

# Simulation, Dynamics and Control of Freeway Traffic

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*Abstract: This paper presents the Simulator of Intelligent Transportation Systems (SITS). The SITS is based on a microscopic simulation approach to reproduce real traffic conditions in an urban or non-urban network considering different types of vehicles, drivers and roads. A dynamical analysis of several traffic phenomena, applying a new modelling formalism based on the embedding of statistics and Fourier transform, is then addressed. The results of using classical system theory tools point out that it is possible to study traffic systems, taking advantage of the knowledge gathered with automatic control algorithms.*

*Keywords: simulation and modelling, traffic control, traffic dynamics, Intelligent transportation systems.*

## 1 Introduction

Nowadays we have a saturation of the transportation infrastructures due to the growing number of vehicles over the last five decades. This situation affects substantially our lives, while people needs, more and more, to move rapidly between different places. The results are traffic congestion, accidents, transportation delays and larger vehicle pollution emissions.

The difficulties concerned with this subject motivated the research community to center their attention in the area of ITS (Intelligent Transportation Systems). ITS applies advanced communication, information and electronics technology to solve transportation problems such as, traffic congestion, safety, transport efficiency and environmental conservation. Therefore, we can say that the purpose of ITS is to take advantage of the appropriate technologies to create “more intelligent” roads, vehicles and users [1][2][3].

Computer simulation has become a common tool in the evaluation and development of ITS. The advantages of this tool are obvious. The simulation models can satisfy a wide range of requirements, such as: evaluating alternative treatments, testing new designs, training personal and analyzing safety aspects [4][5].

Bearing these facts in mind, this paper is organized as follows. Section 2 describes briefly the microsimulation model SITS. Section 3 presents simulation results related with the dynamic behaviour and control of a traffic system. Finally, section 4 presents some conclusions and outlines the perspectives towards future research.

## 2 The SITS Simulation Package

SITS is a software tool based on a microscopic simulation approach, which reproduces real traffic conditions. The program provides a detailed modelling of the traffic network, distinguishing between different types of vehicles and drivers and considering a wide range of network geometries. SITS uses a flexible structure that allows the integration of simulation facilities for any of the ITS related areas. This new simulation package is an object-oriented implementation written in C++. The overall model structure is represented on Figure 1.

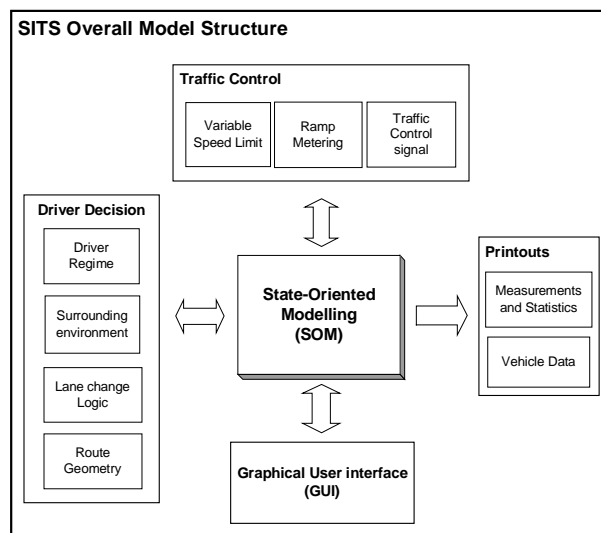


Figure 1 - SITS overall Model Structure

SITS models each vehicle as a separate entity in the network according to the state diagram showing in figure 2. Therefore, are defined five states {1-aceleration, 2-braking, 3-cruise speed, 4-stopped, 5-collision} that represent the possible vehicle states in a traffic systems.

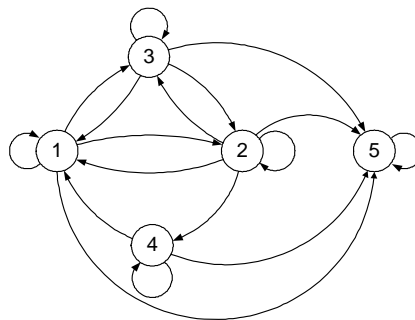


Figure 2 - SITS state diagram

In this modelling structure, so called State-Oriented Modelling (*SOM*) [6], every single vehicle in the network has one possible state for each sampling period. The transition between each state depends on the driver behaviour model and its surrounding environment. Some transitions are not possible; for instance, it is not possible to move from state #4 (stopped) to state #2 (braking), although it is possible to move from state #2 to state #4.

