Low Cost Fuzzy Control Solutions for Embedded Systems

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Abstract: The paper presents low cost fuzzy control solutions (of Takagi-Sugeno type) for embedded systems dedicated to a class of servo-systems. The considered controllers are dedicated to a class of second- and third-order integral-type plants, specific to the field of electrical drives for mobile robots, which can be characterized in their simplified linearized forms by standard models. For these models even conventional linear control structures give satisfaction. There is presented a development method for the fuzzy controllers, based on the fact that fuzzy controllers can be, in some certain conditions, well approximated by linear controllers and, so, the Extended Symmetrical Optimum (ESO) method and the Modified Structure of ESO (MESO) method are applicable in this situation. The fuzzy controllers and the corresponding development method are validated by an application example that can correspond to the speed control of a servo-system meant for a mobile robot.

Keywords: ESO and MESO method, digital implementation, AWR circuit, Takagi-Sugeno fuzzy controller, non-homogenous dynamics, servo-system, mobile robot.

1 Introduction

In control solutions for embedded systems there is a special place for typical control algorithms with or without dynamics that can ensure:

- good control performance for many situations;
- well established and accepted parameter computation methods;
- wide implementation possibilities;
- supplementary features needed by the control, overlapped on the basic control structure and control algorithm.

Due to these features more then 85% of control solutions are low cost automation (LCA) solutions and use typical control algorithms with dynamics [1]. Now, the embedded LCA solutions are implemented with micro-controllers or DSPs.

The ***low cost fuzzy control solutions*** (LCFCSs) for embedded systems suppose:

- the use of "low cost" by the user developed equipment, particularly embedded solutions;
- the use of some algorithms, their design being simple and it does not need advanced control engineering knowledge;
- the use of such control algorithms can be easily implemented with/on this devices.

This paper will present the following aspects:

- some implementation solutions for typical control algorithms (PI, PID), with and without supplementary functions;
- some implementation solutions of low LCFCSs or embedded systems, based on quasi-typical PD and PID control algorithms;
- a method of parameter computation of these controllers for a type of servo-systems which can be characterized by benchmark type models ;
- a systematic design methodology of these controllers for a class of servo-systems specific to mobile robot applications.

2 Continuously and Quasi-Continuously Operating PI (PID) Control Algorithms

Many control applications prefer structures with typical control algorithms with homogenous or non-homogenous information processing on the two input channels (the reference input and controlled output). Such structures have the general form given in Fig.2.1-a - some one with an adequate particularization of the modules, Fig.2.1-b and Fig.2.1-c – and ensure particular control lows regarding to the inputs. There can be established relations between such controllers and the 2-DOF controllers [2].

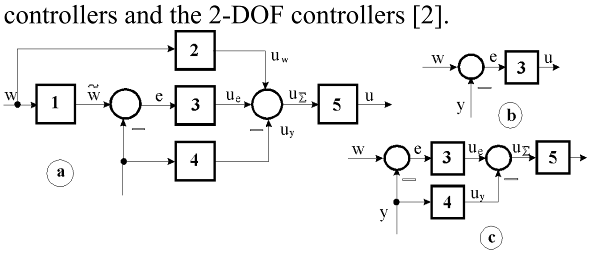

Fig.2.1. Typical controller structures and particularizations regarding the modules

The blocks (1) ... (5) can be described by the transfer functions (t.f.s) defined as follows:

$$u(\lambda) = H_5(\lambda)\cdot u_\Sigma(\lambda),\ \ u_\Sigma(\lambda) = u_w(\lambda) - u_y(\lambda) + u_e(\lambda), \quad (2.1)$$

$$u_e(\lambda) = H_3(\lambda)\cdot e(\lambda)\quad , \qquad e(\lambda)=H_1(\lambda)\cdot w(\lambda) - y(\lambda), \quad (2.2)$$

$$u_w(\lambda) = H_2(\lambda)\cdot y(\lambda \quad , \qquad u_y(\lambda) = H_4(\lambda)\cdot y(\lambda),$$

having: λ=s - in continuous time and λ=z - in discrete time.

In the presence of an integral (I) component and a limitation block in the controller structure, Fig.2.2 (a), the use of the AWR measure (Anti-Windup-Reset) will be recommended. A classical structure for introducing the AWR measure on a controller structure with integral component is presented in Fig.2.2. The AWR measure can be globally implemented with respect to controller output or locally with respect to the I component of the controller.

