Digital Control of Permanent Magnet Synchronous Motors

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Abstract: Permanent Magnet Synchronous Motor (PMSM) variable-speed drive is widely used in the industry because of its particularly high mechanical power density, simplicity and cost effectiveness. Eliminating the mechanical sensor mounted on the shaft of the motors gives further improvement. These drives are referred to as "sensorless" electrical drives. In this paper a novel sensorless algorithm is proposed for the permanent magnet synchronous motor drive with β =180° pole-width rectangular field.

Keywords: Permanent Magnet Synchronous Motors, Digital Control, Sensorless Operation

1. Introduction

Most of the industrial electrical drive-systems are configured with closed-loop control systems to keep the speed on a satisfactory value, where the feedback signal comes from a mechanical speed or position encoder mounted on the shaft of the motor. These sensors have diversified constructions, but their general disadvantages are the following:

- the size of the drive-system is increasing,
- the cost of the drive-system is increasing,
- the reliability is decreasing.

There is an extreme way of eliminating the above disadvantages. This way absolutely passes over the unit inducing the problem (the speed or position encoder) and sets up a speed control system without any mechanical sensor. These drives are generally referred to as "sensorless electrical drives". In this paper we have investigated those drive-systems that are equipped with β =180° pole-width rectangular field permanent magnet synchronous machines.

The economic constraints and new standards legislated by governments place increasingly stringent requirements on electrical systems. New generations of equipment must have higher performance parameters such as better efficiency and reduced electromagnetic interference. System flexibility must be high enough to facilitate market modifications and to reduce development time. All these improvements must be achieved while at the same time decreasing the system cost.

Permanent magnet synchronous motor technology makes it possible to achieve these specifications. Such motors combine high reliability with high efficiency and a lower cost in comparison with other traditional motor types.

2. Permanent magnet synchronous motors

As with most motors the synchronous motor (SM) has two primary parts. The non-moving is called the stator and the moving — usually inside the stator — is called the rotor. Synchronous motors can be built in different structures.

To enable a motor to rotate two fluxes are needed. One from the stator and the other from the rotor. The energized stator windings create electromagnetic poles. By using the appropriate sequence to supply the stator phases a rotating field on the stator is created and maintained. The rotor is attracted by the energized stator phases. This action of the rotor — chasing after the electromagnetic poles of the stator — is the fundamental action used in synchronous motors. The lead between the rotor and the rotating field must be controlled to produce torque. This synchronization implies knowledge of the rotor position.

On the stator side three-phase motors are the most common. These offer a good compromise between precise control and the number of power electronic devices required to control the stator currents.

By the synchronous motor drives there are mainly two ways to generate the rotor flux. One uses rotor windings, fed via slip-rings from the stator, the other is made of permanent magnets and generates a constant flux itself.

As mentioned, to obtain its current supply and generate the rotor flux, a motor fitted out with rotor windings requires brushes. In this case the contacts are made of rings which do not have any commutator segment and referred to as slip-ring. The drawback of this structure that maintenance needs and both the reliability and the lifetime are decreased, while the costs are relatively high.

Replacing common rotor filed windings and pole structure with permanent magnets puts the motor into the category of brushless motors. It is possible to build permanent magnet synchronous motors (PMSM) with any even number of magnet poles. Motors have been constructed with two to fifty or more magnet poles. Greater number of poles create lower rated speed, and usually create a

greater torque for the same level of current. On the other hand — by adding more magnet poles — a point is reached where the torque no longer increases, because of the space needed between the magnet poles. The manufacturing cost also increases with the number of poles. As a consequence, the number of poles is a compromise between torque, cost and volume. A simplified structure of the permanent magnet synchronous motor can bee seen in Fig. 1.



Fig. 1. Simplified structure of the PMSM

The use of permanent magnets enables an efficient use of radial space and totally replaces the rotor windings, therefore suppressing the rotor copper losses.

Advanced rare-earth magnet materials such as Neodymium-Iron-Boron (NdFeB) and Samarium-Cobalt (SmCo) permit a considerable reduction in motor dimensions while maintaining high power density. In low and medium performance motors ferrite material can also be used as magnets. In the case of embedded systems where the space occupied is important, a permanent magnet synchronous motor is usually preferred to a conventional AC synchronous motor with brushes.