Included on the most important elements of SITS are the network components, travel demand, and driving decisions. Network components include the road network geometry, vehicles and the traffic control. To each driver is assigned a set of attributes that describe the drivers behaviour, including desired speed, and his profile (*e.g.*, from conservative to aggressive). Likewise, vehicles have their own specifications, including size and acceleration capabilities. Travel demand is simulated using origin destination matrices given as an input to the model.

At this stage of development the SITS considers different types of driver behaviour models, namely car following, free flow and lane changing logic. SITS considers each vehicle in the network to be in one of two driver regimes: free flow and car-following. The free flow regime prevails when there is either (*i*) no lead vehicle in front of the subject vehicle or (*ii*) the leading vehicle is sufficiently far ahead that it does not influence the subject vehicles behaviour. In the free flow case the driver travels at his desired maximum speed. Car-following regime dictates acceleration/deceleration decisions when a leading vehicle is near enough to the subject vehicle in order to maintain a safe following distance. Accelerations and decelerations are simulated using the Perception-Driver Model (*PDM*) [7]. According with the *PDM*, the driver decides to decelerate/accelerate depending on

two factors: the difference between the distance to the leading vehicle and the critical distance, and his active state.

The lane changing model in SITS uses a methodology that tries to mimic a driver behaviour when producing a lane change. This methodology was implemented in three steps: (i) decision to consider a lane change; (ii) selection of a desired lane; (iii) execution of the desired lane change if the gap distances are acceptable. A driver produces a lane change manoeuvre in order to increase speed, to overtake a slower vehicle or to avoid the lane connected to a ramp. After selecting a lane, the driver examines the lead  $g_b$  and lag  $g_a$  gaps in the target lane in order to determine if the desired change can be executed, as shown in Figure 3.

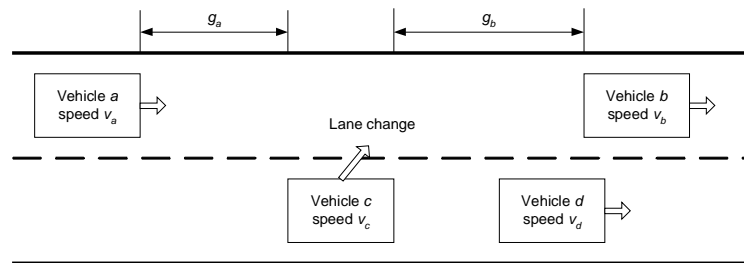


Figure 3 - Lead  $g_b$  and lag  $g_a$  gaps for a lane change manoeuvre of vehicle  $n$ .

If  $g_a$  and  $g_b$  are higher than the critical distances between vehicle  $a$  and  $c$ , and  $c$  and  $b$ , respectively, then the desired lane change is executed in a single simulation sampling interval  $\Delta t$ .

SITS allows also the analysis of signal control devices and different road geometries considering road junctions and access ramps. The simulation model adopted in the SITS is a stochastic one. Some of the processes include random variables such as, individual vehicle speed and input flow. These values are generated randomly according to a pre-defined amplitude interval.

The main types of input data to the simulator are the network description, the drivers and vehicles specifications and the traffic conditions. The output of SITS consists not only in a continuously animated graphical representation of the traffic network but also the data gathered by the detectors, originating different types of printouts.

SITS tracks the movements of individual vehicles to a resolution of  $10^{-2}$  sec and uses five different colours to represent the individual vehicle states; namely, stopped (red), acceleration (green), breaking (yellow), cruise speed (blue) and collision (black), as represented on figure 4.

