Correlated electrons in low dimensions

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Outline

- correlated electrons in quantum dots
- quantum Hall effect
- incompressible quantum liquids
- composite fermions
- nonabelian anyon statistics

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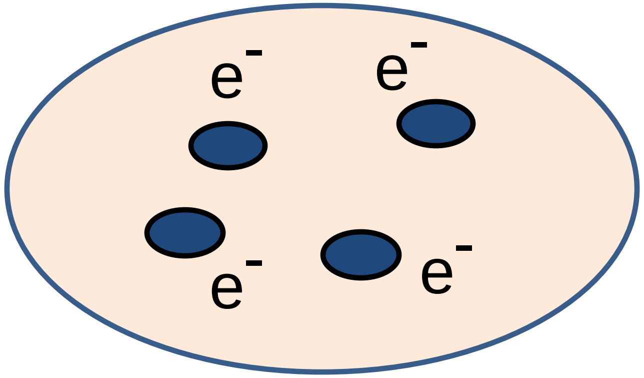
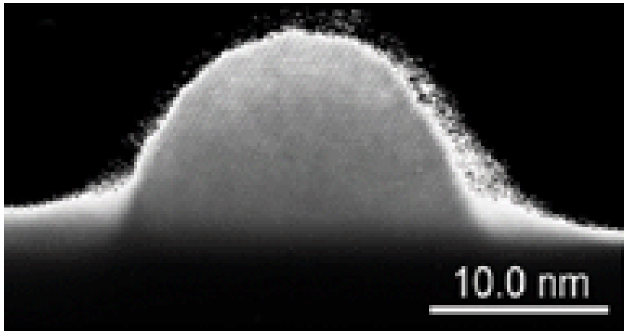
Quantum dots

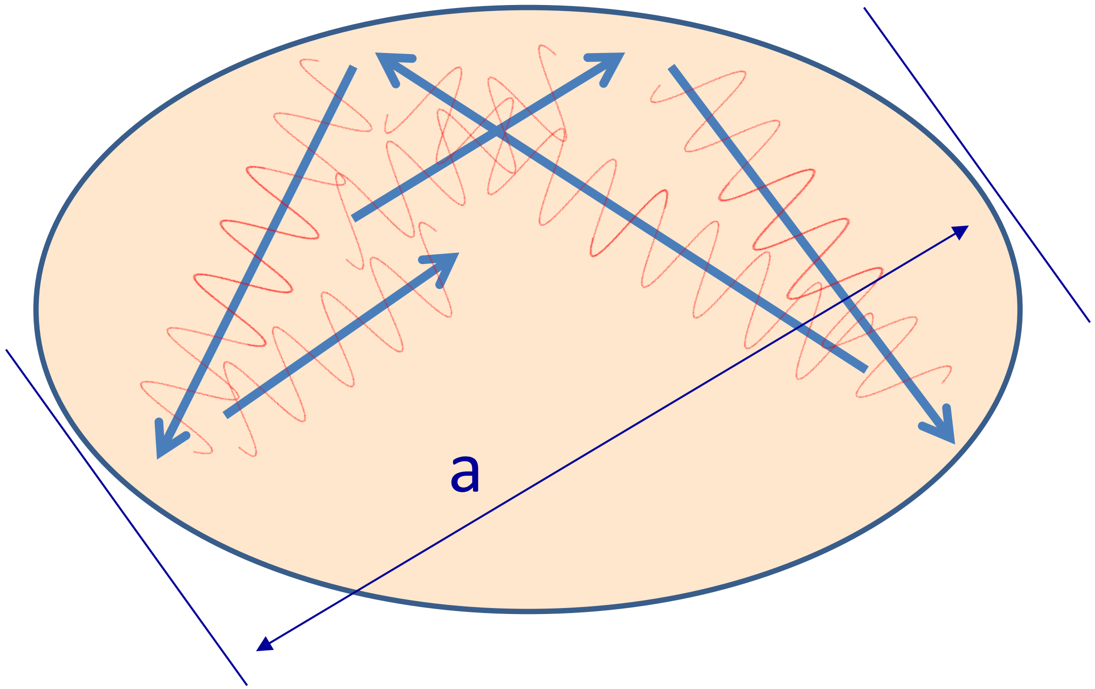
Nanoscale objects containing **small** and **controlled number of electrons** (also called „artificial atoms”)

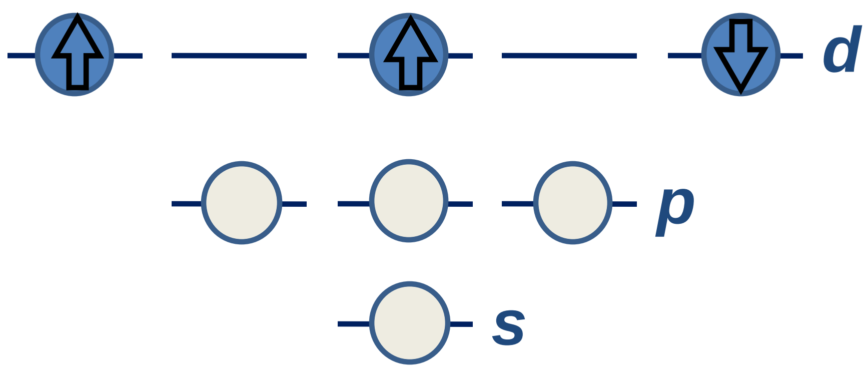




- often made of semiconductors
  (various methods)
- ∅ from few to 100 atoms
  (different shapes)
- N ≤ a few tens of electrons
- coupling to environment
  (electric, magnetic,optical, mechanic)

Quantum dots





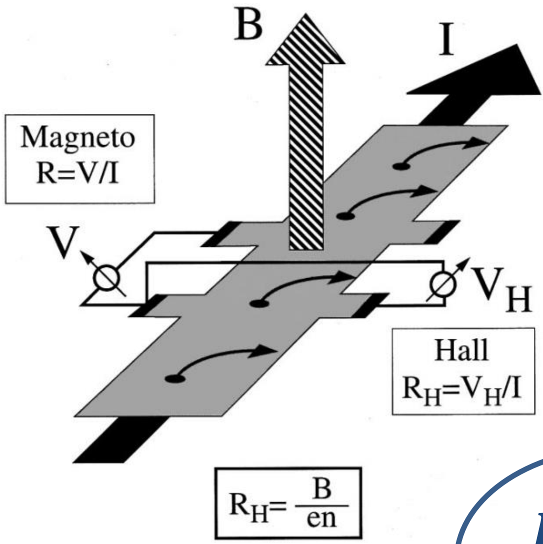
spatial confinement
& electron wave character

→ quantization
(discrete energy levels)

shape/symmetry
→ shell degeneracy
→ Hund rules (e.g. max L)

size, magnetic field,
light (electrons & holes)
→
strong & nontrivial
correlation effects

Hall effect



Ohm's law

$$\boldsymbol{j} = \sigma \boldsymbol{E}$$
$$\boldsymbol{E} = \varrho \boldsymbol{j}$$

