Hybrid Systems – Introduction

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Abstract: Hybrid system has come to mean a system which is an amalgamation of continuous and discrete inputs, outputs states, and dynamic equations. Hybrid systems arise when the continuous and the discrete meet. Particularly, hybrid systems arise from the use of finite-state logic to govern continuous physical processes (as in embedded control systems) or from topological and network constraints interacting with continuous control (as in networked control systems). This paper provides an introduction to hybrid systems, building them and shows some description of modeling language for Hybrid Systems.

1 Introduction

Hybrid systems are those in which a melding of two worlds—the analog and the digital-exists. Hybrid systems are all around us. Such systems arise whenever one mixes logical decision mak-ing with the generation of continuous-valued control laws. These systems are driven on our streets, used in our factories, and flown in our skies. Hybrid systems are systems that involve interaction between discrete and continuous dynamics. Such systems have been studied with growing interest and activity in recent years. Very often, the same phenomenon can be described either by a discrete model or a continuous one, depending on the context and purpose of the model [1]. One reason for the interest is that modeling and simulation of a complex system often require a combination of mathematical models from a variety of engineering disciplines. Practical control systems typically involve switching between several different modes, depending on the range of operation. Basic aspects of hybrid systems were treated in [6,7]. For stability analysis, see [3] and references therein. Related methods were discussed for discrete systems in [2] and on an abstract level for hybrid systems in [4]. So, hybrid systems arise in embedded and networked control.

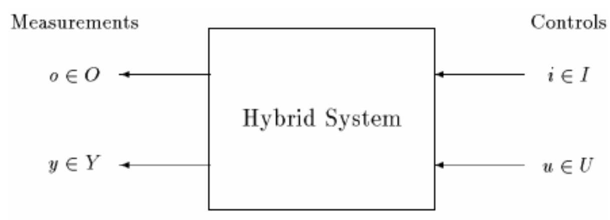

Figure 1
Block diagram of hybrid systems

More specifically, real-world examples of hybrid systems include systems with relays, switches, and hysteresis [5,8]; computer disk drives; transmissions, stepper motors, and other motion controllers [9]; constrained robotic systems [11]; automated highway systems (AHSs) [10]; flight control and management systems [12]; multi-vehicle formations and coordination [13]; analog/digital circuit codesign and verification; and biological applications [14].

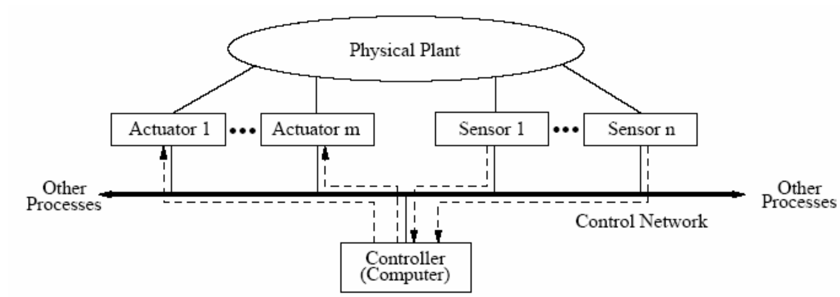

Figure 2
Typical networked control system setup and information flows

Adding to the complexity is the case where sensing, control, and actua-tion are not hardwired but connected by a shared network medium; see Fig. 2. In the autonomous case, the system evolution itself may fall naturally into a finite number of different phases , between which abrupt changes in continuous dynamics (switching) or continuous states (jumps or resets) occur. In the controlled case, a simple finite state machine may be used to regulate a physical process, such as may arise even in a simple thermostat. In more complicated situations, a mixture of autonomous and controlled phe-nomena may be present. See Fig.1.

1.1 The Need for Hybrid Control

To deal with large complex systems engineers are usually inclined to use a combination of continuous and discrete controllers. The reasons why continuous controllers are used are many:

- Interaction with the physical plant, through sensors and actuators, is essentially analog, i.e. continuous, from the engineering point of view.
- Continuous models have been developed, used and validated extensively in the past in most areas that interest control engineers (e.g. electrical and mechanical systems, electromagnetic systems, etc.).
- Powerful control techniques have already been developed for many classes of continuous systems. Moreover, in conjunction with the reliable continuous models, proofs of guaranteed performance can be obtained for these techniques.