The spatial distribution of the rotor flux around the air-gap can be sinusoidal, or rectangular by the permanent magnet synchronous motors. The pole-voltage (also called back-EMF) is also sinusoidal with sinusoidal field motor and is less or more trapezoidal with the rectangular field motors. In this paper the rectangular filed permanent magnet synchronous motors are dealt with.

The spatial distribution of the rectangular flux density is drawn in Fig.2.(a). It is symmetrical $B(x_r) = -B(x_r + \pi)$ and its width β depends on the design. The β width is practically equal to the arc-length of the permanent magnet poles if the pole number of the motor is two.



Fig. 2. (a):Ideal spatial distribution of the flux density with rectangular field (b): Stator winding coil sides

Practically the width of the rectangular field of motors can be β =180°,150°, 120°, 90 and 60°. In this paper the machine with β =180° is examined, which are often referred to as Brushless DC (BLDC) motors. This is due to the following properties.

First let us consider that the permanent magnet machine is operating in generator mode. The rotor rotates with w angular speed whereafter separate pole voltages are induced in the three phase stator winding, which are called u_{pa} , u_{pb} and u_{pc} .

From the phase pole voltages the \overline{u}_p voltage Park-vector and its zero sequence component can be expressed:

$$\overline{\mathbf{u}}_{p} = (2/3) \left(\mathbf{u}_{pa} + \overline{a}\mathbf{u}_{pb} + \overline{a}^{2}\mathbf{u}_{pc} \right) =$$

$$(2/3) \left(\mathbf{K}_{a} + \overline{a}\mathbf{K}_{b} + \overline{a}^{2}\mathbf{K}_{c} \right) \cdot \mathbf{w} = \overline{\mathbf{K}} \cdot \mathbf{w}$$
(1.a)

$$u_{po} = (1/3)(u_{pa} + u_{pb} + u_{pc}) = (1/3)(K_a + K_b + K_c) \cdot w = K_o \cdot w$$
(1.b)

Where \overline{a} is the rotative unit-vector ($\overline{a} = e^{j2\pi/3}$). The \overline{K} vectorial and the K₀ zero sequence voltage factors are both depend on the α rotor position.

Now let us consider that the permanent magnet machine is operating in motor mode. Flowing i_a , i_b and i_c currents in the stator phase windings, the magnetic field generated by the rotor magnets develop m_a , m_b and m_c torques by the a, b and c phases. The resultant torque is the sum of the phase torques:

$$m = m_a + m_b + m_c = K_a i_a + K_b i_b + K_c i_c$$
 (2)

This torque can also be expressed by Park-vectors:

$$\mathbf{m} = (3/2) \cdot \overline{\mathbf{K}} \cdot \overline{\mathbf{i}} + 3 \cdot \mathbf{K}_{0} \cdot \mathbf{i}_{0}$$
(3)

Where $\overline{i} = (2/3)(i_a + \overline{a}i_b + \overline{a}^2i_c)$ is the current Park-vector, $i_o = (1/3)(i_a + i_b + i_c)$ is its zero-sequence component. The \overline{K} vectorial and the K_o zero sequence torque factors are both dependent of the α rotor position, and $\overline{K} \cdot \overline{i}$ denotes scalar product.

It is known from [1] that the voltage factor of (1) and the torque factor of (2) are identical, so the \overline{K} can easily be referred to as machine-factor, and its dimension can be either Vs or Nm/A.

In most cases the stator windings of the motor are Y-connected, so the neutral point is isolated, no zero sequence current can flow, therefore $3 \cdot K_0 \cdot i_0 = 0$. Since zero sequence voltage can produce mechanical power only with zero sequence currents, u_{po} does not take part in the torque production. Hereby the equations can be rewritten that determine the operation of the rectangular field permanent magnet synchronous motor:

$$\overline{\mathbf{u}}_{\mathbf{p}} = \overline{\mathbf{K}} \cdot \mathbf{w} \tag{4.a}$$

$$\mathbf{m} = (3/2) \cdot \overline{\mathbf{K}} \cdot \overline{\mathbf{i}} \tag{4.b}$$

Investigating (4.a) and (4.b) can be seen, that these equations are very similar to the equations of the conventional DC motors, hence the denotations "Brushless DC (BLDC) Motor" or "Commutator-less DC Motor" are often used instead of the much longer denotation "Rectangular Field Permanent Magnet Synchronous Motor".