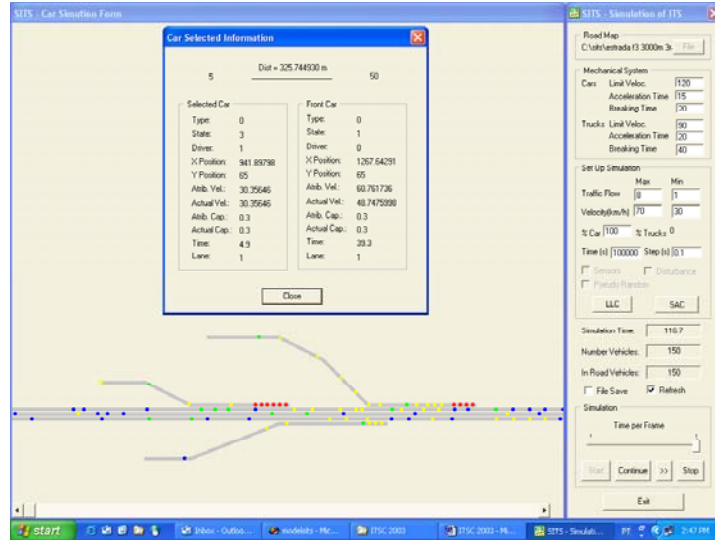


Figure 4 - SITS animated graphical representation

### 3 Dynamics and Control

#### 3.1 Dynamical Analysis

In the dynamical analysis are applied tools of systems theory. In this line of thought, a set of simulation experiments are developed in order to estimate the influence of the vehicle speed  $v(t,x)$ , the road length  $l$  and the number of lanes  $n_l$  in the traffic flow  $\phi(t,x)$  at time  $t$  and road coordinate  $x$ . For a road with  $n_l$  lanes the Transfer Function ( $TF$ ) between the flow measured by two sensors is calculated by the expression:

$$G_{r,k}(s; x_j, x_i) = \Phi_r(s; x_j) / \Phi_k(s; x_i) \quad (1)$$

where  $k, r = 1, 2, \dots, n_l$  define the lane number,  $x_i$  and  $x_j$  represent the road coordinates ( $0 \leq x_i \leq x_j \leq l$ ), respectively, and  $s$  is the Laplace variable. The Fourier transform for each traffic flow is:

$$\Phi_r(s; x_j) = \mathcal{F}\{\phi_r(t; x_j)\} \quad (2)$$

$$\Phi_k(s; x_i) = \mathcal{F}\{\phi_k(t; x_i)\} \quad (3)$$

Figure 5 show an overall schema of the notation adopted in the analysis of traffic dynamics.

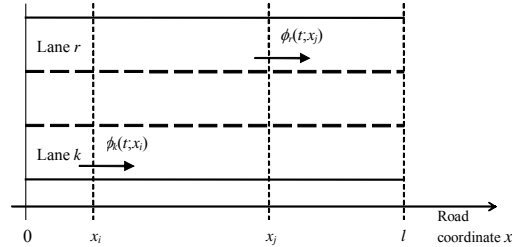


Figure 5 - Overall schema of the notation adopted

It should be noted that, traffic flow is a time variant system but, in the sequel, it is shown that the Fourier transform can be used to analyse the system dynamics.

The first group of experiments considers a one-lane road (*i.e.*,  $k = r = 1$ ) with length  $l = 1000$  m. Across the road are placed  $n_s$  sensors equally spaced. The first sensor is placed at the beginning of the road (*i.e.*, at  $x_i = 0$ ) and the last sensor at the end (*i.e.*, at  $x_j = l$ ). Therefore, we calculate the *TF* between two traffic flows at the beginning and the end of the road such that,  $\phi_1(t;0) \in [1, 8]$  vehicles  $s^{-1}$  for vehicle speed  $v_1(t;0) \in [30, 70]$   $km\ h^{-1}$ , that is, for  $v_1(t;0) \in [v_{av} - \Delta v, v_{av} + \Delta v]$ , where  $v_{av} = 50$   $km\ h^{-1}$  is the average vehicle speed and  $\Delta v = 20$   $km\ h^{-1}$  is the maximum speed variation. These values are generated according to a uniform probability distribution function.

The results obtained of the polar plot for the *TF*  $G_{1,1}(s;1000,0) = \Phi_1(s;1000)/\Phi_1(s;0)$  between the traffic flow at the beginning and end of the one-lane road is distinct from those usual in systems theory revealing a large variability, as shown in Figure 6a). Moreover, due to the stochastic nature of the phenomena involved different experiments using the same input range parameters result in different *TFs*.