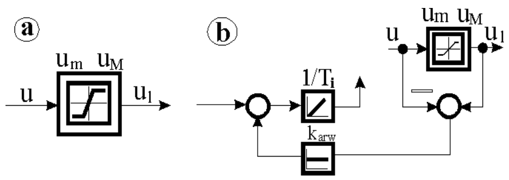

Fig.2.2 Classical structure for introducing ARW measure on the controller structure

The transfer function of the continuous PID controller is written often related to the design procedure, implementation or even discretization. Some well-known forms are given in the relations (2.3) – (2.5):

$$H_R(s) = \begin{cases} k_R[1+1/(sT_I)+k_dT_fs/(1+sT_f)], & (2.3) \\ k_r(1+sT_{r1})(1+sT_{r2})/s, & (2.4) \\ k_p + k_i/s + k_ds. & (2.5) \end{cases}$$

The relations between the coefficients of different explicit formulations, { k_R, T_i, T_d },…{ k_p, k_i, k_d }, can be established easily. Particularly, for a PI controller the following coefficients must be omitted:

- in (2.3) and (2.5): $k_d = 0$,
- in (2.4): $T_{r1} = T_r$ $\quad T_{r2} = 0$.

a. Digital implementation solution of a PID control algorithm in the quasi-continuous form. The classical algorithm

The implementation of a quasi-continuously (QC) operating PID digital control algorithm can be based on the informational diagram presented in Fig.2.3; its particularity consists in the appearance of a supplementary state variables x_k, associated to the I component and the adding of the AWR measure [3].

The *parameters values* $\{K_{pid}, K_i, K_d, K_{arw}\}$ depend on the parameters $\{k_p, k_i, k_d\}$ and $\{k_R, T_i, T_d\}$, respectively, and on the sampling time value, T_e ; to explicitly get the relations the PID control algorithm has to be equated to the digital PID control algorithm. The obtained relations are synthesized in Table 2.1.

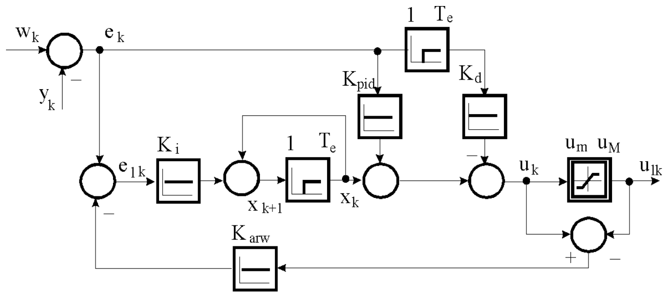

Fig.2.3 A quasi-continuous PID digital control algorithm implementation

Table 2.1. QC Parameters of a PID continuous algorithm

Controller type	Continuous t.f.	K_P	K_I	K_d
PI	$\frac{k_r(1+sT_r)}{s}$	$k_r(T_r - T_e/2)$	k_rT_e	-
PID	$\frac{k_r(1+sT_{r1})(1+sT_{r2})}{s(1+sT_e/2)}$	$k_r(T_{r1}+T_{r2}-T_e)$	k_rT_e	$k_rT_{r1}T_{r2}/T_e -$ $- \{k_r[2(T_{r1}+T_{r2})-T_e]\}/4$

The algorithm and the AWR measure can be implemented on the basis of the block diagram by means of a program that has the following structure written in pseudo-code:

Read w_k, y_k

Compute $e_k = w_k - y_k$

$u_k = x_k + K_{pid}\, e_k - K_d\, e_{k-1}$

if $u_k > u_M$ *then* $u_{lk} = u_M$

else if $u_k < u_m$ *then* $u_{lk} = u_m$

else $u_{lk} = u_k$

Transmit u_k

Re-initializations: $e_{lk} = e_k - (1/K_{pid})(u_k - u_{lk})$

$x_k = x_k + K_i\, e_{lk}$

$e_{k-1} = e_k$

End of program.

This principle is used also to implement control algorithms with non-homogenous information processing with respect to the two input channels including the corresponding AWR measure.

b. Non-homogenous information processing on two channels

The implementation of non-homogenous information processing has two basic requirements:

- an I or PI behavior with respect to the reference channel;
- a PI or PID behavior with respect to the feedback channel.