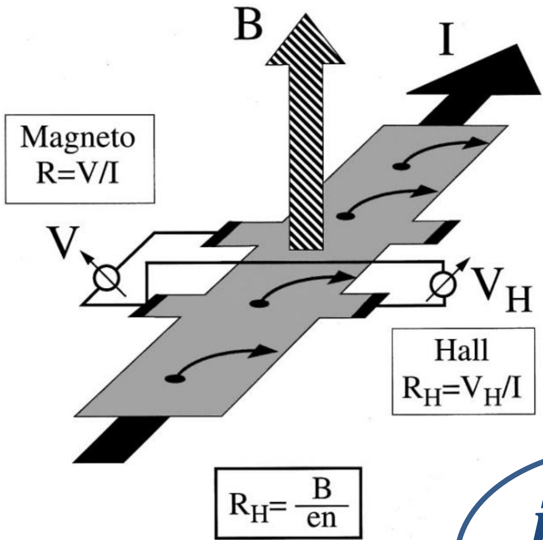
in magnetic field:
Hall effect (1879)

$$\boldsymbol{j} = \hat{\sigma} \boldsymbol{E}$$
$$\boldsymbol{E} = \hat{\varrho} \boldsymbol{j}$$

$$\begin{bmatrix} j_x \\ j_y \end{bmatrix} = \begin{bmatrix} \sigma & -\sigma_H \\ \sigma_H & \sigma \end{bmatrix} \begin{bmatrix} E_x \\ E_y \end{bmatrix}$$

$$\begin{bmatrix} E_x \\ E_y \end{bmatrix} = \begin{bmatrix} \varrho & \varrho_H \\ -\varrho_H & \varrho \end{bmatrix} \begin{bmatrix} j_x \\ j_y \end{bmatrix}$$

Quantum Hall effect

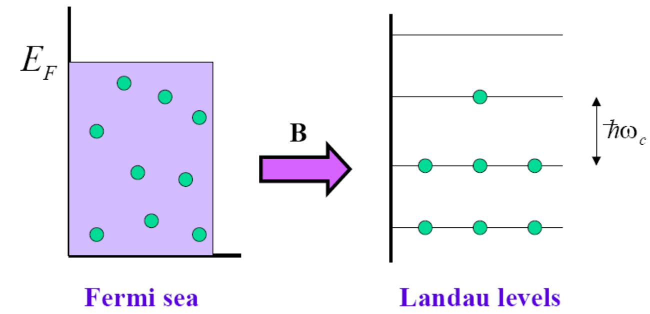


Ohm/Hall law

$$\boldsymbol{j} = \hat{\sigma}\boldsymbol{E}$$
$$\boldsymbol{E} = \hat{\varrho}\boldsymbol{j}$$

extreme cond's

- 2D
- low T
- high B
- low/approp. *n*
- weak disorder



cyclotron energy: $\hbar\omega_c = \hbar eB/mc$

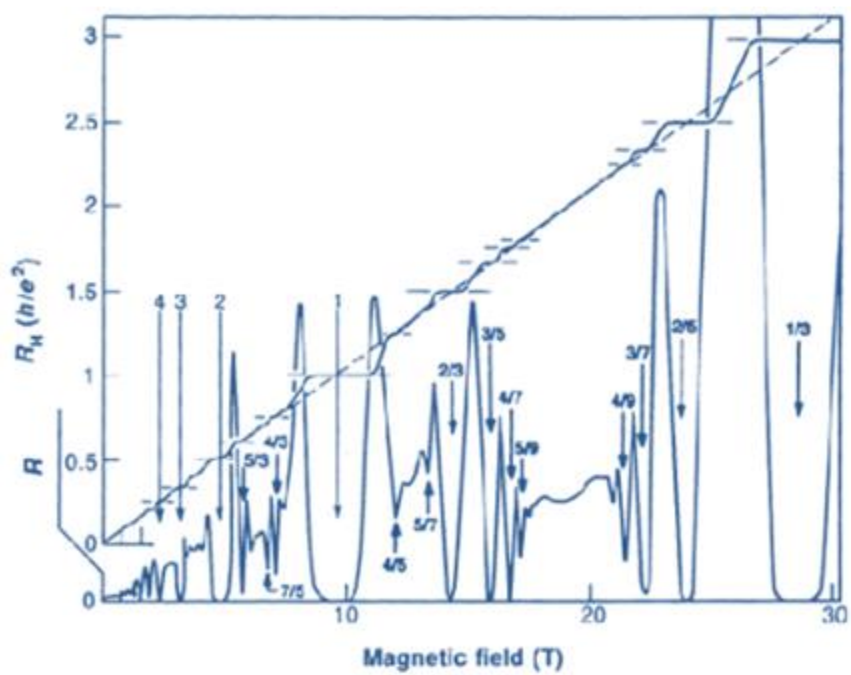
filling factor: $\nu = N/N_\phi = n/n_\phi$

Quantization:

$$\sigma_{xy} = \nu e^2/h\,;\ \sigma_{xx} = 0$$

$$\boldsymbol{\sigma} = \frac{\boldsymbol{\nu}}{\boldsymbol{R_{\mathrm{K}}}}\begin{bmatrix} \mathbf{0} & \mathbf{-1} \\ \mathbf{1} & \mathbf{0} \end{bmatrix};\ \boldsymbol{\varrho} = \frac{\boldsymbol{R_K}}{\boldsymbol{\nu}}\begin{bmatrix} \mathbf{0} & \mathbf{1} \\ \mathbf{-1} & \mathbf{0} \end{bmatrix}$$