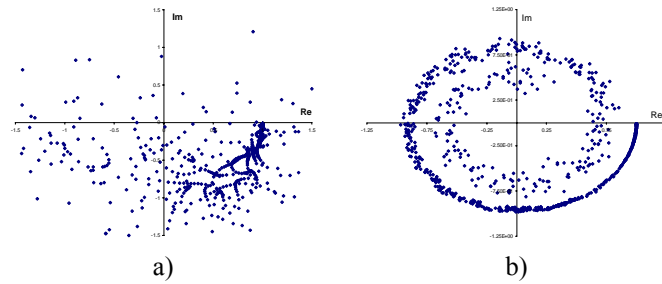


Figure 6 - Polar diagram of a) *TF* for  $n = 1$  experiment and b) *STF* for  $n = 2000$  experiments, with  $\phi_1(t;0) \in [1, 8]$  vehicles  $s^{-1}$  and  $v_1(t;0) \in [30, 70]$   $km\ h^{-1}$  ( $v_{av} = 50$   $km\ h^{-1}$ ,  $\Delta v = 20$   $km\ h^{-1}$ ,  $l = 1000$  m and  $n_l = 1$ ).

This phenomenon makes the analysis complex and experience demonstrates that efficient tools capable of rendering clear results are still lacking. Moreover, classical models are adapted to 'deterministic' tasks, and are not well adapted to the 'random' operation that occurs in systems with a non-structured and changing environment.

In order to overcome the problems, alternative concepts are required. Statistics is a mathematical tool well adapted to handle a large volume of data but not capable of dealing with time-dependent relations. Therefore, to overcome the limitations of statistics, it is adapted a new method, that takes advantage of the Fourier transform by embedding both tools [8].

In this line of thought, the first stage of the new modelling formalism starts by comprising a set of input variables that are free to change independently (*ivs*) and a set of output variables that depend on the previous ones (*ovs*). In a traffic system the *ivs* and *ovs* are defined as  $\phi_k(t; x_i)$  and  $\phi_k(t; x_j)$ , that is the traffic flows at positions  $x_i$  and  $x_j$ , respectively, at time  $t$  and for the  $k^{\text{th}}$  lane ( $k = 1, 2, 3 \dots$ ).

The second stage of the formalism consists on embedding the statistical analysis into the Fourier transform through the algorithm:

- i. A statistical sample is obtained by carrying out a large number ( $n$ ) of experiments having appropriate time/space evolutions. All the *ivs* and *ovs* are calculated and sampled in the time domain.
- ii. The Fourier transform is computed for each of the *ivs* and *ovs*.
- iii. Statistical indices are calculated for the Fourier spectra obtained in *ii*).
- iv. The values of the statistical indices calculated in *iii*) (for all the variables and for each frequency) are collected on a 'composite' frequency response entitled Statistical Transfer Function (*STF*) of each *TF*.

The previous procedure may be repeated for different numerical parameters (*e.g.*, traffic flow, vehicle speed, road geometry) and the partial conclusions integrated in a broader paradigm.

To illustrate the proposed modelling concept (*STF*), the simulation was repeated for a sample of  $n = 2000$  and it was observed the existence of a convergence of the *STF*,  $T_{1,1}(s; 1000, 0)$ , as show in Figure 6b), for a one-lane road with length  $l = 1000$  m  $\phi_1(t; 0) \in [1, 8]$  vehicles  $s^{-1}$  and  $v_1(t; 0) \in [30, 70]$  km  $h^{-1}$ .

The chart has characteristics similar to those of a low-pass filter with time delay, common in systems involving transport phenomena. Nevertheless, in our case we need to include the capability of adjusting the description to the continuous variation of the system working conditions. This requirement precludes the adoption of the usual integer-order low-pass filter and points out the need for a fractional-order *TF*. Therefore, in this case we consider a fractional-order system [9] with time delay:

$$T_{1,1}(s;1000,0) = \frac{k_B e^{-\tau s}}{\left(\frac{s}{p} + 1\right)^\alpha} \quad (4)$$

With this description we get not only a superior adjustment of the numerical data, impossible with the discrete steps in the case of integer-order  $TF$ , but also a mathematical tool more adapted to the dynamical phenomena involved. For fitting (4) with the numerical data it is established a two-step method based on the minimization of the quadratic error. In the first phase  $(k_B, p, \alpha)$  are obtained through error amplitude minimization of the Bode diagram. Once established  $(k_B, p, \alpha)$ , in a second phase,  $\tau$  is estimated through the error minimization in the Polar diagram.

For the numerical parameters of Figure 6a) we get  $k_B = 1.0$ ,  $\tau = 96.0$  sec,  $p = 0.07$  and  $\alpha = 1.5$ .