By omitting the reference filter with exclusively feed-forward character, Fig.2.1 the block 1, this ensures for the automatic control structure (CS):

- missing of a supplementary zero regarding the reference and having essential effect on the CS behavior;
- "conserved" behavior as that of the conventional control structure with respect to the disturbance input.

The non-homogenous information processing structure with respect to the two inputs is presented in Fig.2.4 and Table 2.2, where depending on the information processing type the blocks 1 ... 3 are of P, I or D type.

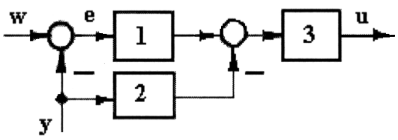

Fig.2.4. The implementation of non-homogenous information processing

Table 2.2. Transfer functions of blocks in Fig.2.4

Case	Channel	Block 1	Block 2	Block 3	Type
(1)	w	I: $(1/sT_i)$	----	P: (k_R)	I: $(1/sT_i)$
	y	I: $(1/sT_i)$	P: (1)	P: (k_R)	PI: $(1+1/sT_i)$
(2)	w	PI: $(1+1/sT_i)$	----	P: (k_R)	PI: $(1+1/sT_i)$
	y	PI: $(1+1/sT_i)$	D: (sT_d)	P: (k_R)	PID

The blocks 1 … 3 in Fig.2.4 are characterized by the transfer functions $H_1(s)$ … $H_3(s)$, respectively:

$$H_1(s) = 1 + \frac{1}{sT_i}, \quad H_2(s) = sT_d \quad , \; H_3(s) = k_R \, . \tag{2.6}$$

Accordingly, for example, in the case (2), the t.f.s with respect to the reference input, $H_{Rw}(s)$, and the t.f. function with respect to the controlled output, $H_{Ry}(s)$, can be computed as:

$$H_{Rw}(s) = \frac{k_r}{s}(1+sT_i) \quad , \; H_{Ry}(s) = \frac{k_r}{s}(1+sT_{r1})(1+sT_{r2}) \, , \tag{2.7}$$

with the connections between the coefficients of the transfer functions in (2.6) and (2.7), for $T_i \geq 4T_d$, expressed in (2.8):

$$k_r = k_R/T_i, \;\; T_{r1}, \, T_{r2} = 0.5[T_i \pm (T_i^2 - 4T_iT_d)^{1/2}] \, , \qquad T_{r2} < T_{r1} \, . \tag{2.8}$$

The quasi-continuous digital implementation is based on the relations (2.3) … (2.5) rewritten according to those specified in Table 2.2, as follows:

$$e(s) = w(s) - y(s), \tag{2.9}$$

case (1): $$u(s) = (k_i/s)e(s) - k_p y(s) \, ; \tag{2.10}$$

case (2): $$u(s) = (k_i/s)e(s) + k_p e(s) - k_d s y(s) \, . \tag{2.11}$$

Relatively simple computations permit:

- the equivalence between the coefficients in (2.6) and (2.8) and in (2.7) and (2.9), respectively;
- the building of the informational block diagram, with including the AWR module and other supplementary functions.
- the control algorithm and the AWR measure implementation, written in pseudo-cod.

3 Takagi-Sugeno Versions of Fuzzy Controllers and Development

a. Basic structures.

For obtaining the structure of the PID Takagi-Sugeno Fuzzy-Controller (TS-FC) it is useful to reorganize the controller structure in Fig.2.4. Accordingly, the Laplace transform of control signal, u(s), can be expressed as function of the Laplace transforms of control error, e(s), and controlled output, y(s), in terms of:

$$u(s) = H_R(s)\, e(s) - k_r T_i s T_d \, y(s) \, , \tag{3.1}$$

with $H_R(s)$ being the t.f. of the linear controller with respect to the control error:

$$H_R(s) = k_r T_I (1 + 1/sT_i) \, , \tag{3.2}$$

where the expression of T_i results in terms of the classical PID (2.8) in the form of:

$$T_i = T_{r2} + T_{r1} \, . \tag{3.3}$$

This controller can be replaced, for the sake of CS performance enhancement, by a PI-fuzzy controller. But, in this conditions, the always used compensation technique of the large time constant of the plant (denoted by T_1) by the large time constant of the controller (denoted by T_{r1}) , $T_{r1} = T_1$, will be no more fulfilled. For avoiding this, there is added a PD term ($1+sT_c$) to the transfer function (3.1) resulting in its modified form, $H_C^*(s)$, of PID type:

$$H_C^*(s) = k_r T_i \left(1 + \frac{1}{sT_i} + 1 + sT_c\right) = \frac{k_r}{s}(1+sT_{c1})(1+sT_{c2}) \, . \tag{3.4}$$