Example 1: *Systems with switches and relays*

Physical systems with switches and relays can naturally be modeled as hybrid systems. Sometimes, the dynamics may be considered merely discontinuous, such as in a blown fuse. In many cases of interest, however, the switching mechanism has some hysteresis, yielding a discrete state on which the dynamics depends. This situation is depicted by the multi-valued function H shown Fig. 3 (left). Suppose that the function H models the hysteretic behavior of a thermostat. Then a thermostatically controlled room may be modeled as follows:

$$\dot{x} = f(x, H(x - x_0), u) \tag{1}$$

where x and x_0 denote actual and desired room temperature. The function f denotes the dynamics of temperature, which depends on the current temperature. Note that this system is not just a differential equation whose right-hand side is piecewise continuous. There is "memory" in the system, which affects the value of the vector field. Indeed, such a system naturally has a finite automaton associated with the hysteresis function *H*, as pictured in Fig. 3 (right). The notation! [condition] denotes that the transition must be taken when "enabled." That is, the event of *x* attaining a value greater than or equal to Δ triggers the discrete or phase transition of the underlying automaton from +1 to −1.

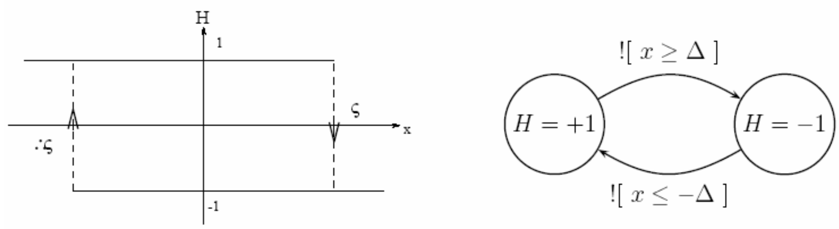

Figure 3
(left) Hysteresis function, *H*, (right) finite automaton associated with *H*

2 From Continuous To Hybrid

This part show ordinary differential equations (ODEs) as a base continuous model.

2.1 Base Continuous Model: ODEs

The base continuous dynamical systems dealt with are defined by the solutions of *ODEs*:

$$\dot{x} = f(x(t)) \tag{2}$$

where $x(t) \in X \subset R^n$. The function $f : X \to R^n$ is called a vector field on R^n. We assume existence and uniqueness of solutions. Actually, the system of ODEs in (2) is called autonomous or time invariant because its vector field does not depend explicitly on time. If it did depend explicitly on time, it would be nonautonomous or time varying, which one might explicitly note using the following notation:

$$\dot{x}(t) = f(x(t), t) \tag{3}$$

An *ODE with inputs and outputs* is given by

$$\begin{aligned} \dot{x}(t) &= f(x(t), u(t)) \\ y(t) &= h(x(t), u(t)) \end{aligned} \tag{4}$$

The functions $u(.)$ and $y(.)$ are the inputs and outputs. Whenever inputs are present, we say that $f(.)$ is a controlled vector field.

Differential inclusions: A differential inclusio*n* allows the derivative to belong to a set and is written as

$$\dot{x}(t) \in F(x(t)) \tag{5}$$

where $F(x(t))$ is a set of vectors in R^n. It can be used to model nondeterminism, including that arising from controls or disturbances.

2.2 Adding Discrete Phenomena

Hybrid systems are those that involve continuous states and dynamics, as well as some discrete phenomena corresponding to discrete states and dynamics. As described above, our focus in this chapter is on the case where the continuous dynamics is given by a differential equation

$$\dot{x}(t) = \xi(t), \quad t \geq 0 \tag{6}$$

then, $x(t)$ is considered the continuous component of the hybrid state, taking values in some subset R^n The vector field $\xi(t)$ generally depends on $x(t)$ and the aforementioned discrete phenomena. Here, $\xi(t)$ in (5) is a controlled vector field which generally depends on $x(t)$, the continuous component $u(t)$ of the control policy, and the aforementioned discrete phenomena. In this section, we identify the discrete phenomena alluded generally arise in hybrid systems. They are as follows:

- autonomous switching
- autonomous jumps
- controlled switching
- controlled jumps

Next, we analyze each of these discrete phenomena in turn.