The parameters  $(\tau, p, \alpha)$  vary with the average speed  $v_{av}$  and its range of variation  $\Delta v$ , the road length  $l$  and the input vehicle flow  $\phi_1$ . For example, Figure 7 shows  $(\tau, p, \alpha)$  versus  $\Delta v$  (with  $v_{av} = 50$  km h<sup>-1</sup>) and  $v_{av}$ , (with  $\Delta v = 20$  km h<sup>-1</sup>).

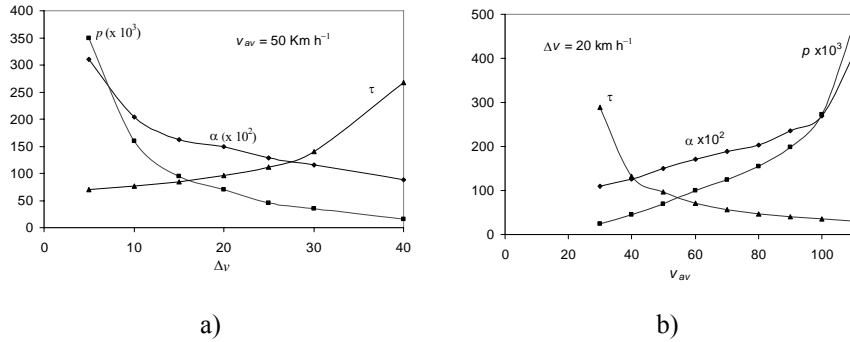


Figure 7 - Parameters  $\tau$ ,  $p$  and  $\alpha$  with  $n_l = 1$ ,  $l = 1000$  m and  $\phi_1(t;0) \in [1, 8]$  vehicles s<sup>-1</sup> versus a)  $\Delta v$  (with  $v_{av} = 50$  km h<sup>-1</sup>) and b)  $v_{av}$  (with  $\Delta v = 20$  km h<sup>-1</sup>).

It is interesting to note in Figure 7a) that  $(\tau, p) \rightarrow (\infty, 0)$ , when  $\Delta v \rightarrow v_{av}$ , and  $(\tau, p) \rightarrow (l v_{av}^{-1}, \infty)$ , when  $\Delta v \rightarrow 0$ . These results are consistent with our experience that suggests a pure transport delay  $T(s) \approx e^{-\tau s}$  ( $\tau = l v_{av}^{-1}$ ), when  $\Delta v \rightarrow 0$  and  $T(s) \approx 0$ , when  $\Delta v \rightarrow v_{av}$  (because of the existence of a blocking cars, with zero speed, on the road). In the case of Figure 7b) we have  $(\tau, p) \rightarrow (\infty, 0)$ , when  $v_{av} \rightarrow \Delta v$ , and  $(\tau, p) \rightarrow (0, \infty)$ , when  $v_{av} \rightarrow \infty$ , which has a similar intuitive interpretation.

In a second group of experiments are analyzed the characteristics of the  $STF$  matrix for roads with several lanes considering identical traffic conditions (*i.e.*,  $\phi_k(t;0) \in [1, 8]$  vehicles s<sup>-1</sup>,  $k = 1, 2$ ,  $l = 1000$ ,  $\Delta v = 20$  km h<sup>-1</sup>). Figure 8a) depicts



the amplitude Bode diagram of  $T_{1,1}(s;1000,0)$  and  $T_{1,2}(s;1000,0)$  for  $v_{av} = 50 \text{ km h}^{-1}$  (*i.e.*,  $v_k(t;0) \in [30, 70] \text{ km h}^{-1}$ ).

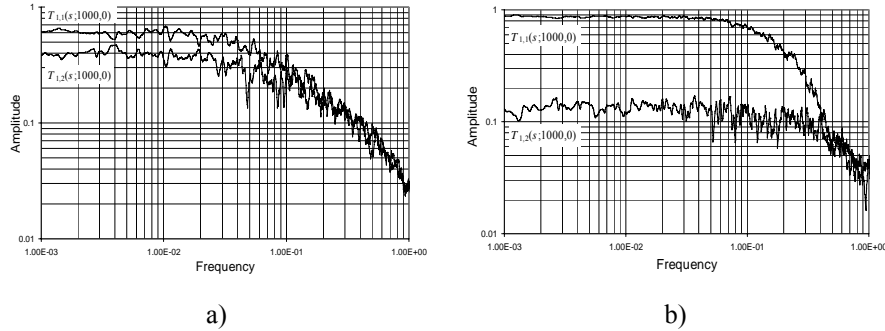


Figure 8 - Bode diagram of  $T_{r,k}(s;1000,0)$  for a)  $v_{av} = 50 \text{ km h}^{-1}$  and b)  $v_{av} = 90 \text{ km h}^{-1}$ ,  $n_l = 2$ ,  $l = 1000 \text{ m}$ ,  $\phi_k(t;0) \in [1, 8] \text{ vehicles s}^{-1}$ ,  $\Delta v = 20 \text{ km h}^{-1}$ ,  $k = 1,2$ .