$$R_{\mathrm{K}} \equiv \frac{h}{e^2} = 25812.807557(18)\ \Omega$$

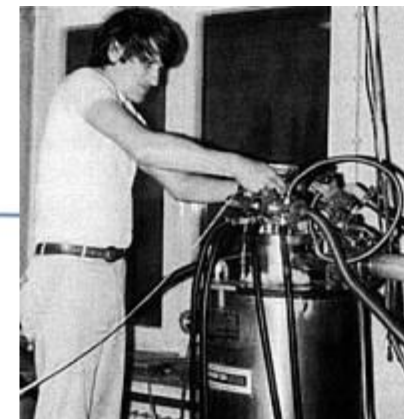


Quantum Hall effect

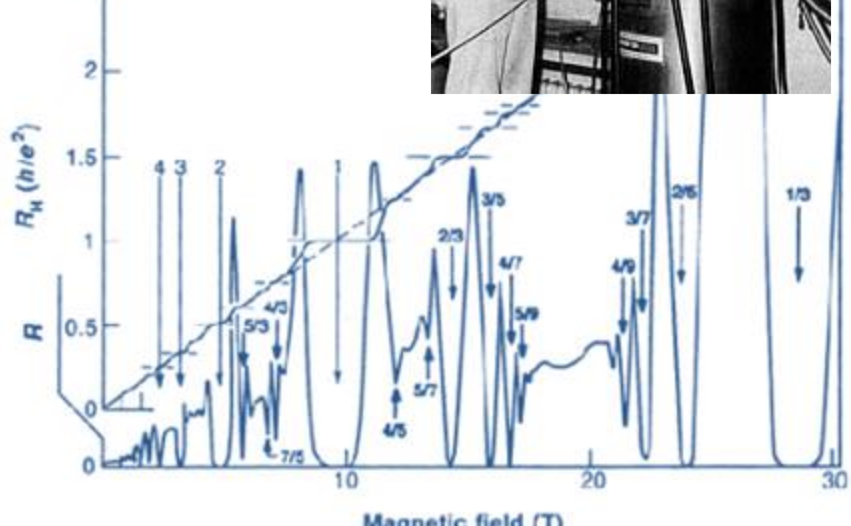
Klaus von Klitzing

High Magnetic Field Laboratory, Grenoble (France)
5 February 1980, 2am

– topological phase of matter



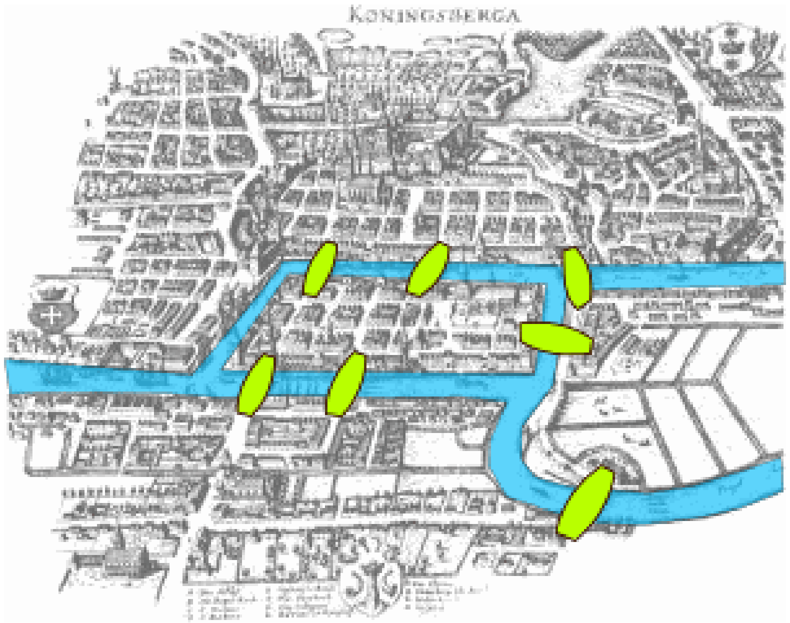
other/related topological states :
- topological insulators (HgTe, Bi_2Se_3)
- topological superconductors (Sr_2RuO_4)



$$\boldsymbol{\sigma} = \frac{\boldsymbol{\upsilon}}{\boldsymbol{R_K}}\begin{bmatrix} \mathbf{0} & \mathbf{-1} \\ \mathbf{1} & \mathbf{0} \end{bmatrix};\ \boldsymbol{\varrho} = \frac{\boldsymbol{R_K}}{\boldsymbol{\upsilon}}\begin{bmatrix} \mathbf{0} & \mathbf{1} \\ \mathbf{-1} & \mathbf{0} \end{bmatrix}$$

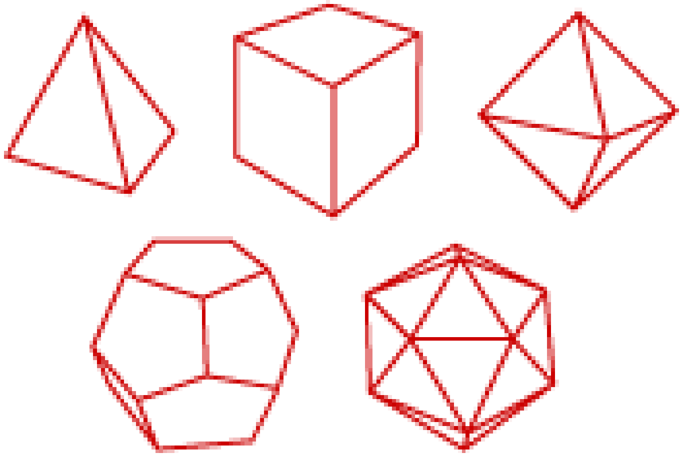
$$R_K \equiv \frac{h}{e^2} = 25812.807557(18)\ \Omega$$

Topology *branch of mathematics concerned with properties of figures unchanged under continuous deformation*



7 bridges of Königsberg
(Leonhard Euler, 1735)

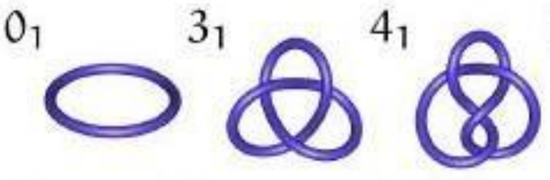
Can one ***cross each bridge exacty once*** *and return to origin?*



$$v - e + f = 2$$

Euler's polyhedron formula
v, *e*, *f* = numbers of vertices, edges, and faces

Knots (Peter Guthrie Tait, 1885)



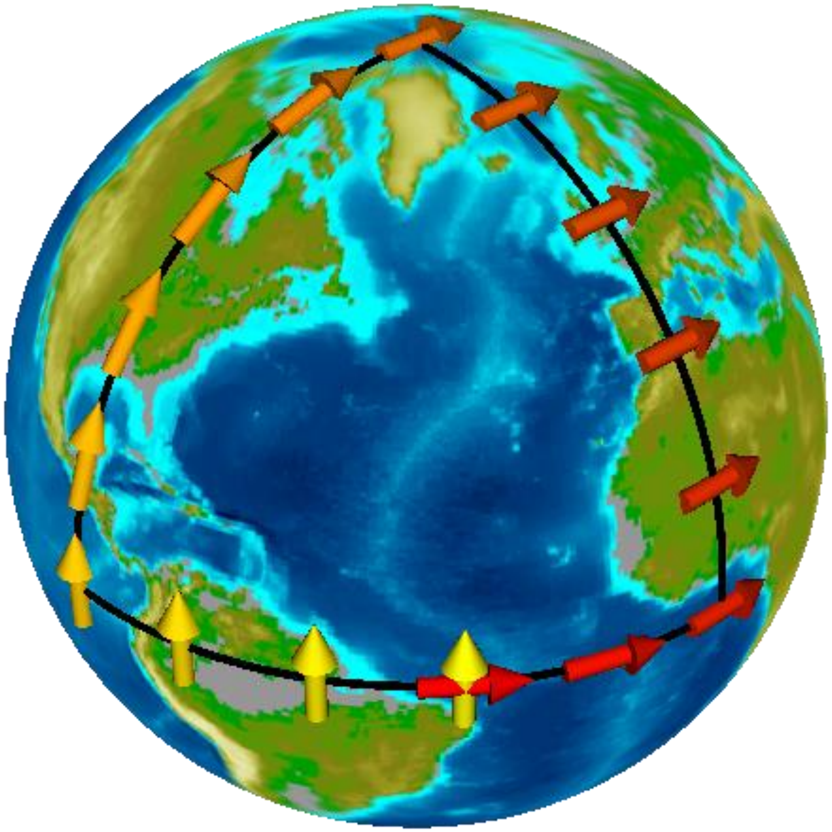
Quantum Hall effect as topological effect

