Possibility Theory Based Approach To Verifying the Fuzzy Rule-Base

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Abstract: This paper deals with the control of a dynamic system where the gains of the conventional PD controller are previously chosen by fuzzy methods in such a way as to obtain the optimal trajectory tracking. The gain factors are determined by solving fuzzy equations, and based on the sufficient possibility measure of the solution. It will be shown, that the rule premise for the given system input in fuzzy control system may also determine the possibility of realizing a rule. This possibility can be used for verifying the rule and for changing the rule-output, too. This leads to the optimization of the output. When calculating the possibility value the possible functional relation between the rule-premise and rule-consequence is taken into account. For defining the rule of inference in Fuzzy Logic Control (FLC) system special class of t-norm is used.

Keywords: fuzzy control, PD controller,control gains

1 Introduction

There is a question that arises during the studying of fuzzified functions: what are those practical problems where given beside certain fuzzified function parameters, an approximation can be provided to other unknown but also fuzzy-type function parameters. If a scruple, linear function relationship is observed, the fuzzification problem of the

$$K_p e + K_d \dot{e} = y \tag{1}$$

type law of the PD-type controller emerges.

The conventional linguistic FLC uses fuzzified quantities $e, \dot{e}$ (error and error change) as inputs and y as output. The rules of this system are

if e is E and $\dot{e}$ is $\dot{\mathrm{E}}$ then y is Y where $E, \dot{E}$ and Y are linguistic terms, whose can be NB(negative big), NM(negative medium), NS(negative small), ZE(zero), PS(positive small), PM(positive medium), PB(positive big). Fuzzy membership functions cover linguistic terms. The scaling and normalization of parameters domains are made by experts.

In law (1) the control gains K_p, K_d could also be modified during operation bringing the controlled system into a desired state. There types of FLCs are called tuning-type controllers. In the literature there are indications regarding the solution of this problem [1].

In this paper a method which using the function relationship between $K_p, K_d, e, \dot{e}, y$ parameters is presented, that creates the rule base on one hand, and furthermore uses this functional dependence in the inference mechanism too.

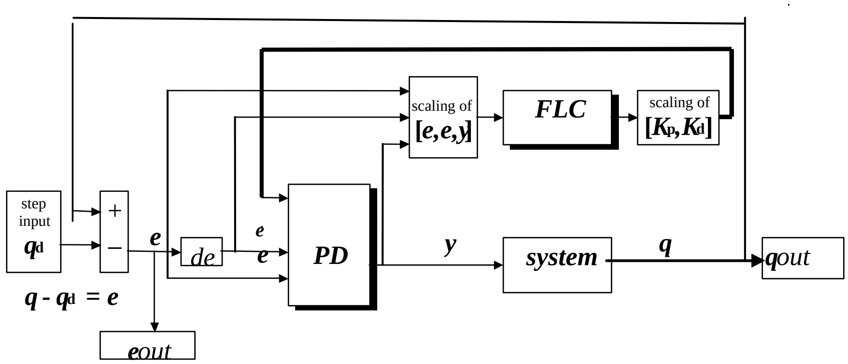

Figure 1.
The architecture of the proposed controller

Figure 1 illustrates how such a tuning FLC can be integrated into the system. The conventional PD controller and the FLC has the same $e, \dot{e}$ as inputs and the tuning FLC also uses the output y of the conventional controller (this is required because of the linear relationship in (1)). The FLC gives two crisp outputs to the PD controller that calculates a new y by using these gains and $e, \dot{e}$ as inputs.

Following the procedure of FLC construction, contains of steps:

st1 determination of fuzzification strategy

st2 the choise of quantities to be fuzzified

st3 fuzzification of these quantities and the rule base construction

st4 choosing of inference mechanism

st5 choosing of defuzzification model, these general steps cover different mathematical procedures depending on the choice of strategy. This paper presents two procedures an an example: a Mamdani-type, in which a novel construction of the rule-base is given, and another one which is said to be *possibility-modified* and the rule possibilities integrated into the rule outputs.

2 General Concept

A fuzzy subset A of universe of discourse X is defined as $A=\{(x,\mu(x))|x\in X, \mu_A : X\rightarrow[0,1]\}$. Denote $\mathcal{FX}$ the set of all fuzzy subsets on X. If the universe is $X = \mathfrak{R}$, and we have a membership function

$$\mu(x)=\begin{cases} g^{(-1)}\left(\dfrac{|x-\alpha|}{\delta}\right), & \text{if } \delta\neq 0 \\ \chi_\alpha(x), & \text{if } \delta = 0 \end{cases} \tag{3}$$

where the characteristic function of A will be denoted by χ_A, and $\alpha\in\mathfrak{R},\delta\geq 0$, then the fuzzy set given by $\mu(x)$ will be called quasi-triangular fuzzy number with the center α and width δ, and we will recall for it by the pair QTFN(α,δ).