We verify that  $T_{1,1}(s;1000,0) \approx T_{2,2}(s;1000,0)$  and  $T_{1,2}(s;1000,0) \approx T_{2,1}(s;1000,0)$ . This property occurs because SITS uses a lane change logic where, after the overtaking, the vehicle tries to return to the previous lane. Therefore, lanes 1 and 2 have the same characteristics leading to identical *STF*.

Figure 8b) presents the amplitude Bode diagram of  $T_{1,1}(s;1000,0)$  and  $T_{1,2}(s;1000,0)$  for  $v_{av} = 90 \text{ km h}^{-1}$  (*i.e.*,  $v_k(t;0) \in [70, 110] \text{ km h}^{-1}$ ).

Comparing Figure 8a) and these results, we conclude that the transfer matrix elements vary significantly with  $v_{av}$ . Moreover, the *STF* parameter dependence is similar to the one-lane case represented previously [10]. Fig. 9a) and 9b) show the variation of parameters ( $k_B$ ,  $p$ ,  $\alpha$ ) for  $T_{1,1}(s;1000,0)$  versus  $v_{av}$  (with  $\Delta v = 20 \text{ km h}^{-1}$ ) and  $\Delta v$  (with  $v_{av} = 50 \text{ km h}^{-1}$ ), respectively, for  $n_l = 2$ .

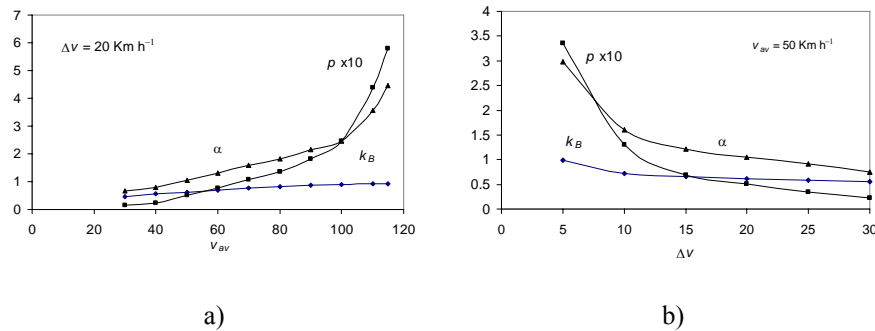


Figure 9 - Parameters ( $k_B$ ,  $p$ ,  $\alpha$ ) versus a)  $v_{av}$  and b)  $\Delta v$ , for  $T_{1,1}(s;1000,0)$  with  $n_l = 2$ ,  $l = 1000 \text{ m}$  and  $\phi_1(t;0) \in [1, 8] \text{ vehicles s}^{-1}$ .

We conclude that:

- i.* The time delay  $\tau$  is independent of the number of lanes  $n_l$ .
- ii.* For a fixed set of parameters we have for each *STF* gain  $\times$  bandwidth  $\approx$  constant.
- iii.* For each row of the transfer matrix, the sum of the *STF* gains is the unit.
- iv.* The gains and the poles of the diagonal elements of the *STF* matrix are similar. The gain of the non-diagonal elements, that represent dynamic coupling between the lanes, are lower (due to *iii*), but the corresponding pole are higher (due to *ii*).
- v.* The fractional order  $\alpha$  increases with  $v_{av}$ . Nevertheless, the higher the number of lanes the lower the low-pass filter effect, that is, the smaller the value of  $\alpha$ .