Quantization of quantum Hall effect is <u>topological</u>

Phase transition <u>not</u> described by spontaneous symmetry breaking

In contrast, at low temperature (ground state) <u>symmetry is higher</u>

Order parameter: <u>non-local</u>, <u>Chern number</u> (<u>topological invariant</u>) ↔ <u>geometry/curvature</u> of Hilbert space



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Wave vector $\boldsymbol{k}$ in closed loop →
Berry phase of Bloch w-fun $u_m(\boldsymbol{k})$:
$\oint A_m \, ds_k; \; A_m \equiv i \langle u_m | \nabla_k | u_m \rangle$

Berry curvature: $\mathcal{F}_m = \nabla_k \times A_m$

Stokes theorem:
$\oint A_m \; ds_k = \iint \mathcal{F}_m \, d^2 k$

Chern number (~total curvature):
$n_m = (2\pi)^{-1} \iint_{occ} \mathcal{F}_m \, d^2 k$

TKNN (Thouless, Kohmoto, Nightingale, den Nijs) 1982:
$\sigma_{xy} = \boldsymbol{n} e^2 / h$ (over occup. LLs)

Gauss & Bonnet (1848):
$\iint K dA + \int k_g ds = 2\pi \, \chi_M$

For a closed surface:
$(2\pi)^{-1} \oiint K \, dA = 2\,(1 - g)$

For torus (2D Brillouin zone):
$g = 1 \;\; \Rightarrow \;\; n = 0$

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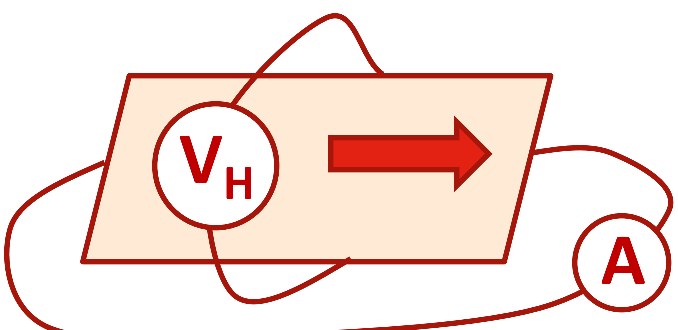
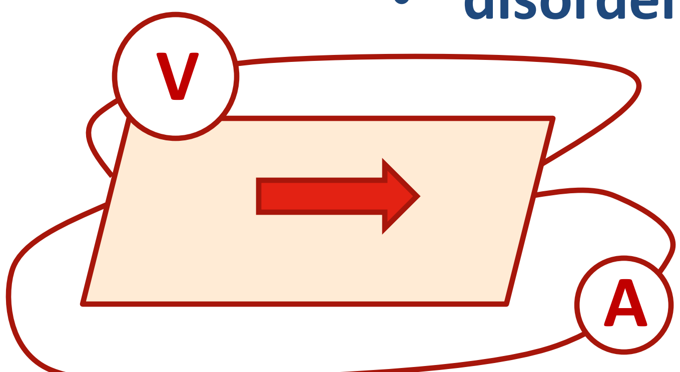
properties depend on topological invariant (Hall conductance σ_{xy} ~ Chern number)

Effect (exact quantization) is independent of:

- **material**
- **type of structure**
- **sample geometry**
- **disorder**
- **magnetic field**
- **temperature**





