Mathematical Model and Regulation of Non-Stationary Heat Condition

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Abstract: Non-stationary temperature condition of a room and its corresponding thermal balance is a response of system constructions that form the room considering the excitation parameters: temperature outside, sun rays flatling on circuit constructions and released glass-covered room, speed and course of wind flow and inner source of heat as lighting, technological equipments and so on. These excitation parameters cause propagation of heat by conduction, radiation and convection in a room. For mathematical modeling of non-stationary thermal condition of a room on computer we need to obtain an appropriate mathematical model of non-stationary thermal balance in a room. During the formation of the mathematical model we were inspired by the work of Jaroslav Řehánek [5].

Keywords: thermal comfort, heating, non-stationary condition, identification, mathematical model, control

1 Introduction

The main goal of heating is to form such inner environment that would fit a man the most. The aim of heating is to eliminate ones body heat loss in cool seasons by creating balance between the body and environment. Conditions that affect thermal comfort are mainly: air temperature, mid temperature of the walls, air moisture, air movement, air purity, heat resistance of dress and human activity. Heating affects only the air temperature and the mid temperatures of the walls. We call them effective temperature. Other conditions can be affected only by air conditioning which is many times used technical aid, for achieving comfort in present days.

Development of the theory of mathematical modeling of non-stationary heat condition of a room by the use of appropriate mathematical model enables us to simulate the course of the inner temperature and the courses of internal surface temperatures with set outer climatic conditions. The other option is to simulate non-stationary heat output needed for maintaining constant inner air temperature depending on non-stationary climatic conditions outside.

2 Modeling, Models and Universal Simulation Model of a Room's Heat Condition

Modeling is one of the theoretical methods of knowledge. It is characteristic by the fact that it is a cognitive process, where the original object that we want to know is replaced by a model. The model then substitutes the unknown object to a certain extent when the obtained information we consider equivalent with the information otherwise obtained by the study of the original.

For mathematical modeling of non-stationary heat condition of a room, it is important to form an appropriate mathematical model of non-stationary heat balance. Two methods simulating heat condition of a room such as heat simulation and simulation of non-stationary temperature of internal air and the courses of internal surface temperatures of constructions by calculations in selected time periods are going to be modeled. There is no consideration of the influence of heating capacity of the heating construction on the temperature in the room. With these simulations we can solve calculations for thermal stability in the room in summer or winter time. The article continues with a more detailed calculation of the internal air temperature with regard to the heating capacity. The simulation of non-stationary heating capacity expects constant internal air temperature in the room. In order to retain constant values of air temperature there is going to be found a corresponding non-stationary heating capacity needed for maintenance depending from non-stationary external climatic conditions.

2.1 Anticipations and Simplifying Conditions

Modeling of non-stationary thermal conditions focuses on a room. Mathematical model respects thermal flows in the system under some circumstances. Mitalas´s and Stephenson´s model is considered as a basic work in the area of mathematical modeling of thermal condition in the room with regard to non-stationary external climatic conditions during longer time period. Laplace´s transformation is used to solve Fourier´s differential equation of heat conduction. Mathematical models are divided into different levels of complexity using simplifying anticipations and other methods of solving systems of differential and linear equations.

During realization of the mathematical model for simulation of non-stationary thermal conditions these simplifying anticipations and conditions are applied:

- internal air temperature in exact time is constant in the entire room capacity, i.e. isothermal internal air volume,
- We consider only one-dimensional propagation of heat by conduction through building construction – there is identical temperature at a specific time on the whole surface of the construction.

2.2 Thermal Condition of the Room

Mathematical model of non-stationary thermal condition includes qualitative and quantitative links between external climatic conditions affecting the building with its thermo-technical characteristics and internal temperature characteristics.

Thermal condition in the room can be characterized by accumulative room temperature

$$\theta_M(t) = \theta_i(t) + \theta_p(t), \tag{2.1}$$

where

$\theta_M(t)$ is a summary temperature in the room in exact time t [°C],

$\theta_i(t)$ is the air temperature in the room in exact time t [°C],

$\theta_p(t)$ average temperature of internal surfaces in the room in exact time t [°C].

Internal air temperature and mid surface temperature on the internal constructions then completely characterize the thermal condition of the room determined by the model simulator.

3 Mathematical Model of the Selected Object

The aim is to acquire a mathematical model of non-stationary heat balance of a room in a kindergarten (KG) by analytic identification. Next step is to propose an effective control of heating with reaching heat comfort and maximal savings.