Autonomous Switching

Autonomous switching is the phenomenon where the vector field $\xi(t)$ changes discontinuously when the continuous state $x(.)$ hits certain “boundaries” [15].

Example 2: Consider the problem of controlling a household furnace. The temperature dynamics may be quite complicated, depending on outside temperature, humidity, luminosity; insulation and layout; whether incandescent lights are on, doors are closed, vents are open, people are present; and many other factors. Thus, let’s just say that when the furnace is On, the dynamics are given by

$\dot{x} = f_1(x(t))$, where $x(t)$ is the temperature at time t; likewise, when the furnace is Off, let's say that the dynamics are given by $\dot{x} = f_0(x(t))$. The full system dynamics are that of a *switched system*:

$$\dot{x} = f_{q(t)}(x(t)) \tag{7}$$

where $q(t) = 0$ or 1 depending on whether the furnace is Off or On, respectively.

Autonomous Jumps

An autonomous jump is the phenomenon where the continuous state $x(.)$ jumps discontinuously on hitting prescribed regions of the state space [16]. We may also call these autonomous impulses. The simplest examples possessing this phenomenon are those involving collisions.

Example 4: (Bouncing Ball). Consider the case of the vertical motion of a ball of mass m under gravity with constant g. The dynamics are given by

$$\begin{aligned} \dot{x} &= v \\ \dot{v} &= -mg \end{aligned} \tag{8}$$

Further, upon hitting the ground (assuming downward velocity), we instantly set v to $-\rho v$, where $\rho \in [0,1]$ is the coefficient of restitution. We can encode the jump in velocity as a rule by saying

If at time t, $x(t) = 0$ and $v(t) < 0$, then $v(t^+) = -\rho v(t)$.

In this case, $v(.)$ is piecewise continuous (from the right), with discontinuities occurring when $x = 0$. This "rule" notation is quite general, but cumbersome. We have found it more desirable to use the following equational notation:

$$v^+(t) = -\rho v(t) \quad (x(t), v(t)) \in \{(0, v) \mid v < 0\} \tag{9}$$

Here, we have used Sontag's evocative discrete-time transition notation [17] to denote the "successor" of $x(t)$.

Controlled Switching

Controlled switching is the phenomenon where the vector field $\xi(.)$ changes abruptly in response to a control command, usually with an associated cost. This can be interpreted as switching between different vector fields Controlled switching arises, for instance, when one is allowed to pick among a number of vector fields:

$$\dot{x} = f_q(x), \quad q \in Q \approx \{1,2,...,N\}. \tag{10}$$

Here, the q that is active at any given time is to be chosen by the controller. If one were to make the choice an explicit function of state, then the result would be a closed-loop system with autonomous switches.

Example 5: (Satellite Control). In satellite control, one encounters

$$\ddot{\theta} = \tau_{eff} v \tag{11}$$

where θ and $\dot{\theta}$ are the angular position and velocity and $v \in \{-1,0,1\}$ depending on whether the reaction jets are full reverse, off, or full on.

Controlled Jumps

A controlled jump is the phenomenon where the continuous state $x(.)$ changes discontinuously in response to a control command, usually with an associated cost We also call these jumps controlled impulses.

Conclusions

In this paper, we have shown some basis about hybrid systems. For an introduction to hybrid systems simulation, see [18]. A survey of the hybrid systems literature is well beyond the scope of this chapter. For early surveys and more details on hybrid systems modeling, see [19, 20]. For a recent monograph on switching systems, see [21]. Analysis and control techniques for hybrid systems have been developed. See [19] for details and [22] for a summary.

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