The binary operation T: $\mathrm{I}^2\rightarrow\mathrm{I}$, (I=[0,1]) is a t-norm. The t-norm is Archimedian if and only if it admits the representation $T(a,b)=g^{-1}(g(a)+g(b))$, where the generator function $g:\mathrm{I}\rightarrow\mathfrak{R}^+$ is continuous, strictly decreasing, $g(0)=1, g(1)=0$ and $g^{(-1)}(x)$ is the pseudo-inverse of the function g. The generalization of this representation is $T_{gp}(a,b)=g^{-1}\left(g^p(a)+g^p(b)\right)^{\frac{1}{p}}$.[4]

Let be T_{gp} an Archimedian t-norm given by generator function g^p, $p\in[1,\infty)$.

The membership function of the t-norm of fuzzy sets is defined as follows

$$\mu(x)\cap\nu(x)=T(\mu(x),\nu(x)) \in \mathcal{FR} \tag{4}$$

The Mamdani type controller applies the rule: if x is $\mu(x)$ then y is $\nu(y)$, where x is the system input, y is the system output, x is $\mu(x)$ is the rule-premise, y is $\nu(y)$ is the rule-consequence. $\mu(x)$ and $\nu(y)$ are linguistic terms and they can be described by QTFN-s.

For a given input fuzzy set $\mu'(x)$, in a mathematical-logical sense, the output fuzzy set $\nu'(y)$, will be generated with a Generalized Modus Ponens (GMP).

At every fixed $x \in \Re$ a T-fuzzification of the function value of the parametric function $f(a_1,a_2,...a_k, x)$ by the fuzzy parameter vector $\mu_a = (\mu_1, \mu_2, ...\mu_k)$ is a fuzzy set of $\mathcal{FR}$.

Let EQ be a non-fuzzy equality relation on universe. The T-fuzzification of EQ is a fuzzy set on $\mathcal{FR} \times \mathcal{FR}$

$$E\widehat{Q}(\mu(x), \nu(y)) = \sup_{x=y} T(\mu(x), \nu(y)) = \sup_{x} T(\mu(x), \nu(x)) \tag{5}$$

The (g,p,δ) fuzzification of a linear function $l(\alpha, x) = \alpha_1 x_1 + \alpha_2 x_2 + ... + \alpha_n x_n$ by the fuzzy vector parameter $\mu_a = (\mu_1, \mu_2, ...\mu_n)$ (where the coefficients α_i are uncertainly parameters, and replaced by QTFN(α_i, δ_i), and the fuzzification of function will be defined by T_{gp} norm), is given in [4].

The (g,p,δ) fuzzification of a linear equality $\alpha_1 x_1 + \alpha_2 x_2 + ... + \alpha_n x_n = \alpha_0$ by the fuzzy vector parameter $\mu_a = (\mu_1, \mu_2, ...\mu_n)$ is:

$$E\widehat{Q}(\widehat{l}(\mu_a, x), \chi_0) = \widehat{l}(\mu_a, x)(0) =$$

$$\sigma(x) = g^{(-1)}\left(\frac{|l(\alpha, x)|}{\|diag(\delta) \cdot x\|_q} \right) = g^{(-1)}\left(\frac{|l(\alpha_1 x_1 + \alpha_2 x_2 + ... + \alpha_n x_n - \alpha_0)|}{\|diag(\delta) \cdot x\|_q} \right) \tag{6}$$

$\sigma(x)$ will be called *possibility measure* of equality [2],[3].

3 Construction of the Mamdani-type FLC for Control Law, Example

st1 Let us chose a Mamdani-type linguistic model for the problem (1).

st2 $e, \dot{e}, y$ quantities are uncertain, fuzzified, and comprise the FLC and the rule-inputs. K_p, K_d are also uncertain, fuzzified but they comprise the outputs. The rule type for the scaling of the gain papameters K_p, K_d is

$$\text{if } \left(e \text{ is E} \quad \text{and} \quad \dot{e} \text{ is } \dot{\text{E}} \quad \text{and} \quad y \text{ is Y} \right) \quad \text{then} \quad \left(K_p \text{ is } \text{K}_\text{p} \quad \text{and} \quad K_d \text{ is } \text{K}_\text{d} \right)$$

shortly

$$\text{if} \quad \text{E} \cap \dot{\text{E}} \cap \text{Y}(\underline{e}) \quad \text{then} \quad \text{K}_\text{p} \cap \text{K}_\text{d}(\underline{k}) \quad \text{or} \quad \text{if} \quad \text{E} \cap \dot{\text{E}} \cap \text{Y}(\underline{e}) \quad \text{then} \quad \text{K}(\underline{k}) \ .$$