### 3.2 Traffic Control

Based on the previous dynamic description, we study a new traffic control concept. In this perspective, it is adopted a Variable Speed Limit Indicator (VSLI) to control the vehicle speed. The reference speed  $v_{ref}$  is displayed by the VSLI, at a given position  $x_{VSLI}$ , while a Feedback Sensor (FS) is placed at distance  $x_{FS}$  ahead of the VSLI. The reference speed is given by:

$$v_{ref} = \rho v_{max} + (1 - \rho) v_{average}, \quad 0 < \rho < 1 \quad (5)$$

where:

$v_{max}$  is the maximum speed allowed in the lane

$v_{average}$  is the average traffic speed of the FS readings

Therefore, in the perspective of control theory  $v_{ref}$  consists on the ‘reference signal’,  $v_{average}$  is the ‘feedback’ and equation (5) is the ‘controller’.

In the experiments we have  $x_{VSLI} = 200$  m,  $x_{FS} = 5$  m,  $l = 1000$  m,  $n_l = 1$ ,  $v_{max} = 100$  Km h<sup>-1</sup>,  $\rho = 0.5$  and the speed limit displayed by the VSLI is computed with a sampling interval  $T_s = 100$  sec.

The dynamics of the closed-loop system (*i.e.*, with controller (5)) differs somehow from the previous open-loop case as can be verified in Figure 10. An analytical expression for the *STF*, fitting closely the resulting data, requires a large number of poles and zeros. In order to easy the comparison of the open and closed loop dynamics, in the sequel it is adopted expression (4), since we can still get a reasonable curve fitting.

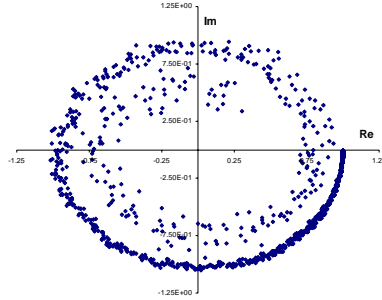


Figure 10 - Polar diagram of the STF  $T_{1,1}(s; 1000, 0)$  for a closed-loop case with  $\phi_1(t; 0) \in [1, 8]$  vehicles  $s^{-1}$  and  $v_1(t; 0) \in [30, 70]$  km  $h^{-1}$  ( $v_{av} = 50$  km  $h^{-1}$ ,  $\Delta v = 20$  km  $h^{-1}$ ,  $l = 1000$  m and  $n_l = 1$ ).

Figure 11 depicts the variation of the STF parameters ( $\tau$ ,  $p$ ,  $\alpha$ ) versus  $v_{av}$ , for the open and closed loop cases, with  $\Delta v = 20$  km  $h^{-1}$  and  $\phi_1(t; 0) \in [1, 8]$  vehicles  $s^{-1}$ .

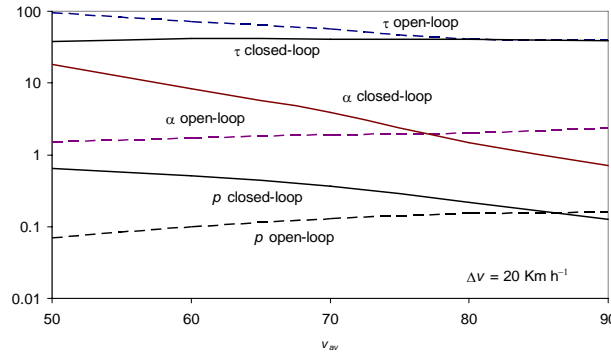


Figure 11 - Parameters  $\tau$ ,  $p$  and  $\alpha$  versus  $v_{av}$  for the open-loop and the closed-loop cases, with  $\Delta v = 20$  km  $h^{-1}$ ,  $n_l = 1$ ,  $l = 1000$  m and  $\phi_1(t; 0) \in [1, 8]$  vehicles  $s^{-1}$

It can be observed that reducing  $v_{av}$ , yields:

- i.* The closed-loop time delay  $\tau$  remains almost constant which is justified by the VSLI control effect.
- ii.* The closed-loop pole  $p$  increases corresponding to a larger bandwidth.
- iii.* The variation of closed-loop fractional order  $\alpha$  seems related with the elimination of noise associated with uncontrolled (*i.e.*, with large  $\Delta v$ ) traffic. Nevertheless, a clear understanding of the complete phenomena is still under research.

### 3.3 Entropy Analysis

A complementary perspective for the analysis of the traffic flow along the road can be quantified through the entropy  $H(x) = N^{-1} \sum_i f_i \ln(f_i)$ , where  $f_i = n_i/N$ ,  $N$  is the total number of vehicles used in the simulation and  $n_i$  is the absolute frequency. In our experiments was adopted  $N = 2048$  [11].

