Cellular Neural Network Approach in Assignment Problem for Minimizing the Cost Function

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ABSTRACT: *A Cellular Neural Network approach to combinatorial optimization problems is presented. An assignment problem for minimizing the cost function has been considered. This approach could be applicable to map combinatorial optimization problems such as industrial control, operation research and production planning. The energy function has been presented as a function of optimization for the network decision. This function can be applied in other optimization problems.*
Keywords: *Cellular Neural Networks, assignment problem, Lyapunov function, optimization problems.*

I. INTRODUCTION

In the area of decision-making, many applications substantially involve the concepts of scheduling, such as communications routings, industrial control, operation research, production planning, etc. However, the optimal solution for a large problem can be a time-consuming. The assignment problem has a great importance for manufacturing planning and more recently for flexible manufacturing systems [1]. This can be referred as, for example, assigning tasks to machines, workers to jobs, salesmen to regions, requirements to suppliers, etc. The main characteristic of these problems is that only one candidate, namely: task, worker, etc is assigned to each position, machine, job, etc.

Many papers which consider the applications of neural networks in combinatorial optimisation have been produced. Hopfield and Tank proposed a neural network model to solve these problems in their pioneering paper [9], and in particular they presented a mapping of the traveling salesman problem to neural networks. Since then numerous modifications of Hopfield network architecture have been presented aimed at improving the network performance particularly for the traveling salesman problem. But most of these attempts have lacked many theoretical principles. Fine-tuning of the coefficient of networks for neural presentation of the problem had been accomplished by trial and error and the neural representations themselves have been made heuristically or without care. Accordingly, Hopfield have failed in many combinatorial optimisation problems [10], sometimes did not represent a feasible solution to the problem and there were difficulties in selecting appropriate parameters which can effectively balance all the terms in the energy function. Many researchers have tried to optimally select the parameter, while others have focused their attention on reducing the number of the terms in the energy function. Even if a suitable energy function representation has been found, which guarantees the feasibility of the solutions; the quality of these solutions is unlike to be comparable to these obtained using conventional techniques [11].

The assignment problem can be illustrated by the following example: given a set of N tasks, N machines, and cost $c_{ij} > 0$ to carry out task j by machine i. The goal of the assignment problem is to find an output $y_{ij} \in \{0,1\}$, which minimizes the total cost for carrying out all the tasks under the constraints: each task must be carried out once and by only one machine and each machine must carry out only one task. Here y_{ij} means that machine i does or does not carry out task j.

Consider the case of N = 4, the cost matrix is given by

$$U = \begin{bmatrix} 0.9 & 0.4 & 0.2 & 0.1 \\ 0.3 & 0.1 & 0.1 & 0.6 \\ 0.1 & 0.4 & 0.8 & 0.9 \\ 0.6 & 0.1 & 0.2 & 0.3 \end{bmatrix} \quad (1)$$

The task is to find an optimal solution Y_o that meet the constraints and minimize the cost function:

$$C = \sum_{i,j=1,\dots,N} y_{ij} u_{ij} \quad (2)$$

For this example, the output matrix, which represents the optimal decision, is given by:

$$Y_o = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \quad (3)$$

while the global minimum cost function is:

$$C = \sum_{i,j=1,\dots,N} y_{ij} u_{ij} = 0.4 \, .$$

Here are some other feasible solutions, which differ from the optimal solution:

$$Y_1 = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \quad Y_2 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$Y_3 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad Y_4 = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \tag{4}$$

The cost functions for the previous solutions are given by:

$$C_1 = 0.5 < C_2 = 0.7 < C_3 = 0.9 < C_4 = 1.$$

Observe that we have not written the feasible solutions of large cost function.

II. CELLULAR NEURAL NETWORK FOR ASSIGNMENT PROBLREM

The early applications of the Cellular Neural Network (CNN) were to solve the image processing and pattern recognition problems [4,5,6]. Since 1988 many researchers have made significant contributions to the CNN paradigm by various generalizations. In [3] a new approach of CNN has been presented for communication problems, which performs the spirit of our paper.

A cell c_{ij}, $(i = 1,\dots,N, j = 1,\dots,N)$, of the CNN is controlled by the following state equation

$$\begin{aligned} \dot{x}_{ij} &= -x_{ij} + \alpha y_{ij} + A \sum_{l=1,\dots,N,l\neq j} y_{il} + A \sum_{k=1,\dots,M;k\neq i} y_{kj} \\ &+ \beta u_{ij} + B \sum_{l=1,\dots,N,l\neq j} u_{il} + B \sum_{k=1,\dots,N,k\neq i} u_{kj} + I \end{aligned} \tag{5}$$

where $y_{ij} = f(x_{ij})$ is a piecewise linear output function with $y(x) = 0$ if $x \le 0$, $y(x) = x$ if $0 < x < 1$ and $y(x) = 1$ if $x \ge 1$. The variables x_{ij}, y_{ij}, u_{ij} refer to the state, output and input voltage of a cell $c(i, j)$. Constant α is the self-feedback of the cell. While A, B are the feedback and control templates of the cells sharing the same row and column of the cell $c(i, j)$. Parameter β is the self-control template of the cell. Constant I is the independent current source. The following set of inequalities represent the necessary conditions for the CNN templates [3]:

$$\left.\begin{aligned} &\rho = \frac{\beta}{2B} > 0 \\ &I > 0 \\ &\alpha > 1 \\ &A < 1 - \alpha - I < 0 \\ &0 < B < \Gamma = \frac{1}{2}\frac{1-\alpha-I-A}{\rho+N-1} \end{aligned}\right\} \tag{6}$$