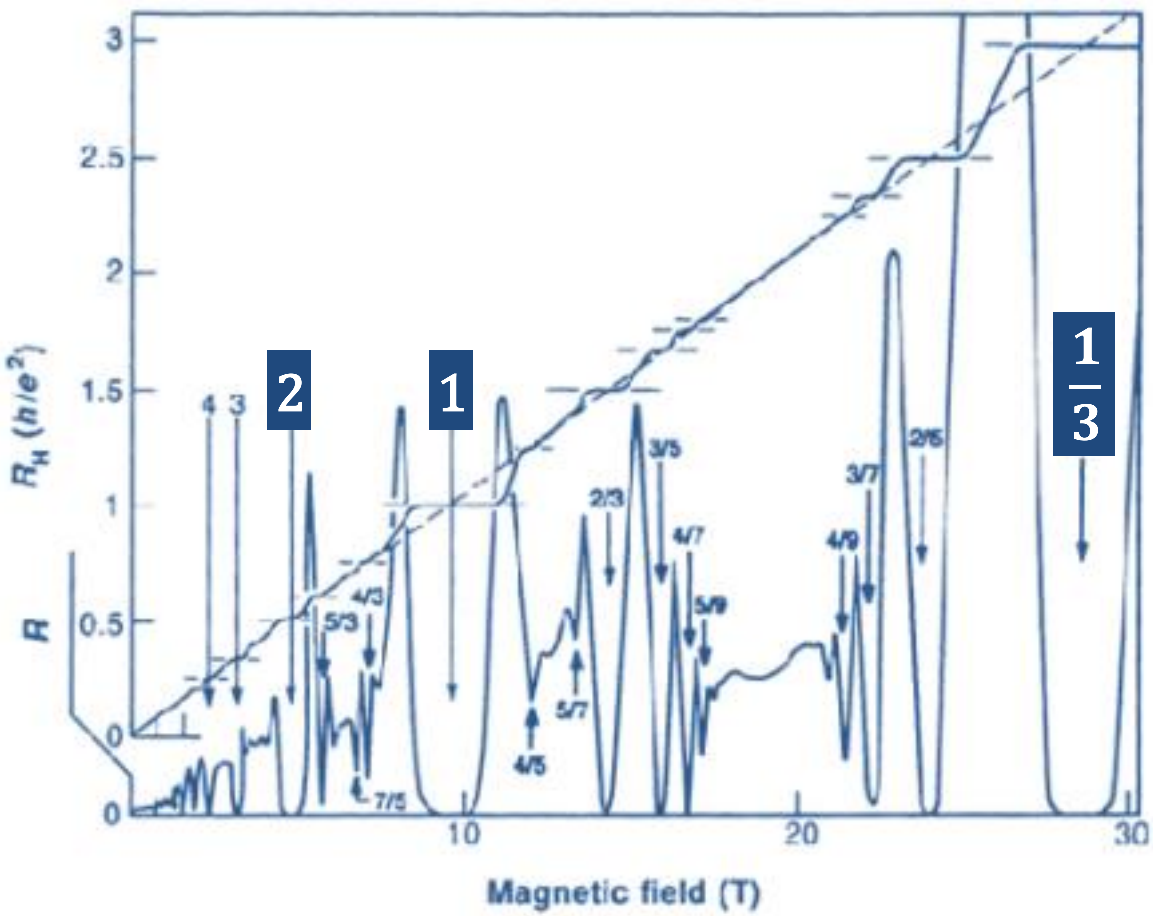
Fractional quantum Hall effect

Störmer, Tsui, Gossard ’82

Integral QHE: quantization of R_H near exact filling of Landau levels

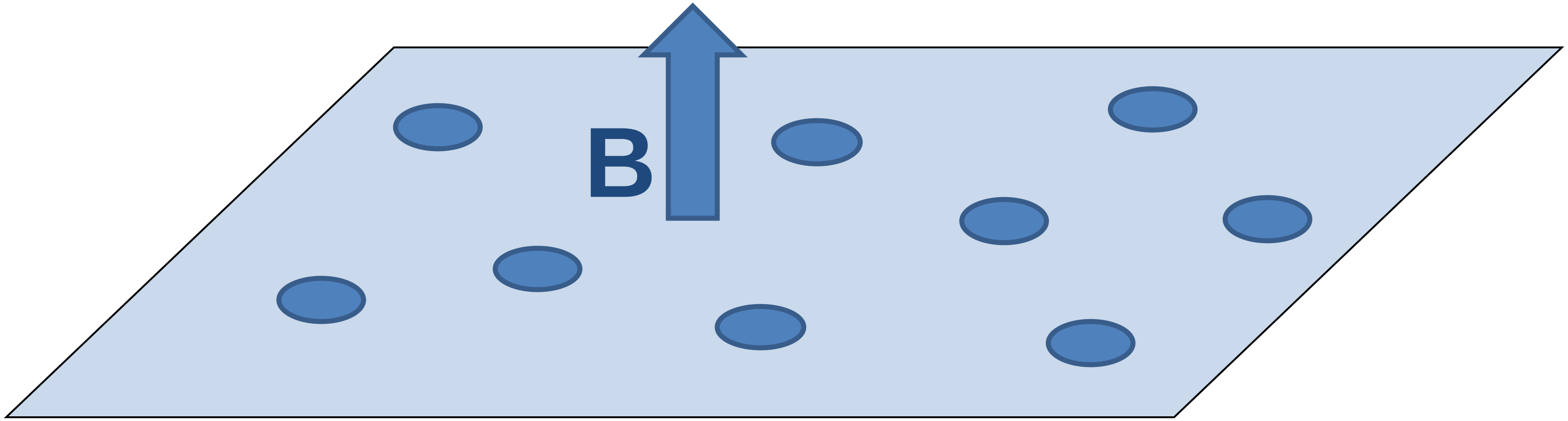
Fractional QHE: similar behaviour near certain fractional fillings

new physics: quantum liquid, fractional excitations, anyon statistics



Quantum liquid

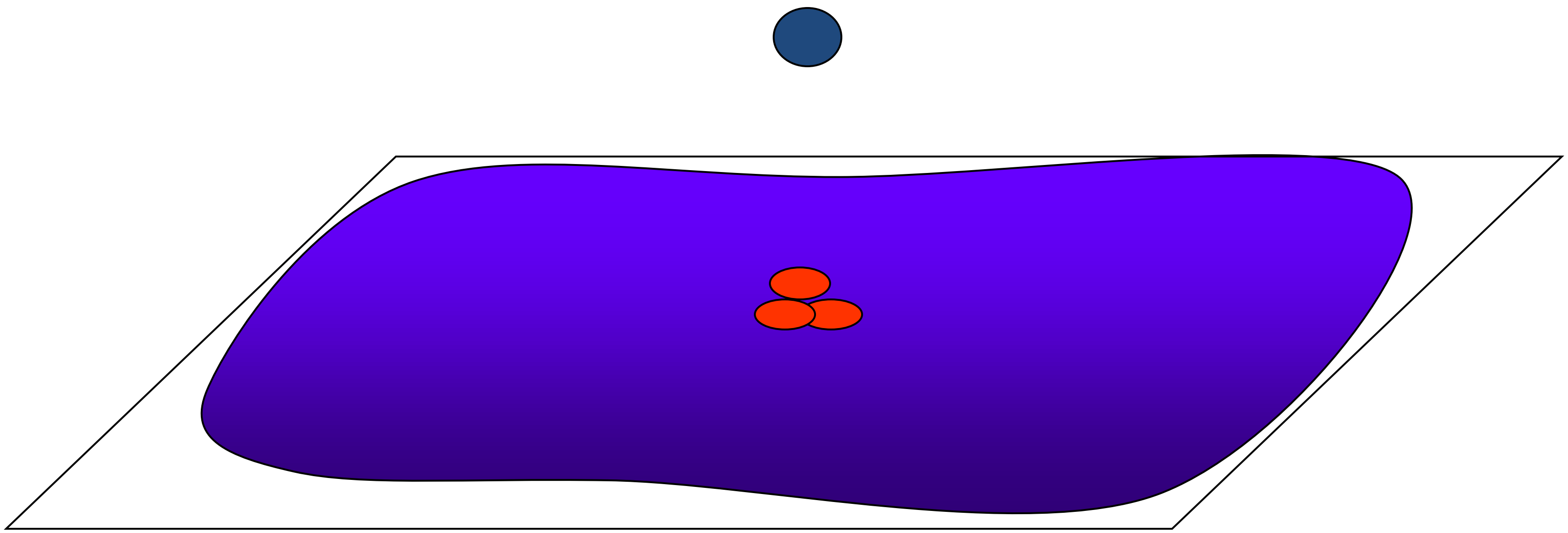
electrons in two dimensions (very thin layer)
in high magnetic field (motion further restricted/quantized to LLs)
interacting with one another via Coulomb forces
at approprate (low) density
become strongly correlated and **condense into a liquid**



new phase of matter: quantum[1] liquid[2]

[1] made of quantum particles (electrons)
[2] isotropic and incompressible

Fractionally charged (quasi)particles



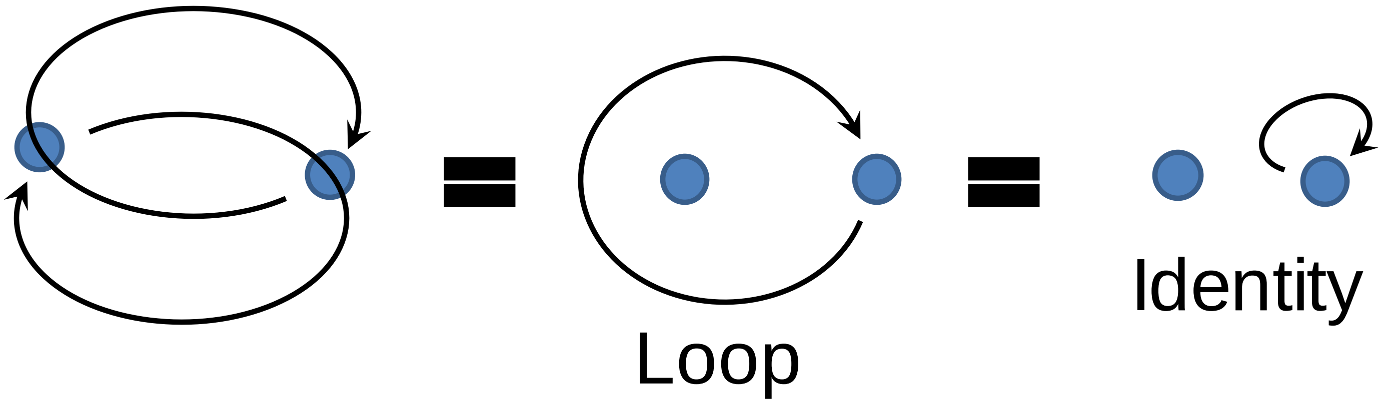
additional electron entering the liquid splits into 3 fractionally charged quasiparticles