A set of simulations analyses the variation of  $H(x)$  for different ranges of vehicle speed. Figure 12 shows the results for an average vehicle speed of  $v_{av} = 50 \text{ km h}^{-1}$  in the ranges  $v_1(t;0) \in [20, 80]$ ,  $v_1(t;0) \in [30, 70]$ ,  $v_1(t;0) \in [40, 60]$ ,  $v_1(t;0) \in [45, 55]$  and  $v_1(t;0) \in [48, 52] \text{ km h}^{-1}$ . The entropy decreases along the road because the faster vehicles have to diminish their speeds to match the speed of the slower vehicles. Figure 12 also identifies two transients, namely the input and output transients.

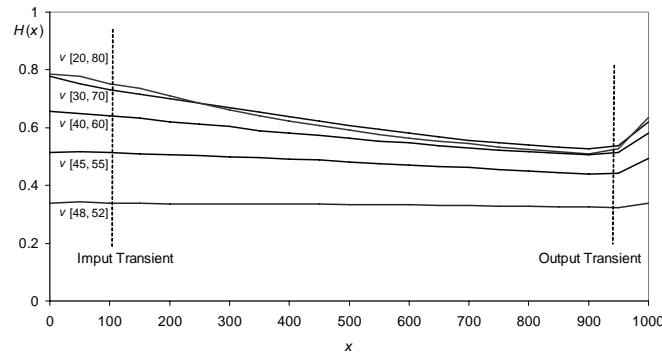


Figure 12 - Entropy  $H(x)$  of the traffic velocity vs. the position  $x$  for  $n_l = 1$ ,  $l = 1000 \text{ m}$ ,  $v_{av} = 50 \text{ km h}^{-1}$  and  $\phi_1(t;0) \in [1, 8] \text{ vehicles s}^{-1}$

It can be observed that, for small  $\Delta v$ ,  $H(x)$  remains almost constant and the transients are difficult to detect. This is justified by the fact that the vehicles have a small variation of speed, which originates a minimal interference among the vehicles. Also relevant are the rise of the output transient and the convergence of entropy, for larger  $\Delta v$ .

In Figure 13 is compared the variation of entropy  $H(x)$  for the open and closed loop cases. We conclude that we have a strong variation caused by the control effect of the VSLI placed at  $x = 200 \text{ m}$ . Therefore, for  $200 < x < 450$  we observe a transient, while for  $x > 450$  the value of  $H(x)$  remains almost constant that is, we have a steady-state regime.

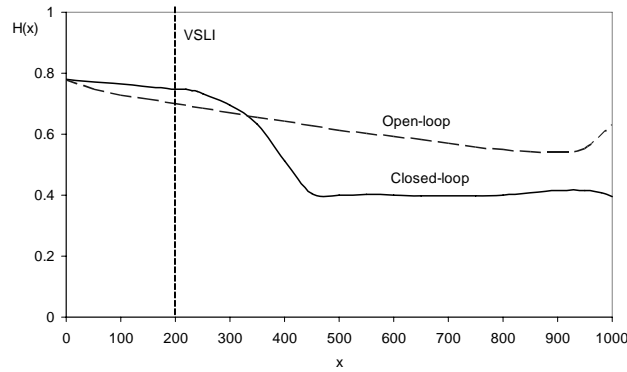


Figure 13 - Entropy  $H(x)$  of the traffic velocity vs. the position  $x$  for  $n_l = 1$ ,  $l = 1000$  m,  $v_1(t;0) \in [30, 70]$  km h<sup>-1</sup> with  $\phi_1(t;0) \in [1, 8]$  vehicles s<sup>-1</sup>

## Conclusions

In this paper it was described a software tool based on a microscopic simulation approach, to reproduce real traffic conditions in an urban or non-urban network. At this stage of development the SITS considers different types of driver behaviour model, namely car following, free flow and lane changing logic. Several experiments were carried out in order to analyse the dynamics of the traffic systems. In this perspective it was adopted a formalism based on the tools of systems theory. Moreover, the new dynamic description integrated the concepts of fractional calculus lead to a more natural treatment of the continuum of the  $TF$  parameters intrinsic in this system. Motivated by the dynamical analysis a new control algorithm was also developed and its performance was analysed. The results pointed out that it is possible to study traffic systems, including the knowledge gathered with automatic control algorithms.

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