The $K_a,\,K_b$ and K_c phase machine factors as well as the \overline{K} vectorial machine factor can be seen in Fig.3.



Fig. 3. Characteristics of a β=180° width rectangular field machine,
(a) phase machine factors, (b) effective phase machine factors, (c) machine factor vector, (d) adjusted phase currents, (e) current vector

It can be seen from (4.a) that the phase machine factors are proportional to the u_{pa} , u_{pb} and u_{pc} phase pole voltages. As mentioned before u_{po} does not take part in the torque production, therefore the so called effective pole-voltages can be written as:

 $u_{pa}^{,} = u_{pa}^{,} - u_{po}^{,}$

 $u_{pb}^{,} = u_{pb}^{,} - u_{po}^{,}$

 $u_{pc}^{,} = u_{pc} - u_{po}$

These effective phase pole voltages express the same Park-vector as the concrete u_{pa} , u_{pb} and u_{pc} phase pole voltages. This is due to the fact that Park vectors eliminate zero sequence components of the three phase signals. Effective phase machine factors (K_a', K_b', K_c') can also be written, due to their proportionality to the phase pole voltages.

3. Adjusted operation of the β =180° motor

To get pulsation-free torque the adjustment of the field and the supply is necessary. Such form of three phase currents have to be found, that meets (4.b).

But investigating (2) an easier way can be found to determine the required current shape.

If such currents are applied to each stator windings that are maximal when its phase machine factor is maximal, and are zero when the phase machine factor is turning polarity, then (2) gives pulsation-free torque. It can be recognized that only two of the three phases are conducting at the same time, therefore the same current flows through both of them. The third phase is non-energized.

These currents can be expressed by Park-vector, which can be seen in Fig.3.e. This Park-vector has six discrete positions without a continuous path. Checking the Park vector of the machine factors, it can be found that the (4.b) scalar product of these two Park-vectors are constant giving pulsation-free torque.

It can be noticed from Fig. 3. e-d. that the required rotor position information is very rough, only six discrete sector boundaries have to be recognized.

4. Control strategy and sensorless operation

Rectangular field permanent magnet synchronous motor speed control requires three control layers to be performed. The innermost one determines the rotor position to commute correctly the stator currents according to as shown in Fig. 3.d. Once the rotor position is known, the magnitude of the stator current has to be generated and controlled. Assuming that the torque is proportional to the current flowing through the stator coils, the control of torque is equivalent to the control of the input current. The most outer control loop is the speed control loop.

Mostly mechanical angular position sensors provide the rotor position.

The well-known incremental encoders and Hall-effect sensors represent the most chosen solutions to obtaining the rotor position. Incremental encoders provide a very high angular resolution as well as an accurate derived speed feedback information in any speed range. They are ideally suited to a highly precise speed and (mostly) position control.

Nevertheless, sensors such as those based on Hall-effect are often preferred to incremental encoders when the rated speed range is not too low, even if they do not provide performances comparable to the incremental encoders' ones. The reasons for this are the rough position information required by the operation mentioned in the previous section and the cost effectiveness of these sensors.

This way of implementation can be seen in Fig. 4. The commutation information is obtained from the actual status information comes from the Hall-sensors, and the speed is calculated from the time passes between two successive Hall-sensor state change, see the "sector decoder" block in Fig. 4.



Fig. 4. Conventional control of a ractangular field permanent megnet synchronous motor drive



Fig. 5. Sensorless Control of a ractangular field permanent megnet synchronous motor drive

Even the cheap Hall-effect sensors can be removed and replaced with microcomputer software with no additional integrated circuits (operational amplifiers, comparators, FPGAs), resulting in a cost effective drive. This solution gives equivalent position information to the Hall-sensor outputs.

The "sector decoder" block of Fig. 5. has the additional feature to the "sector decoder" of fig. 4. that it performs the calculation of the commutation signals from the terminal voltages of the motor eliminating the hall-sensors and facilitating the sensorless operation.

