

Assume that a society decides that a situation where only 1% of the population is highly annoyed is definitely sustainable. When more than 5% of the people are highly annoyed this situation is no longer considered sustainable anymore. Translation into a fuzzy sustainable limit S_{HA} is performed by using:

$$\mu_{S_{HA}} : X \to [0,1] :$$
$$\mu_{S_{HA}}(x) = \sup_{h \in X_{HA}} (\min[L_{HA}(h), DE(x,h)]) \tag{11}$$

For some indicators it is better to set multi-dimensional noise limits. An example is the unacceptable limit for sleep disturbance L_{sd} where not only the $L_{A,eq,night}$ is important but also SEL per event and number of events N. A multi-dimensional fuzzy set can be constructed. For simplicity only the *core* c_{sd} and the *support* s_{sd} are described:

$$c_{sd} = \{(l,s,n) \in X_{LAeq,night} \times X_{SEL} \times X_N : s > 95 \vee (s > 80 \wedge n > 15) \vee l > 65\} \tag{12}$$
$$s_{sd} = \{(l,s,n) \in X_{LAeq,night} \times X_{SEL} \times X_N : s > 80 \vee (s > 65 \wedge n > 10) \vee l > 45\} \tag{13}$$

The interpretation of these sets is that the situation is marked as *certainly unacceptable* whenever the SEL is larger than 95 dB or the SEL is larger than 80 dB and the number of events was larger than 15 or when the $L_{A,eq,night}$ is larger than 65 dB. Just *possibly unacceptable* occurs for levels determined by s_{sd}.

V. Linguistic Evaluation

Indicator values shown on the same scale as the noise limits already clarify the meaning of these values. This is useful for communication of computed noise levels to policy makers and public. In classical environmental impact assessment, results are often summarized by using a few labels only (e.g. *good*, *bad*, emphneutral). In particular when the calculated indicator value is fuzzy, extending the summarizing (linguistic) evaluation is not trivial. Therefor it is useful to derive a more formal system for this purpose. Let F_E be a collection of n suitable labels $\{E_0, E_1, ...E_n\}$ used to translate the fuzzy indicator value into a linguistic expression. Assume that the labels in F_E partition the universe of discourse uniformly by a set of triangular, equidistant fuzzy sets defined on the label universe $U_E = [0,1]$. More suitable fuzzy sets are discussed in [10]. Two fuzzy linguistic rules can be defined based on the fuzzy sets S and U introduced in previous section, where X is a fuzzy value of the indicator I over universe U_I:

$$R_0 : \text{"if } X \subseteq S \text{ then } E \text{ is } E_0\text{"} \tag{14}$$
$$\dots$$
$$R_n : \text{"if } X \subseteq U \text{ then } E \text{ is } E_n\text{"} \tag{15}$$

Labels E_0 and E_n could for instance be *very bad* and *very good*. When the antecedent of neither rule is exactly matched a suitable mechanism has to be used to select the most appropriate linguistic label. The *certainty qualification* [5] is the best suited interpretation of rules in the context at hand. It reads: the more X is in S, the more E is in E_0 or the more E_0 is a suitable label for the indicator I. The membership functions of E'_0 and E'_n where $X \in X_I$:

$$E'_0 : U_E \rightarrow [0,1] : E'_0(y) = \max(1 - \text{inc}(X, S), E_0(y)) \quad (16)$$
$$E'_n : U_E \rightarrow [0,1] : E'_n(y) = \max(1 - \text{inc}(X, U), E_n(y)) \quad (17)$$

Where the Kleene-Dienes implication is recognized. The inclusion used in equations (16) and (17) is defined as:

$$\text{inc}(X, L) = \inf_{y \in U_I} (\max(L(y), 1 - X(y))) \quad (18)$$

The membership function resulting from evaluation of the rules must be combined conjunctively to generate the final membership function $E' = \text{AND}(E'_0, E'_n)$. The conjunction is implemented using the Lukasiewicz t-norm:

$$E' : U_E \rightarrow [0,1] : E'(X) = \max(0, E'_0(X) + E'_n(X) - 1) \quad (19)$$

This choice results in an evaluation that corresponds to intuition, but more importantly it also yields the expected simplification for crisp indicator values. The fuzzy set E_i most closely related to E' has to be chosen as the final summarizing result of the EIA. The label with the largest similarity with E' is selected. A broad selection of similarity measures can be found in literature. The choice for this application is again driven by the desire to obtain an acceptable simplification in the case of crisp indicator values. The similarity is measured by:

$$\text{sim}(E', E_i) = \frac{\sup_{u \in U_E}(T(E'(u), E_i(u)))}{\sup_{u \in U_E}(S(E'(u), E_i(u)))} \quad (20)$$

The Zadeh t-norm min and t-conorm max are chosen because they lead to significant simplifications when X is crisp. X can be made crisp when it is accurately known or the imperfection on it is much smaller than that present in the fuzzy sets S and U.