(the extra electron blends into the liquid, and the excessive local charge shows as 3 new quasiparticles, moving independently)

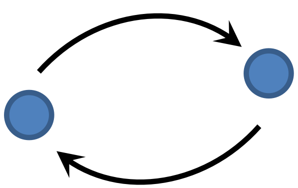
Fractional and non-Abelian (quasi)particles

in 3D:

double exchange *(P^2):*



single exchange:



$P = -1$: fermions → Pauli exclusion principle

$P = +1$: bosons → Bose-Einstein condensation

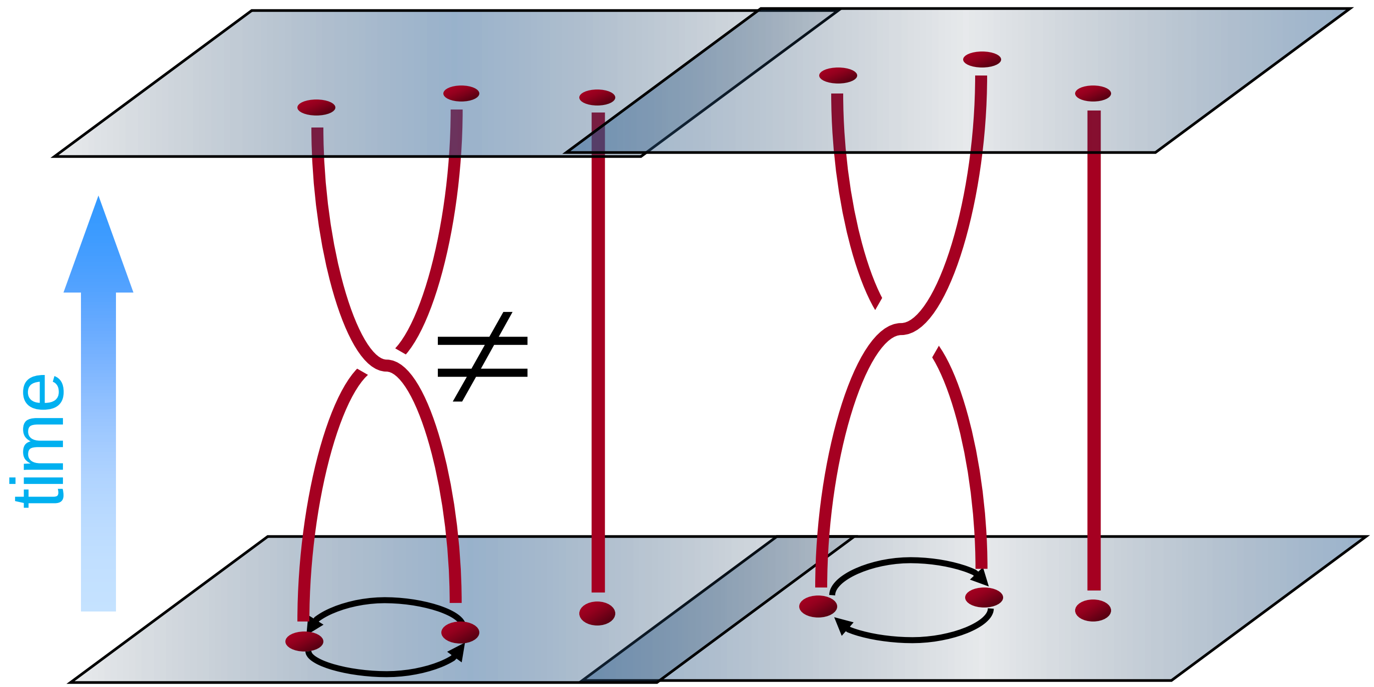
in 2D:

above argument fails, $P = e^{i\theta}$ (**anyons**) or matrix (**non-abelions**)

→ **topological quantum computation**

Particles with „memory of trajectory”

braiding non-Abelions (particles with non-Abelian braid group)



braid/exchange „counter-clockwise”

braid/exchange „clockwise”

final quantum state depends on past trajectories (not only on final positions) → application as protected element of quantum memory

Composite fermions

Jain '89

2D electrons + magnetic field → degenerate energy levels (Landau)

interactions → correlations (**complicated behavior**)

→ incompressible quantum liquid, FQHE

(quantization of R_{xy} & vanishing of R_{xx}) at particular conditions

simpler description/understanding
in terms of a new (hypothetical) particle:

electron (electric charge) **solenoids** (magnetic flux) **composite fermion** (charge+flux)

Composite fermions

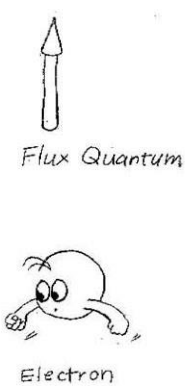
Jain ’89

composite fermion (CF) = electron + correlation hole

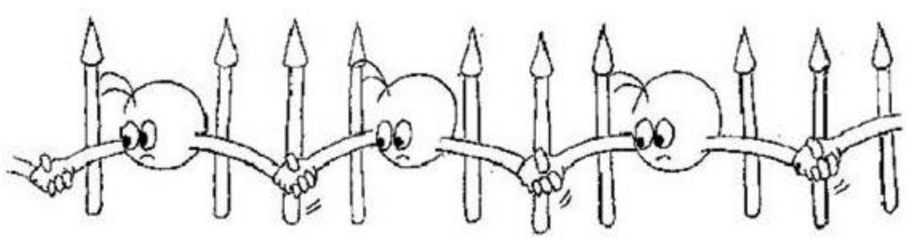
= e + 2 vortices of many-body wave function

= e + 2 magnetic flux quanta hc/e

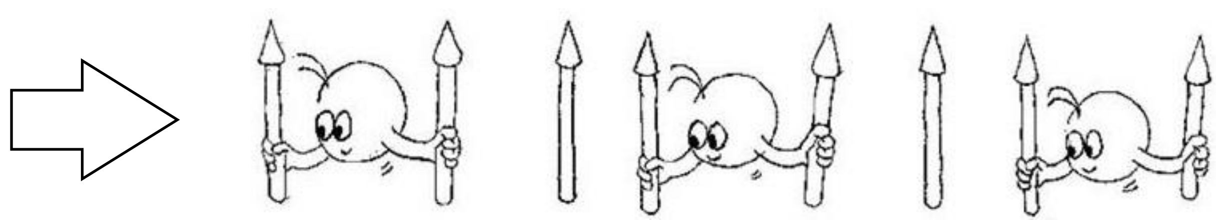
interaction → emergent (essentially) free quasiparticles