The current regulation can be realized either analog with operational amplifiers or comparators, or can be implemented in software as proposed by Fig.5.



Fig. 6. Some important details magnified from Fig.3.

5. The algorithm of the sensorless operation

The important information of Fig 3. is redrawn in Fig. 6. for the further investigations.

Let us assume the rotor is rotating with a constant angular speed, and its angular position is such that phases "a" and "b" are conducting, phase "c" is nonenergized. In this case we are somewhere in the shaded region of Fig. 6. This means, that at the rightmost side of the shaded region a commutation from phase "b" to phase "c" has to be performed. The goal is to determine that point where this commutation becomes necessary.

First let us recall the *Millmann's Theorem* in mind which is known from the subject of Theoretical Electrotechnics and is a special case of the analysis method called Node Potentials.

Please consider Fig. 7. to recall Millamann's Theorem in mind.



Fig. 7. Help figure to recall Millmann's Theorem in mind

Millmann's Theorem says: the voltage between the two neutral points is as follows:

$$\overline{U}_{00'} = \frac{\overline{\overline{U}_1} + \overline{\overline{U}_2} + \overline{\overline{U}_3}}{\frac{1}{\overline{Z}_1} + \frac{1}{\overline{Z}_2} + \frac{1}{\overline{Z}_3}}$$
(5)

Now consider Fig. 8. that shows the voltage-relationship when we are in the shaded region of Fig. 6.



Fig. 8. Voltage-relationship of Fig.6.

 U_a , U_b and U_c are the voltages applied to the adherent stator phase winding. U'_{pa} , U'_{pb} and U'_{pc} are the effective pole voltages, U_{p0} is the zero-sequence component of the pole voltages, Z is the impedance of the stator coils. The neutral point labeled 0' is the neutral point of the motor, and the neutral point labeled 0 is the negative rail of the power electronic equipment. Hence the voltages applied to the stator phases can vary from zero to U_{DC} , which is the DC bus voltage of the power electronic equipment.

In this analysis all voltages are measured from negative rail of the power electronic device, for example U_{00} is equal the voltage of the neutral point of the motor (U_N). Applying the Millmann's Theorem, U_N can be written:

$$\overline{\mathbf{U}}_{\mathrm{N}} = \frac{\frac{\overline{\mathbf{U}}_{\mathrm{a}} - \overline{\mathbf{U}}_{\mathrm{p0}}}{\overline{Z}} + \frac{\overline{\mathbf{U}}_{\mathrm{b}} - \overline{\mathbf{U}}_{\mathrm{p0}} - \overline{\mathbf{U}}_{\mathrm{p0}}}{\overline{Z}} + \frac{\overline{\mathbf{U}}_{\mathrm{c}} - \overline{\mathbf{U}}_{\mathrm{pc}} - \overline{\mathbf{U}}_{\mathrm{p0}}}{\overline{Z}}}{\frac{1}{\overline{Z}} + \frac{1}{\overline{Z}} + \frac{1}{\overline{Z}}}$$

Some rearrangements can be made with the previous equation, and please note, that since the complex impedance is eliminated, instantaneous values of the voltages can be written:

$$u_{N} = \frac{(u_{a} - u'_{pa} - u_{p0}) + (u_{b} - u'_{pb} - u_{p0}) + (u_{c} - u'_{pc} - u_{p0})}{3}$$
$$u_{N} = \frac{u_{a} + u_{b} + u_{c}}{3} - \underbrace{\frac{u'_{pa} + u'_{pb} + u'_{pc}}{3}}_{=0} - \frac{3 u_{p0}}{3}$$

Since the u'_{pa} , u'_{pb} and u'_{pc} effective pole voltages do not contain zero sequence component, their sum is zero, therefore it can be written:

$$u_{\rm N} = \frac{1}{3} (u_{\rm a} + u_{\rm b} + u_{\rm c}) - u_{\rm p0}$$
(6)

Our goal is to determine that point where this commutation becomes necessary. This means, that at the rightmost side of the shaded region of Fig. 6. a commutation from phase "b" to phase "c" has to be performed. One possible method is to determine the zero-crossing point of the voltage of the non-energized coil (u_{pc}) in this case). After it crossed from positive to negative we have to wait a time determines 30° shift of angular rotation, and that time we have to force the commutation.