The disadvantages of the used heating in the KG came from equithermic regulation of delivery heating station (DHS) with parameters appropriate for flats. The KG was overheated. There were realized measurements of temperatures during three weeks in the building in 2003. There were measured such parameters as air temperature in five rooms $\theta_{1\text{-}5}$, afferent temperature of heated water in pipe close after the regulator θ_{uk} and the external temperature θ_{ext}. These measured parameters helped us to construct and control the mathematical model of a room in the KG. We were modeling two neighbouring rooms A1 and A2, A1 is in the corner.

3.1 Thermal Conditions of the Modeled Room

For the given non-stationary external climatic conditions we have determined the time courses and the courses of surface temperatures inside the construction by calculations. We have taken into consideration the influence of the heating capacity of the heating construction on the thermal condition of the room. External climatic conditions involve measured external air temperature θ_{ext}, and the earth temperature θ_{zeme} which can be considered constant the whole year.

Room A1 has a ceiling (S9) that functions as an accumulating symmetrically "is" heated (cooled) building construction overflowed by the stream $\Phi_{is}^{s9}(t)$, from two external (S1, S2) asymmetrically accumulating "es" heated (cooled) constructions overflowed by $\Phi_{es}^{s1}(t)$ and $\Phi_{es}^{s2}(t)$, two binding (S3, S4) accumulating constructions [6] overflowed by $\Phi_{s3}^{1,2}(t)$ between rooms A1 and A2 and $\Phi_{s4}^{1,3}(t)$ between A1 and A3 (vestibule), the floor with $\Phi_{z1}(t)$, windows and doors with $\Phi_{ok1}(t)$ and $\Phi_{dv1}(t)$, aired flow $\Phi_{v1}(t)$ and thermal flow produced by the heating system $\Phi_{r1}(t)$. Room A2 has a ceiling (S10) functions as accumulating symmetrically heated (cooled) building construction overflowed by $\Phi_{is}^{s10}(t)$, from one external (S6) accumulating asymmetrically heated (cooled) construction with $\Phi_{es}^{s6}(t)$, from one internal (S7) accumulating symmetrically heated (cooled) with $\Phi_{is}^{s7}(t)$, from two connected (S5, S8) accumulating constructions overflowed by $\Phi_{s5}^{2,1}(t)$ and $\Phi_{s8}^{2,3}(t)$, from the floor with $\Phi_{z2}(t)$, from window and doors with $\Phi_{ok2}(t)$ and $\Phi_{dv2}(t)$, aired flow $\Phi_{v2}(t)$ and from the thermal flow produced by the heating system $\Phi_{r2}(t)$.

While constructing the model we were coming out from thermal balance where on the left side we listed up members of profiting character and on the right side members of loss character in the balance equation. From the given anticipations the thermal balance of the room A1 is:

$$\begin{aligned}&\Phi_{s3}^{1,2}(t)+\Phi_{s4}^{1,3}(t)+\Phi_{is}^{s9}(t)+\Phi_{r1}(t)=\\&=\Phi_{es}^{s1}(t)+\Phi_{es}^{s2}(t)+\Phi_{ok1}(t)+\Phi_{dv1}(t)+\Phi_{v1}(t)+\Phi_{z1}(t),\end{aligned} \tag{3.1}$$

and the thermal balance of the room A2 is:

$$\begin{aligned}&\Phi_{s5}^{2,1}(t)+\Phi_{s8}^{2,3}(t)+\Phi_{is}^{s7}(t)+\Phi_{is}^{s10}(t)+\Phi_{r2}(t)=\\&=\Phi_{es}^{s6}(t)+\Phi_{ok2}(t)+\Phi_{dv2}(t)+\Phi_{v2}(t)+\Phi_{z2}(t),\end{aligned} \tag{3.2}$$

After substitution of corresponding thermal flows into this equation and some simplifying modifications, appropriate signature, we obtain the system of equations:

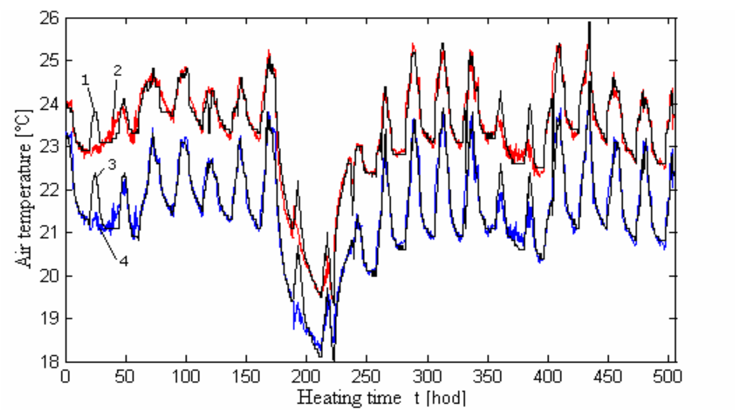
$$\theta_1(t)=\frac{A_{s3}^{1,2}.h_{s3}^{1,2}.E_2^{1,2}}{P}\theta_2(t)+\frac{A_{r1}.U_r}{P}\theta_r(t)+\frac{A_{ok1}.U_{ok}+V_1.c}{P}\theta_{ae}(t)+$$

$$\frac{A_p.U_p}{P}\theta_{zeme}(t)+\frac{A_{s4}^{1,3}.h_{s4}^{1,3}.E_2^{1,3}+A_{dv1}.U_{dv}}{P}\theta_3(t)+\frac{O}{P},$$

$$\theta_2(t)=\frac{A_{s5}^{2,1}.h_{s5}^{2,1}.E_2^{2,1}}{R}\theta_1(t)+\frac{A_{r2}.U_r}{R}\theta_r(t)+\frac{A_{ok2}.U_{ok}+V_2.c}{R}\theta_{ae}(t)+$$

$$+\frac{A_p.U_p}{R}\theta_{zeme}(t)+\frac{A_{s8}^{2,3}.h_{s8}^{2,3}.E_2^{2,3}+A_{dv2}.U_{dv}}{R}\theta_3(t)+\frac{T}{R}. \quad (3.3)$$

From the system of equations (3.3) we can calculate temperatures of the rooms A1 and A2 for given time from the beginning of the inflow, from constant heating or cooling, or time course of temperatures. We have decided for the time course of temperatures, to be able to compare it with measured values of temperatures in rooms A1 and A2. We have been continuously dividing the whole cycle of heating (three weeks) into inflow time, stable condition and time of cooling [7], due to known measured values. At the beginning of each running period, initial conditions for each air temperature in rooms and temperatures of wall surfaces were defined. We have chosen sampling period (time between two calculated temperatures), so as the measurements were made, i.e. t = 15 min. After solving the system of equations (3.3) for our heating period we have got the time course **Figure 1**. This course is designated as Model_1.



Legend: 1 – measured air temperature in the room A1,

2 – model_1 of room A1,

3 – measured air temperature in the room A2,

4 – model_1 of room A2.

Figure 1

Results attained by solving the system of equations

To be able to transcribe the system of equations (3.3) into state space, we needed to choose the state, regulating and fault parameters properly and choose proper matrices for them. We have expected the same sampling period as in case of Model_1. Unlike the Model_1, air temperatures in rooms were not directly obtained, but they were dependent upon previous conditions, i.e. firstly from the internal air temperature in the room in step "k" the temperatures corresponding to surface wall temperatures in the room are calculated and consequently in step (k+1) air temperatures of rooms A1 and A2 from these temperatures and other still functioning parameters are going to be modeled. On the ground of these assumptions we obtained Model_2 in state space:

$$\begin{bmatrix} \theta_1(k+1) \\ \theta_2(k+1) \end{bmatrix} = \begin{bmatrix} \frac{H1}{P1} & \frac{A_{s3}^{1,2}.h_{s3}^{1,2}.E_2^{1,2}}{P1} \\ \frac{A_{s5}^{2,1}.h_{s5}^{2,1}.E_2^{2,1}}{R1} & \frac{H2}{R1} \end{bmatrix} . \begin{bmatrix} \theta_1(k) \\ \theta_2(k) \end{bmatrix} + \begin{bmatrix} \frac{A_{r1}.U_r}{P1} \\ \frac{A_{r2}.U_r}{R1} \end{bmatrix} .\theta_r(k) +$$

$$+ \begin{bmatrix} \frac{A_{ok1}.U_{ok} + V_1.c}{P1} & \frac{A_p.U_p}{P1} & \frac{A_{s4}^{1,3}.h_{s4}^{1,3}.E_2^{1,3} + A_{dv1}.U_{dv}}{P1} & \frac{O}{P1} \\ \frac{A_{ok2}.U_{ok} + V_2.c}{R1} & \frac{A_p.U_p}{R1} & \frac{A_{s8}^{2,3}.h_{s8}^{2,3}.E_2^{2,3} + A_{dv2}.U_{dv}}{R1} & \frac{T}{R1} \end{bmatrix} . \begin{bmatrix} \theta_{ae}(k) \\ \theta_{zeme}(k) \\ \theta_3(k) \\ 1 \end{bmatrix},$$

$$\begin{bmatrix} y_1(k) \\ y_2(k) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} . \begin{bmatrix} \theta_1(k) \\ \theta_2(k) \end{bmatrix}, \tag{3.4}$$

whose figure of matrix inscription is:

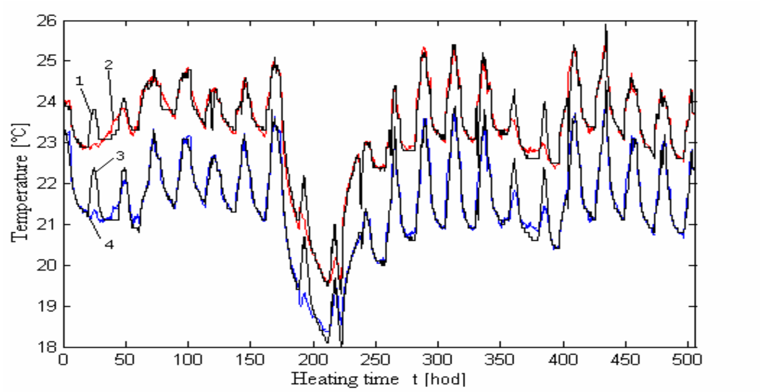
$$\vec{\theta}(k+1) = \mathbf{F}.\vec{\theta}(k) + \mathbf{g}.\vec{u}(k) + \mathbf{M}.\vec{p}(k),$$

$$\vec{y}(k) = \mathbf{C}.\vec{\theta}(k). \tag{3.5}$$

The input of the system is surface temperature of the heating construction, we considered to be equal to mid temperature of heated water, system conditions are air temperatures in the modeled rooms, known disturbances represent external climatic conditions and air temperature of the neighbouring room A3 (hall) and the output of the system represent temperatures in rooms A1 and A2.

We have added the measured external air temperature and the calculated mid temperature of heated water to the input of the model. We have set the needed initial conditions and calculated the relevant air temperatures in rooms A1 and A2. Obtained results of this mathematical model are shown in **Figure 2**.

The model of rooms was made in MATLAB program. We can see the visible reciprocal thermal dependence of rooms and the existence of binding thermal flow, dependant upon thermo-technical characteristics of constructions dividing the rooms and that is confirmed in the figure. It also appears within long stable periods of heating, only during weekends, when the temperatures obtained by the model aren't able to follow temperatures that are really measured. This happens due the fact that the model doesn't incorporate temperatures invoked by the influence of solar energy on the building. If comparing model_1 and 2 there are visible differences, i.e. less temperature variations caused actually by calculation of actual air temperature in the room also depending upon previous air temperature. To be more detailed, air temperature in room A1 in step (k+1) is dependant upon air temperature of A1 in step k. On the contrary, calculating model_1 air temperature in the room in arbitrary chosen time is independent from previous temperature parameters in the given room.



Legend: 1 – measured air temperature in the room A1,

2 – model_2 of room A1,

3 – measured air temperature in the room A2,

4 – model_2 of room A2.

Figure 2

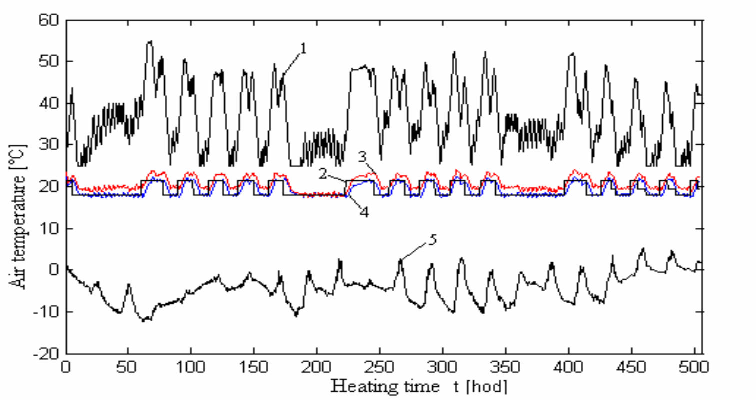
Measured and obtained temperatures in the output of mathematical model_2

4 Proposal of Mathematical Model Regulation

For temperature regulation of modeled rooms we have proposed two types of regulation. First type based on simple standard logic the other is proposed according to optimal regulation. The purpose of temperature regulation (heating) is elimination of energy consumption by heating and to provide monitoring of user adjusted parameters of temperature courses using a regulator at the time of running. The emphasis is put on achieving desired temperature exactly on the set time i.e. immediate.