(Fig. Kwon Park)



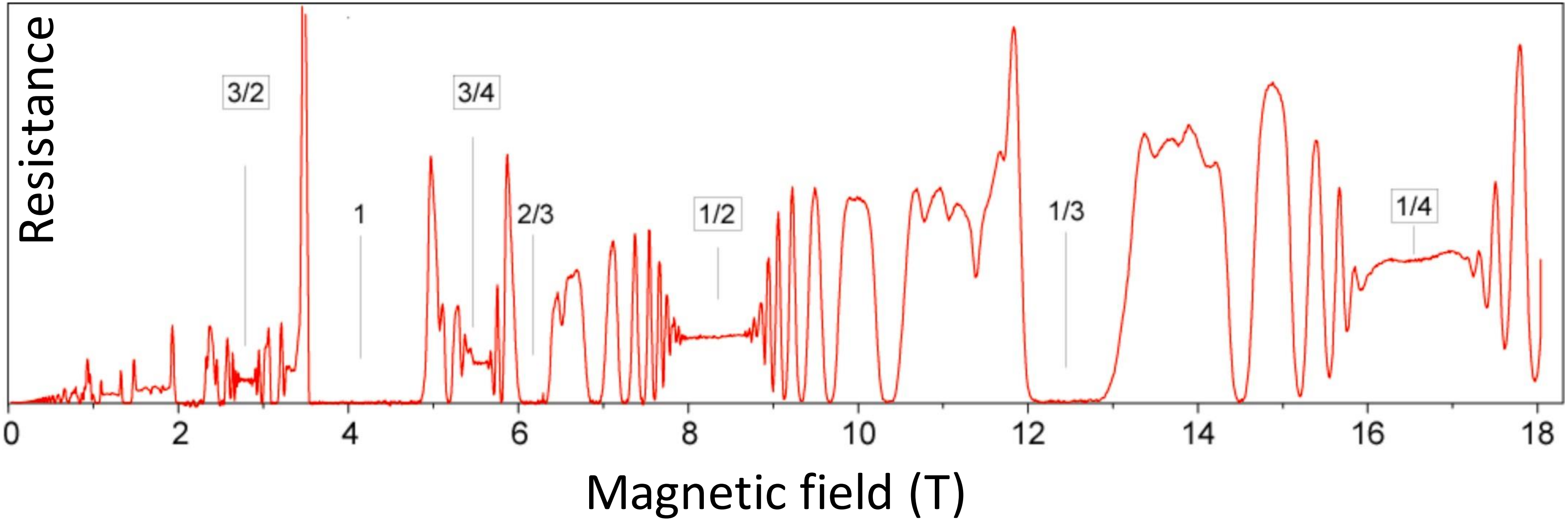
interacting electrons
in strong magnetic field



almost free CFs in reduced magnetic field

Composite fermions – experimental evidence

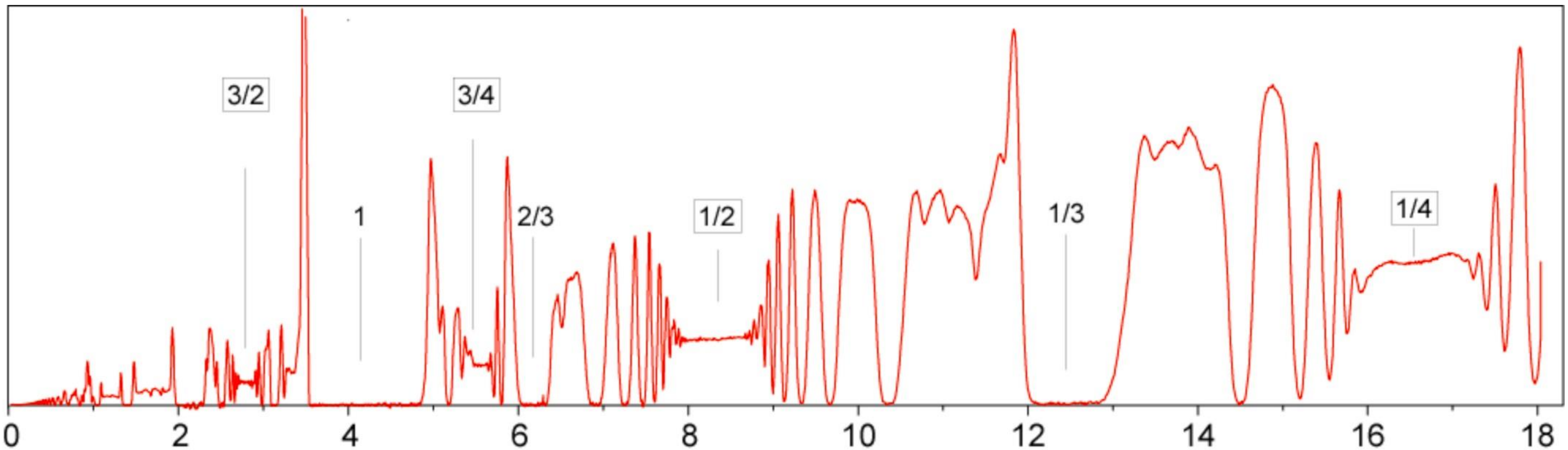
The plot of electric resistance (*R*) vs magnetic field (*B*), in which *R*=0 signifies QHE, is strikingly self-similar



R=0 → QHE (R_H=const), quantum liquid, etc.

Composite fermions – experimental evidence

The plot of electric resistance (*R*) vs magnetic field (*B*), in which *R*=0 signifies QHE, is strikingly <u>self-similar</u>