Please note the following considerations to determine the zero crossing point of u_{pc} . We have to know u_a , it is not a problem, it equals to the mean value of the voltage applied by the power electronic equipment, u_b equals to zero (this is used by applied control method when two phases are energized), and u_c is measurable. The voltage across the stator impedance of phase "c" is zero, because phase "c" is non-energized, no current flows through it.

From Fig. 6. it can be recognized, that u_{p0} also equals to zero in that instant when u_{pc} crosses the zero. Since $u_N=u_c-u_{pc}$ (see Fig. 8.), therefore in this instant u_N equals to the measured value of u_c .

Now we can rearrange Eqn. (6):

$$u_{N} = u_{c} = \frac{1}{3} (u_{a} + u_{b} + u_{c}) - u_{p0} = \frac{1}{3} (u_{a} + u_{c}) = \frac{1}{3} u_{a} + \frac{1}{3} u_{c}$$
$$\frac{2}{3} u_{c} = \frac{1}{3} u_{a}$$
$$\overline{U}_{a} = \overline{U}_{N} = \frac{1}{2} \overline{U}_{a}$$

Eqn. (7) says, that u_{pc} voltage is crossing the zero in that instant, when the measured voltage of the non-energized phase u_c is half of the applied voltage u_a .

(7)

Similar relationships exist with the non-shaded regions of Fig. 6.

Please remember that the speed is calculated from the time passes between two successive Hall-sensor state change in the conventional drive. In the sensorless algorithm Hall-sensor state changes are replaced with the zero crossing instant of the non-energized phases, speed is calculated from these signals, as well as the 30° shift in angular position which is required between the zero crossing and the commutation.

6. Reliability of the proposed sensorless algorithm

The transient behaviour of the system is very important, as this algorithm responds to changes of the load. Let us therefore assume that the motor slows down and see how the control reacts. The computed 30° shift time will be too short in comparison with actual shift time required. The early arrival of the commutation point will tend to accelerate the motor. There will be an opposite controller reaction if the motor accelerates, therefore a kind of natural robustness is added to the speed control loop.

7. Drawbacks of the proposed method

- The initial rotor position is not known therefore it is not known which two phases have to be energized at starting.
- There are no measurable pole voltages at low speed, so the zero crossing point of the non-energized phase is undeterminable. Therefore a special starting strategy is required.
- The required 30° shift time between the zero crossing and the commutation can be determined correctly only in steady state. The natural robustness mentioned in the previous section become a very ugly disadvantage, when the motor speed tries change according to for example the change of the speed reference signal or at startup.

8. Starting strategy

As mentioned in the previous section, there are no measurable pole voltages at low speed, so a special starting strategy is required.

The initial rotor position is not known, therefore at starting two predetermined phases are energized. This forces the rotor jumping into a known position determined by the energized coils. This brings the motor to a steady state blocked and short circuited state, the current is only limited by the phase resistance of the stator coils, therefore a very reliably operating current regulator or current limitation is needed.

After we have known the position of the rotor, limited voltage is applied to the stator windings, because at low speed the pole voltages are also low, so high currents can flow. The current vector is stepped in open loop control operation, and its stepping speed is increased slowly enough to be able to be followed by the stator. This forces the rotor accelerating. As the speed becomes faster, increasing voltage can be applied its terminals. Finally the system can be turned to closed-

loop control operation, when the rotor reaches a speed where the pole voltages are high enough to determine the zero crossing point of the non-energized phase.

This starting strategy also has a very ugly disadvantage. When the two predetermined phases are energized at staring, it can not be determined which direction the rotor jumps away. This can cause counter-movements to the desired direction at the starting instant.

9. Conclusion

The novel proposed method is easy to implement, and can be the basis of a costeffective, highly reliable synchronous motor drives. Easy construction permanent magnet synchronous motors, simple low cost microcomputers (mainly DSPs) and integrated power modules can build up low and medium power, medium performance electrical drives, which can be used with for example electrical vehicle (wheel-chairs for disabled people, electrical mopeds, golf-cars etc.).

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