The justification for the mentioned condition has been presented in [3] for special class of communication problems. We adopt the condition (6) for the related part of the parameters α, A, I and we leave the parameters β, B to be discussed later in the next section.

III. LYAPUNOV FUNCTION AS A FUNCTION OF OPTIMIZATION

In fact we will not devote much attention to the main equation state of (5). Instead, we move to the equivalent Lyapunov function for this approach as it depicted in [3]. This function was derived from the basic Lyapunov function formula [4] but with some modification.

$$\begin{aligned} E(t) &= -\frac{1}{2} \sum_{i,j=1,\dots,N} y_{ij} \Big[(\alpha-1) y_{ij} + A \sum_{l=1,\dots,N;l\neq j} y_{il} \\ &+ A \sum_{k=1,\dots,N;k\neq i} y_{kj} + 2I \Big] \\ &- \sum_{i,j=1,\dots,N} y_{ij} \Big[\beta\, u_{ij} + B \sum_{l=1,\dots,N,l\neq j} u_{il} + B \sum_{k=1,\dots,N,k\neq i} u_{kj} \Big] \end{aligned} \tag{7}$$

The properties of Lyapunov function (7) were broadly illustrated in [2]. As we are interested in the solution whether it is optimal or not through the energy function point of view, we will focus on the saturation region where the output of the cell equals one. Hence, we take into account the following equation

$$E = -\frac{1}{2} N(\alpha - 1) - NI - 2B \sum_{i,j} u_{ij} - (\beta - 2B) \sum_{i,j} u_{ij} y_{ij} \tag{8}$$

As we concern about the minimization of the cost, the optimal solution must represent the minimum cost and globally minimum energy function.

We put the energy function (8) under close scrutiny and discuss the following points

a) We rewrite (8) as follows

$$E = -\frac{1}{2} N(\alpha - 1) - NI - 2B(\sum_{i,j} u_{ij} - Cost) - \beta.Cost \tag{9}$$

Let us assume that $B > 0$ and $\beta < 0$, then we get

$$E = -\frac{1}{2} N(\alpha - 1) - NI - 2B(\sum_{i,j} u_{ij} - Cost) + \beta.Cost \tag{10}$$

It is obvious from (10) that the minimum cost means the minimum of E, in other words the optimal solution corresponds to the globally energy minimum. Hence the equation (10) judges the quality of the obtained output in terms of energy minimization. Moreover, by increasing B the energy function will be minimized. Note that B should be within a limit in order to meet the constraints imposed by the assignment problem

$$B < \frac{1 - \alpha - A - \beta - I}{2(N-1)} \tag{11}$$

For the sake of simplicity, we assume $\beta = -1$. We get

$$E = -\frac{1}{2}N(\alpha - 1) - NI - 2B(\sum_{i,j} u_{ij} - Cost) + Cost \qquad (12)$$

By using the values of A =-30, $\alpha = 2$, I=1, x (0) = 1 from [3], we have examined the behavior of the network for the sizes 4x4, 6x6 and 8x8 for 100 trials where the input was randomly chosen according to a uniform distribution on the interval between zero and one. Recall that B should be near to border of the limitation in (11). The results of the simulation are shown in the Table.I.
Fig.1 shows the CNN output of size (4x4) with the input given by (1). Observe that the obtained output is the optimal decision (3), which minimizes the cost function.
We introduce a quality of solution formula as follows

$$Q = 100 \times (1 - \frac{C_N - C_O}{C_O}) \qquad (13)$$

where C_O is the globally minimum cost, C_N is the global cost obtained by the CNN; If Q=100% then the network converged to an optimal solution.

