Toward a Unified Theory of Uncertainty[1]

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Summary

In science, and especially within probability community, it is an almost universally held view that probability theory is the theory of uncertainty, and that no other theory of uncertainty is needed. To quote Professor Dennis Lindley, an eminent probabilist, "The only satisfactory description of uncertainty is probability. By this I mean that every uncertainty statement must be in the form of a probability; that several uncertainties must be combined using the rules of probability; and that the calculus of probabilities is adequate to handle all situations involving uncertainty…probability is the only sensible description of uncertainty and is adequate for all problems involving uncertainty. All other methods are inadequate…anything that can be done with fuzzy logic, belief functions, upper and lower probabilities, or any other alternative to probability can better be done with probability." (Lindley, 1987) A quick counterpoint. An important facet of uncertainty that is disregarded in Professor Lindley's comment is that, in many real-world settings, uncertainty is not probabilistic. For example, interval analysis deals with uncertainty, but no probabilities are involved.

In recent years, a variety of theories of uncertainty have been advanced, some of which are mentioned in Professor Lindley's statement. In the main, such theories are generalizations of standard probability theory rather than alternative approaches to uncertainty.

Fundamentally, uncertainty is an attribute of information. In general, information may be represented as a collection of so called generalized constraints—constraints which include probabilistic constraints as a special case. The concept of a generalized constraint is the centerpiece of a theory of uncertainty which is

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outlined in the following—the unified theory of uncertainty (UTU). In this theory, the point of departure is the concept of partiality—a concept which has a position of centrality in human cognition. Thus, in human cognition almost everything is partial, that is, is a matter of degree. For example, we have partial knowledge, partial understanding, partial truth, partial certainty, partial possibility, partial belief, partial causality, partial information, partial preference, partial independence and partial satisfaction. In the unified theory of uncertainty, there are three partialities that stand out in importance, (a) partiality of certainty (likelihood); (b) partiality of truth (verity); and (c) partiality of possibility. In the perspective of partiality, various theories of uncertainty may be viewed as dealing with specific mixtures of (a), (b) and (c). In the unified theory of uncertainty, all mixtures are allowed. In a sense, the three principal partialities act like the three primary colors: red, blue and green. Just as in the case of colors, the primary partialities are assumed to be perception-based. What should be underscored is that likelihood (probability), truth (verity) and possibility are distinct concepts, although they are points of tangency between them. It is of historical interest to note that Leibniz linked probability to possibility by defining probability as a grade of possibility (probabilitas is gradus: possibilitatis, 1660).

A concept which plays a pivotal role in the unified theory of uncertainty is that of a generalized constraint (Zadeh 1986). More specifically, if X is a variable, information about X is expressed as a generalized constraint of the form X isr R, where X is the constrained variable, R is the constraining relation and r is an indexing variable which defines the modality of the constraint, that is, the way in which R constrains X. The principal modalities are: r=blank (possibilistic); r=p (probabilistic); r=v (veristic); r=u (usuality); r=rs (random set); r=fg (fuzzy graph); and r=ps (Pawlak set). Constraints are governed by rules of constraint propagation. For example, the conjunctive rule is of the general form: if X isr R and Y iss S then (X,Y) ist T. As an illustration, in the Dempster-Shafer theory, the principal rule is: if X isp P and (X,Y) is R, then Y isrs Q. A more specific example of constraint propagation is the following. Assume that we start with a query: How old is Monika? The available perception-based information is: (a) Monika is about ten years older than Carol; and (b) Carol has two children; Robert, who is in mid-twenties, and Helen, who is in mid-thirties. In this example, the propositions “Robert is in mid-twenties” and “Helen is in mid-thirties,” translate into generalized constraints on the ages of Robert and Helen, respectively. Furthermore, an information item drawn from a world knowledge database is: childbearing age ranges from about sixteen to about forty two. Rules of constraint propagation applied to these propositions lead to a generalized constraint on Carol’s age, and then to a generalized constraint on Monika’s age. The calculus of generalized constraints is a basic part of the unified theory of uncertainty. Translation from propositions expressed in a natural language to generalized constraints is carried out through the use of what is referred to as Precisiated Natural Language (PNL).

A special form of generalized constraint which plays an important role in the unified theory of uncertainty is referred to as a bimodal distribution. More specifically, let X be a real-valued random variable. In a generic form, a bimodal distribution of X is a, collection of ordered pairs expressed as (P_i,V_i), i=1, …; n, where the V_i are granular values of X, and P_i is a granular value of probability of the event X is V_i. Calculus of bimodal distributions involves a generalization of the Dempster-Shafer theory of evidence and belief.

A special case of generalized constraint propagation is the generalized extension principle. A simple example is: if X is R and Y=f(X), then Y is f(R); where f is a function. Computation of f(R) reduces to a variational problem.

In the unified theory of uncertainty, standard probability theory is viewed as a theory of probabilistic constraints; standard logic is viewed as a theory of veristic constraints; and possibility theory is viewed as a theory of possibilistic constraints. In the unified theory of uncertainty, all modalities of constraints are allowed, but primary concern is focused on combinations of probabilistic, veristic and possibilistic constraints.

An important aspect of the unified theory of uncertainty relates to the way in which the basic concepts within the theory are defined. In existing theories of uncertainty, most or all concepts are bivalent. In the unified theory of uncertainty, bivalence is abandoned. As a consequence of abandonment of bivalence, concepts, as a rule, become a matter of degree. A basic concept which undergoes this transformation in the unified theory of uncertainty is that of independence. A basic mode of definition within the unified theory involves formulation of a definition in a natural language, followed by a precisiation of the definition through the use of PNL.

What is important to note is that the fact that standard probability theory, PT, deals only with probabilistic constraints, is a serious limitation of PT—a limitation which is widely unrecognized, as Professor Lindley’s comment shows. A consequence of this limitation is that PT is not equipped to operate on perception-based information expressed in a natural language.

In addition to the concept of a generalized constraint, there are two other concepts which play key roles in the unified theory of uncertainty. The first is that of granularity, meaning that information about values of variable is—or is allowed to be—granular, with a granule being a clump of values which are drawn together by indistinguishability, equivalence, similarity, proximity or functionality. For example, granular values of age may be labeled as young, old, not very young, middle-aged, etc. In general, a granule is characterized by a generalized constraint.

Another basic concept is that of a protoform, which is an abbreviation of prototypical form. Briefly, if A is an object, then a protoform of A, written as PR(A), is its abstracted summary—a summary which places in evidence the deep semantic structure of A. In the unified theory of uncertainty, deduction is protoform based, with a rule of deduction comprising a symbolic part and a computational part

Both in spirit and in substance, the unified theory of uncertainty represents a wide-ranging departure from existing theories. At this juncture, the theory is in its initial stages of development. I believe that the theory is a step in the right direction. What should be underscored is that its full development will be a challenging task.