The connections between the coefficients in (3.4) are:

$$T_{c1} + T_{c2} = 2T_i, \qquad T_{c1}T_{c2} = T_i T_c \, . \tag{3.5}$$

By imposing for compensation of the large time constant of the plant, T_1, by the large time constant of the controller:

$$T_{c1} = T_1 \, , \tag{3.6}$$

and by using (3.3) and (3.5), the expression of the time constant of the PD term, T_c, will be:

$$T_{c1} = T_i(T_i+2T_c)/(T_i+T_c) \, . \tag{3.7}$$

The TS-FC structure results by using the fact that from (3.1) and (3.4) it results that:

$$H_R^*(s) = H_R(s) + k_c T_i (1+sT_c) \, . \tag{3.8}$$

The PI term in (3.8) will be replaced by a PI-FC and, for a further implementation, the PD term from (3.7) will be replaced by a lead-lag element. Then, by using the dependence (3.1) and by replacing the derivative element by a real derivative element, the TS-FC structure will be that from Fig.3.1, which highlights the non-homogenous information processing in the controller together with the hybrid information processing. The controller dynamics is introduced by differentiating the control error (e_k) and integrating the increment of control signal ($\Delta u_{1k} = u_{1k} - u_{1k-1}$). The considered TS-FC is a type III fuzzy system according to [5], and in the development phase the input membership functions are initially of regularly distributed triangular type with an overlap of 1 (e_k and $\Delta e_k = e_k - e_{k-1}$ – the increment of control error are the two inputs), Fig.3.2.

The FC uses the MAX and MIN operators in the inference engine, assisted by the rule base expressed by the decision table from Table 1, and employs the weighted average method for defuzzification [6]. However, it is the user's option to use different operations in the inference engine and different defuzzification methods (depending on the application involved); this will result in several versions of TS-FCs.

The strictly positive parameters of the TS-FC (Fig.3.2 and Table 3.1) will be determined in the sequel by means of the proposed development method: {k_c, T_i, T_d, T_{f1} and T_{f2}} for the linear part and {B_e, $B_{\Delta e}$, γ} for the FC block. The role of the parameter γ is to introduce additional nonlinearities that can be useful for CS performance enhancement especially when controlling complex plants, where the plant can be seen as a sub-system [7].

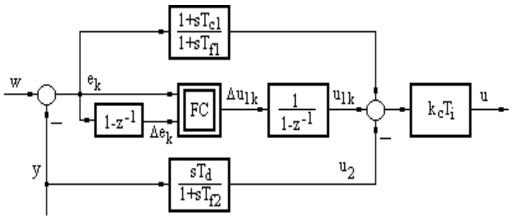

Fig.3.1. Structure of TS-FC

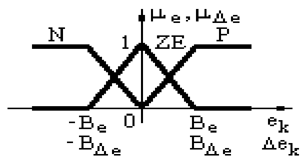

Fig.3.2. Accepted membership functions of input linguistic variables

Table 3.1. Decision table of FC

		e_k		
		N	ZE	P
Δe_k	P	Δu_{1k}	Δu_{1k}	$\gamma \cdot \Delta u_{1k}$
	ZE	Δu_{1k}	Δu_{1k}	Δu_{1k}
	N	$\gamma \cdot \Delta u_{1k}$	Δu_{1k}	Δu_{1k}

The fuzzy control system structure is presented in Fig.3.3, where the conventional controller is replaced by the TS-FC.