4.1 Rational Regulation

This type of regulation as it results from the title is based on logical consideration, i.e. if the required temperature is higher than the actual air temperature in the room, there is incremented the temperature of afferent heated water. If the required temperature is lower than the actual air temperature in the room, there is decremented the temperature of the heating water. Initial temperature of heating water was calculated according to equation [1] and we had to respect limits of active intervention caused by DHS. On **Figure 3** we can see the obtained temperatures in rooms using rational regulation. The zoom up of temperatures is fairly slow and delayed, because of slow increase of heating water temperature, but the savings are clear.



Legend: 1 – θ_{uk} temperature of the heating water,

2 – θ_{A2} air temperature in room A2,

3 – θ_{A1} air temperature in room A1,

4 – $\theta_{pozadovana}$ required temperature in rooms A1 & A2,

5 – θ_{ext} external temperature.

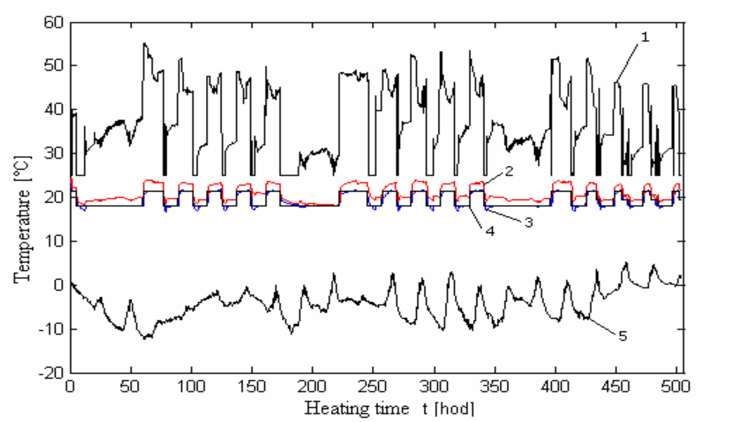
Figure 3

Rational regulation of the model

4.2 Optimal Regulation

The basis of the proposed optimal regulation is LQ regulation (Linear quadratic control) [2] and optimal regulation with integration component [4].The present disturbances are known and directly incorporated into the model, therefore they are not considered in computation of K matrix. K matrix is solved by means of LQ regulation. To obtain a regulator with integration component convenient for elimination of persisting regulation divergence we needed to use matrix K and convenient equation [4] to obtain condition regulator K1 and amplification of K2.

After obtaining the needed variables and inserting into condition regulation scheme with model [1], we reach the outcomes of optimal air temperature regulation in the room (**Figure 4**).



Legend: *1 – θ_{uk} temperature of the heating water,*

2 – θ_{A2} air temperature in room A2,

3 – θ_{A1} air temperature in room A1,

4 – $\theta_{pozadovana}$ required temperature in rooms A1 & A2,

5 – θ_{ext} external temperature.

Figure 4

Optimal regulation of the model

It is visible from obtained results that this kind of control stabilizes on the required temperature faster, but requests even faster [jumping] variation and higher temperatures of heating water. It means if we want fast achieving of required temperatures in rooms we need to add more energy which causes increasing of financial charges for heating.

Conclusion

The task was to obtain a mathematical model of non-stationary heat condition of a particular room and to propose algorithms of heat regulation and to simulate the proposed regulation on achieved model.

The article presents two different models describing the same object, by which we can simulate the course of temperatures of inner air with consideration of heating output. Both models are different by certain simple mathematical and theoretical features.

The proposed heating controls had to fulfill three requests. Firstly to provide the required temperature on exact time, this is coherent with the second request, to secure thermal comfort in the room. The last request was to eliminate consumption of energy. To fulfill all the three requests we rather use rational control with sufficient onset of the heating start, using properly chosen required temperature.

We need to emphasize that we will not reach significant savings with regulation, than with accurately proposed isolation. If the elementary demands on thermo-technical characteristics of the building are not observed, there is no need to propose and install a complex regulating construction.

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