When X is crisp and $E_0 \cap E_n = \emptyset$:

$$\mu_X : U_I \rightarrow [0,1] : \begin{cases} 0 & x \neq q \\ 1 & x = q \end{cases} \quad (21)$$

then the situation is evaluated as $S(q)$ *sustainable* or $U(q)$ *unacceptable*. In the example of the $L_{A,eq}$ with a value of 53 dBA this leads a linguistic evaluation where the situation is labeled as $S(53dBA)$ *sustainable* or $U(53dBA)$ *unacceptable*.

VI. Macroscopic Assessment

Environmental noise impact assessment in general compares different scenario's or policy options. Often, this results in a macroscopic evaluation comparing the number of people affected rather than in an estimate of the impact on an individual. This aggregation of the impact over a population can be readily extended using the fuzzy (linguistic) evaluation proposed in the previous sections.

Consider a population where s_j represents the j^{th} subject in a population U_p of size n. The number of people whose living situation is evaluated as *very bad* can be used as decision

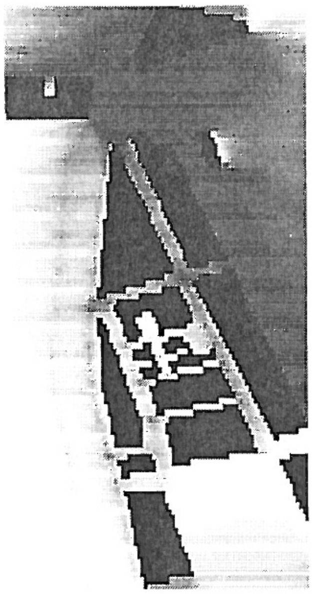

Fig. 6. Fuzzy noise map using fuzzy noise limits for visualization, green is *sustainable*, red is *unacceptable*

criterion. In general the membership of the fuzzy set Q_{E_i} of subjects evaluated with label E_{s_j} is written as:

$$Q_{E_i} : U_p \rightarrow [0,1] : \mu_{Q_{E_i}} = \frac{\text{sim}(E_i, E_{s_j})}{\max_i(\text{sim}(E_i, E_{s_j}))} \quad (22)$$

The rating of a scenario can be multi-dimensional by considering more than one set Q_{E_i}. The relative cardinality of these sets is an indicator usable in the final assessment of a scenario. The uncertainty on the evaluation can be computed by using the support and core of the fuzzy sets E_{s_j} in equation (22).

A. Examples

The first example reconsiders the same area of figure 2. The computed fuzzy $L_{A,eq}$ is now used in conjunction with the fuzzy noise limits of figure 4. Labels E_0 and E_n are represented by the colors *green* and *red*. The intensity of a color represents the similarity of the evaluation E_i of the fuzzy noise levels to the respective labels. The color with largest intensity was given priority in the visualization. Whitish colors show areas of large uncertainty concerning the label to use in that particular location.

The second example illustrates the use of the technique on a large population. The measured noise exposure near 250 dwellings is compared for the years 1996 and 2001. For each location the evaluation is performed with respect to the fuzzy noise limits and aggregated using the technique expound in section VI. Table I shows the percentage of dwellings with *sustainable* and *unacceptable* exposure. The bracketed percentage is the crisp aggregation for absolute certainty and just non-zero possibility for the given evaluation.

TABLE I

AGGREGATED FUZZY NOISE LIMIT EVALUATION OF 250 DWELLINGS IN FLANDERS

	1996	2001
sustainable	18.4% (1.8%-40.2%)	14.8% (2.0%-38.9%)
unacceptable	12.0% (2.4%-26.8%)	13.5% (2.0%-30.5%)

VII. CONCLUSION

This paper shows that uncertainty and vagueness is present in the process of noise impact assessment. To account for the uncertainty and other imperfection in the noise mapping step the use of fuzzy numbers is proposed. With a good choice of representation and the use of underlying convex fuzzy sets the additional computational cost is very low.

Fuzzy noise limits are used to represent the uncertainty on the effects of noise as well the inherent vagueness introduced by social, economic and cultural preferences that shape the final noise limits. A specific scale with two fuzzy and one crisp limit is highlighted. The two fuzzy noise limits describe the sustainable and the unacceptable situation. The third crisp noise limit represents a policy target. A linguistic label summarizing the EIA result can formally be associated with a situation. For a whole population an aggregation is made making the methodology useful for noise monitoring, scenario assessment, and policy support.

ACKNOWLEDGMENT

Part of this research has been conducted with support of the IWT-Vlaanderen, Institute of Science and Technology Flanders.

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