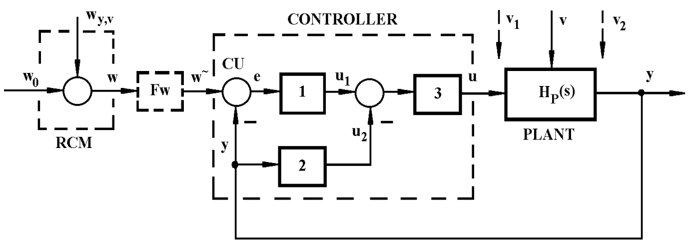

Fig.3.3. Structure of conventional control system

For the development of M-PI-FC and TS-PI-FC it is firstly necessary to discretize the continuous linear PI controller with the transfer function (3.9):

$$H_{PI}(s) = 1 + \frac{1}{sT_i} \; . \qquad (3.9)$$

The use of Tustin's discretization method leads to the incremental version of the quasi-continuous digital PI controller:

$$\Delta u_{Ik}=K_P\Delta e_k + K_I e_k\ ,\ K_P=1- T_e/(2T_i)\ ,\ K_I= T_e/T_i\ , \qquad (3.10)$$

where T_e represents the sampling period, and k stands for the current sampling interval. For ensuring the quasi-PI behavior of the FC block in the conditions of the accepted rule bases, there is applied the modal equivalences principle [8], resulting in:

$$B_{\Delta e} = (K_I/K_P)B_e\ . \qquad (3.11)$$

Therefore, there are two degrees of freedom in the development of the FC block, represented by the strictly positive parameters B_e and γ.

By using all aspects presented before, the proposed development method for the TS-FC consists of the following development steps to be proceeded:

(1) The development of the classical (linear) controller, PI or PID in its homogenous or non-homogenous form, for example regarding to the application, using the ESO [9], MESO [10] or an adequate algebraic method for the development of a 2-DOF controller;

(2) The development of the TS FC in its non-homogenous form.

4 Application to servo-system control

a. Linear controller development strategy

For many servo-system applications used in mobile robots [10], the plant (Fig.3.3) can be characterized by a t.f. of the form (4.1):

$$H_P(s) = \frac{k_P}{s(1+sT_1)(1+sT_\Sigma)}\ , \qquad (4.1)$$

According to the MESO method [7], by imposing the optimization conditions given by the ESO method [9], the closed-loop transfer function with respect to the reference input results as:

$$H_w(s)_{opt} = \frac{1}{1+\beta T_\Sigma s+ \beta^{3/2}T_\Sigma^2 s^2+ \beta^{3/2}T_\Sigma^3 s^3}\ , \qquad (4.2)$$

and the tuning parameters are:

$$k_c = 1/(\beta^{3/2}T_\Sigma^2 k_P)\ ,\quad T_{r2} = \beta T_\Sigma\ ,\quad T_{r1} = T_1\ . \qquad (4.3)$$

where β ($4 < \beta \le 20$, recommended) is the design parameter with the value in accordance with different goals to be reached.

The control system performance indices (σ_1, φ_r , $\hat{t_1}=t_1/T_\Sigma$ and $\hat{t_s}=t_s/T_\Sigma$ in normalized form) can be illustrated by the connections as function of β in Fig.4.1. With respect to the classical structure with controllers tuned by the ESO method, the tuning by the MS-ESO method proves to have the advantages illustrated in Fig.4.1. On the other hand, by comparing the ESO and MS-ESO methods with other analytical tuning methods, it can be seen that the ESO and MS-ESO offer at least more complete results.

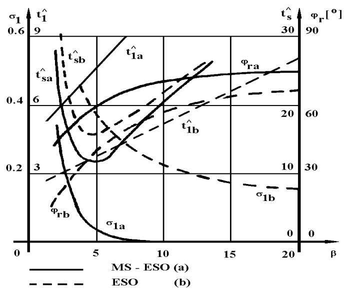

Fig.4.1. Control system performance indices versus β

b. TS-FC controller development strategy

In the case of using TS-FC, the compensation problem (3.6) is not actually because the controller becomes nonlinear and T_{r1} must be variable depending on the operating point. This is the reason why there is proposed here a FC comprising a PI-fuzzy controller in parallel with a lead-lag module for the compensation of T_1. The TS-FC was chosen due to its property of blending several linear controllers depending on the regions of the input space resulting in bumpless transfer from one linear controller to another.