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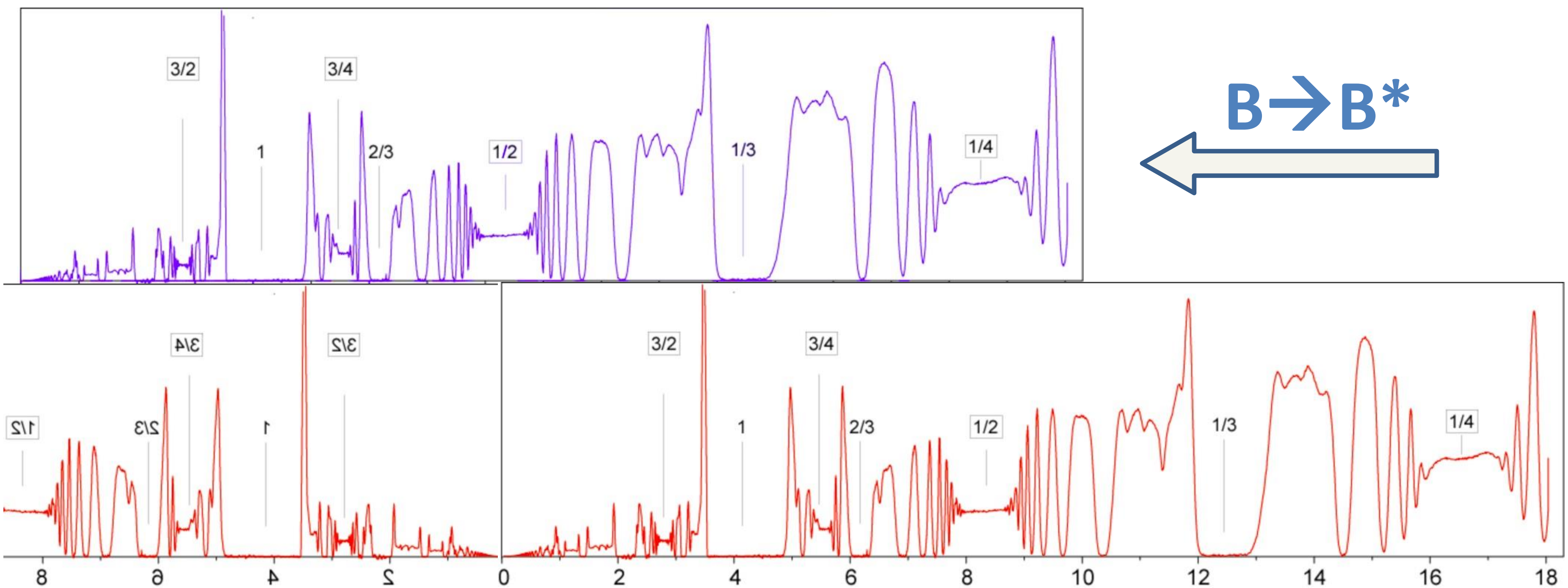
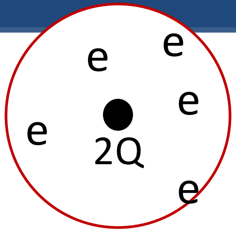
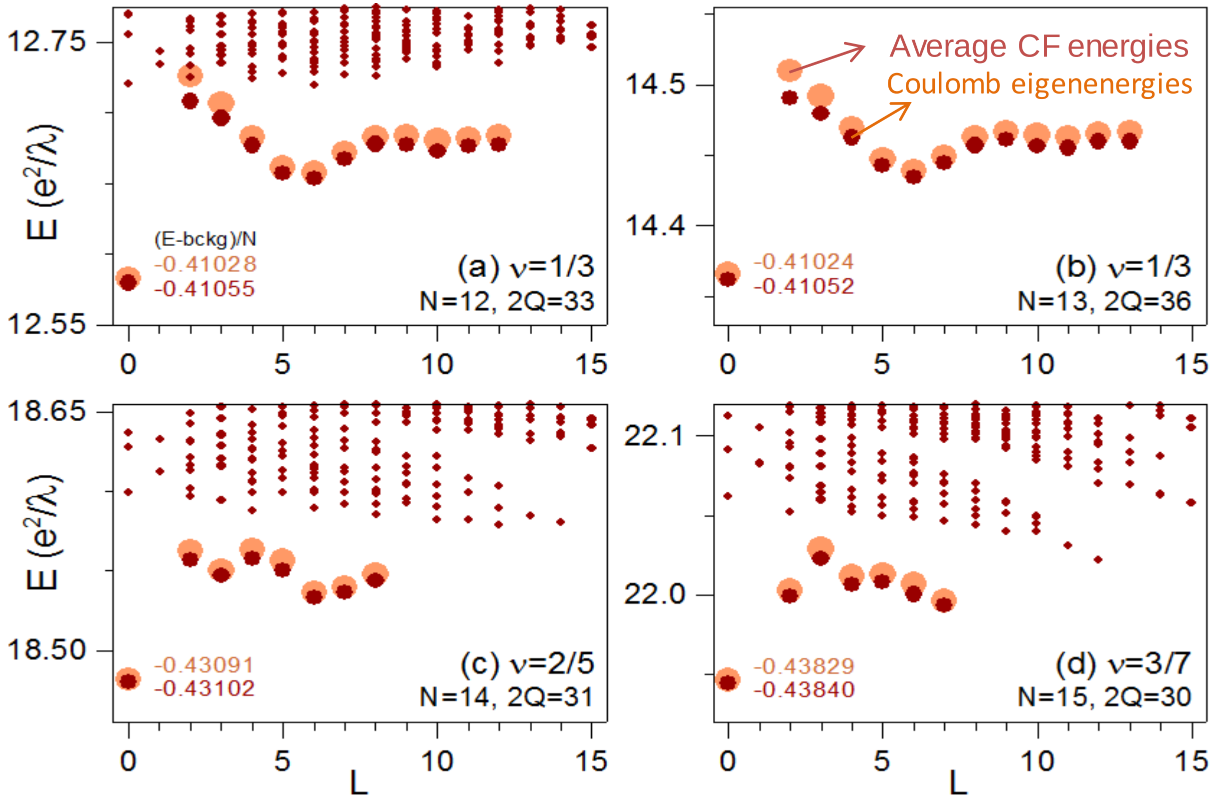


Composite fermions – numerical evidence

(own work)





E = total Coulomb energy, L = total angular momentum

Labels = correlation energy per particle

Haldane model:

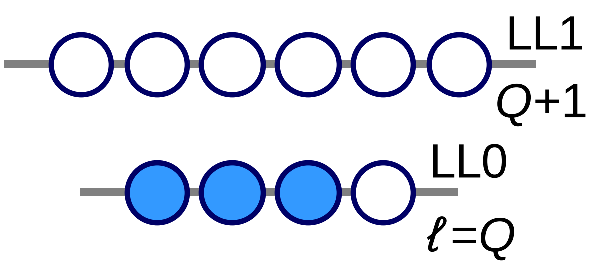
N electrons on sphere

field B from monopole 2Q

2Q=flux through surface

LL degeneracy=2Q+1

$\nu \sim N/2Q$



generalized (monopole) spherical harmonics:

$Y_{Q,l,m}(\theta,\varphi)$, $|m| \leq \ell$

Coulomb interaction V=1/r

Own research: correlations in low dimensions

1. Interacting electrons in semiconductor quantum dots

2. Fractional quantum Hall effect / quantum liquids

- **topological effects**
- microscopic mechanisms of condensation
- interaction with light (optical properties)
- **composite fermion theory**
  - CFs with residual interaction
  - CFs with spin (→ skyrmions)
  - CFs with additional freedom (flavor)
  - realistic systems (thickness, finite magnetic field, disorder)
- interacting fermions on artificial lattices

3. Two-dimensional crystals (graphene, MoS_2, etc.)

Summary of keywords and concepts

interacting electrons in extreme conditions

(2D, atomic perfection, low T, high B)

→ strong correlations

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topology – cause of universality / exactness of a macroscopic (transport, electric) **phenomenon:**

quantum Hall effect

quantum phase of matter: quantum liquid

new particles: anyons, non-Abelions, and

composite fermions