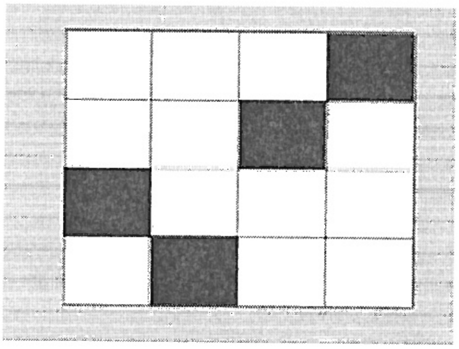

Fig.1. Output of the CNN of size 4x4. Black and white pixels correspond to +1 and 0 output saturation levels

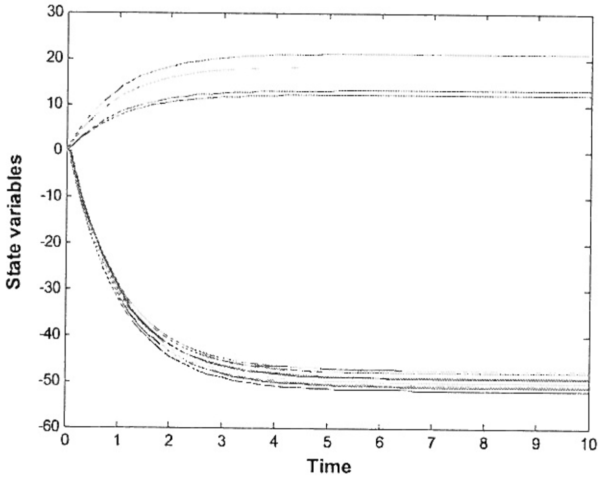

Fig.2. Transient motion of the CNN of size 4x4

Table I: Results of the Computer Simulation for $\beta = -1$

CNN Size	4x4	6x6	8x8
B	4.7	2.8	2
Convergence to an optimal solution	95%	78%	58%
Average quality of the non-optimal solutions	85.7%	79.4%	84%

b) Observe that the third term of (8) must reflect the objective of the given task. In other words, it must represent the minimization of cost function and as a result the minimization of the energy function. So we introduce the following condition

$$B > \frac{\beta}{2} \qquad (14)$$

We assume $\beta = 1$. Parameter B should be near the border of limitation of (11) so that the energy function will be minimized. We have carried out 100 trials of the same random input in the previous simulation. Meanwhile, the same values of A, α and I are valid for this part of simulation. Table.II shows the results for defined β and B. The network with an input given by (1) has the same output that depicted in Fig.1 and the transient motion is identical to Fig.2.

Table II: Results of the computer simulation for $\beta = 1$

CNN Size	4x4	6x6	8x8
B	4.4	2.8	1.8
Convergence to an optimal solution	94%	75%	54%
Average quality of the non-optimal solutions	84.5%	79.5	83.4%

The simulation results caused in the following notes

1. All the solutions are feasible, which meet with the required constraints.
2. There is an outrageously decrease in the rate of the convergence to an optimal solution when the size of the network increases. In fact, this can be considered as a drawback because of the size influence. The fall in the rate of the convergence to an optimal solution could be attributed to the increasing of the competitor cells as well as the possibility of having the same input value which renders the process cumbersome due to the increasing of the eligible solutions. Subsequently, this severe competition moves the solution far from the optimality. However, the rate of optimal solutions is promising in comparison with those obtained by neural networks.
3. The network with determined coefficients in a) has a better performance than b) particularly in terms of the convergence to the optimal solution when the size of the network increases.
4. The convergence time to a solution performs a potential feature of this approach to be integrated in high-speed systems. Following [3], normalized time constant $\tau = 1$ s has been considered for the transient motion in Fig.2.

The computation time for the CNN with an input of (1) is 0.4τ . Thus if we assumed $\tau = 1\,\mu s$, the decision would be obtained in 0.4 μs .

5. CNN is always stable and finds a locally optimum output in the steady state. However, the output may not be a globally optimum solution in terms of the generalized network energy because of the existence of multiple minima at which the energy is minimized locally [7].
6. The energy function represents the goal function of the optimization. Generally, when the energy function is minimized, the corresponding set of outputs represents the solution. The output state converges towards one of the minima of the computational energy.
7. It is difficult to find the optimal solution because of the inherent local minimum problem [8]. However, some modification for the coefficients, which determine the energy function, could be a promising method. This modification should be defined according to a theoretical basis rather than trial and error.
8. Due to the drawbacks of neural networks in scheduling applications in terms of the feasibility of the solutions and stability, there was a need to develop other models of neural networks for scheduling and combinatorial optimization problems. Because CNN is composed of analog elements, it is a high-speed real-time processing system and each cell can be represented by a separate analog processor. This offers a potential attribute, which could be profoundly exploited in the combinatorial optimization problems field. However, the CNN network still stagnant in comparison with other scheduling algorithms and further research is needed to shed light on its capabilities.
9. We have applied this approach to limited sizes of assignment problems because our objective was to present a new theoretical framework of CNN application to scheduling problem. Further research will be carried out in order to compare the performance of this approach with other algorithms in terms of quality of solutions and computational time in addition to the VLSI implementation.

IV. CONCLUSION

We have presented a Cellular Neural Network approach for assignment problem that minimizes the cost function. We have introduced the Lyapunov function of the Cellular Neural Networks as a function of optimization for combinatorial optimization. This function has become a crucial, decisive part for optimization.

Cellular Neural Networks can be depicted as an intelligent decision-making for tackling problems of this kind. Furthermore, we expect to see more future applications of these networks in high-speed computing systems.

V. REFERENCES

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