By using the presented aspects, the development method for the TS-FC consists of the following development steps to be proceeded

(a) express the simplified mathematical model of controlled plant in the form (4.1);

- ***Proceed the I development phase*** corresponding to the linear part of the TS-FC and comprising the steps (b) … (f):

(b) choose the value of the design parameter β as a compromise between the desired / imposed control system performance by using the diagrams (Fig.4.1);
(c) obtain the tuning parameters $\{k_r , T_{r1} , T_{r2}\}$ of the linear controller with non-homogenous information processing by applying the design relation (4.3);
(d) compute T_i by using (3.3) and T_d by using the following relation obtained from (2.8) and (3.5);
(e) apply (3.7) to compute the time constant T_c;
(f) choose the filtering time constants T_{f1} and T_{f2} ;

- ***Proceed the II development phase*** corresponding to the nonlinear part of the TS-FC (the FC block) and comprising the steps (g) and (h):

(g) choose the sampling period, T_e , discretize the linear PI controller (3.9) and compute the parameters $\{K_P, K_I\}$ of the resulted QC digital PI controller;
(h) choose the values of the parameters B_e and γ of the FC block, in accordance with the experience of an expert in control systems, to obtain the value of $B_{\Delta e}$.

5 Case Study

For the validation of the presented FC and of development method, there is considered a case study that correspond to the speed control of a servo-system characterized in its linearized version by the transfer function (4.1), in normalized form, with the parameters k_P=1, T_Σ=1 sec and T_1=5 sec.

Firstly, the linear part of the TS-FC is developed by proceeding the I phase of the development method. Therefore, there is chosen the design parameter β = 16, and from (4.3) there will result: k_r = 0.0156, T_{r1} = 16 sec, T_{r2} = 5 sec. There will be obtained as follows T_i = 21 sec, T_d = 3.8095 sec and T_{r1} = 8.8095 sec, respectively. Finally, the filtering time constants, T_{f1} = 0.3 sec and T_{f2} = 0.3 sec are chosen.

During the phase II is developed the FC block as part of the TS-FC. For the sampling period T_e = 0.02 sec, the parameters take the values K_P = 0.9995 and K_I = 0.00095. By choosing B_e= 0.3, results in $B_{\Delta e}$= 0.00028; the value of γ was chosen γ = 0.95, using the recommendation proposed in [11].

Part of the digital simulation results is presented in Fig.5.1 and Fig.5.2 for the CS with non-homogenous linear PID controller and for the CS with the developed TS-FC controller, respectively. The considered simulation scenario was: a unit step of reference input w(t), followed by a – 0.5 step modification of disturbance input v_2 (after 75 sec), with continuous line for y(t), and dotted line for u(t).

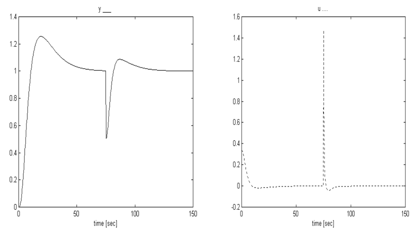

Fig.5.1. Controlled output and control signal versus time for CS with non-homogenous linear PID controller

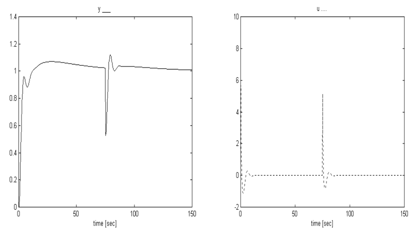

Fig.5.2. Controlled output and control signal versus time for CS with with TS-FC controller

5 Conclusions

The paper presents low cost fuzzy control solutions expressed in terms of a TS-FC with non-homogenous dynamics with respect to the two input channels and its development method meant for controlling a class of second- and third-order systems with integral character (servo-system).

The proposed development method comprises useful development steps, and it is based on applying the tuning relations specific to the MS-ESO method to the original non-homogenous linear PID controller.

There was developed a TS-FC for a case study, and the presented digital simulation results validate the presented FC and development method. The CS performance enhancement ensured by the proposed fuzzy controller proves that the TS-FC can successfully cope with control of plants in the case of servo-systems. CS performance can be further improved by using reference filters Fw.

The presented TS-FC is very simple and easy to be implemented on embedded systems and used as low cost automation solutions in control of complex plants [10], [13].

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