(Details see in [4])

st3 Experts can provide those $[-L_e, L_e], [-L_{\dot{e}}, L_{\dot{e}}]$ intervals where $e, \dot{e}$ quantities exist. For simplification and generalization of the problem these $[-L_e, L_e], [-L_{\dot{e}}, L_{\dot{e}}]$ intervals are normalized and transformed into interval [-1,1]. During the scaling operation $e, \dot{e}$ receive 7-7 linguistic terms, there being determined by (3) type fuzzy numbers, for example

$$\mathrm{E}(e) = \begin{cases} g^{(-1)}\left(\dfrac{|e - e_c|}{de}\right), & \text{if de} \neq 0 \\ \chi_{e_c}(e), & \text{if de} = 0 \end{cases}$$

$e \setminus \dot{e}$	**NB**	**NM**	**NS**	**ZE**	**PS**	**PM**	**PB**
NB	NB	NB	NB	NM	NM	NS	ZE
NM	NB	NB	NM	NM	NS	ZE	PS
NS	NB	NM	NM	NS	ZE	PS	PM
ZE	NM	NM	NS	ZE	PS	PM	PM
PS	NM	NS	ZE	PS	PM	PM	PB
PM	NS	ZE	PS	PM	PM	PB	PB
PB	ZE	PS	PM	PM	PB	PB	PB

Table 1

These 49 possibilities would increase seven times if the *y* quantity was normalized and scaled likewise. It should be noted, however, that $e, \dot{e}, y$ quantities are not independent from each other. The relationship generally used by experts in such controllers, (see Table 1 for the y quantity), can be applied for completing input parameters into the rule. Finally we have 49 different rule inputs. The scaling of *y* is the same on normalized interval [-1,1].

For the rule outputs also linguistic terms are defined which are obtained within the domain of K_p, K_d by scaling. The $[-L_{K_p}, L_{K_p}], [-L_{K_d}, L_{K_d}]$ intervals and the scaling are determined by experts. For the given $\mathrm{E}, \dot{\mathrm{E}}, \mathrm{Y}$ the suitable $\mathrm{K_p}, \mathrm{K_d}$ rule outputs are chosen based on experience meta-rules or tiresome experimental work.

In our case the $\mathrm{K_p}, \mathrm{K_d}$ output fuzzy domains will be determined as such for which the possibility of law (1) is the greatest, in case of given $\mathrm{E}, \dot{\mathrm{E}}, \mathrm{Y}$.

First let us assign linguistic terms to $\mathrm{K_p}, \mathrm{K_d}$ (like by $e, \dot{e}, y$) on $[-L_{K_p}, L_{K_p}], [-L_{K_d}, L_{K_d}]$ interval. The possible K_p is $\mathrm{K_p}$ and K_d is $\mathrm{K_d}$ (i.e. $\mathrm{K_p} \cap \mathrm{K_d}$) domain-number is 49.

Define the possibility measure:

$$\sigma\left(K_p, K_d\right) = g^{(-1)}\left(\frac{\left|K_p e_c + K_d \dot{e}_c - y_c\right|}{\left\|diag(\delta)\cdot\left[K_p, K_d, 1\right]^T\right\|_q}\right) \tag{7}$$

for each rule-premise. The possibilistic rule is defined as follows:

if $\mathrm{E}\cap\dot{\mathrm{E}}\cap\mathrm{Y}(\underline{e})$ then $\sigma\left(K_p, K_d\right)$ or if $\mathrm{E}\cap\dot{\mathrm{E}}\cap\mathrm{Y}(\underline{e})$ then $\sigma(\underline{k})$

In principle, any $K_p \cap K_d$ intersection can be assigned as output to the rule-premise, but in our case the one with the greatest possibility is used, i.e.

$$poss(\,i\,max, j\,max) = \max_{i,j}\left(\min_{\underline{k}}\left(\sigma(\underline{k})\cap \mathrm{K}_{ij}(\underline{k})\right)\right), \quad \mathrm{i, j} = 1,\ldots 7. \tag{8}$$

is the greatest. The suitable output is $\mathrm{K}_{i\,max, j\,max}(\underline{k})$. $\left(\mathrm{K_p}\cap\mathrm{K_d}\in\left\{\mathrm{K_{ij}}, i, j = 1,2\ldots 7\right\}\right)$

Example 1

Let be $\left[-L_{K_p}, L_{K_p}\right] = \left[-L_{K_d}, L_{K_d}\right] = \left[-400, 400\right]$.

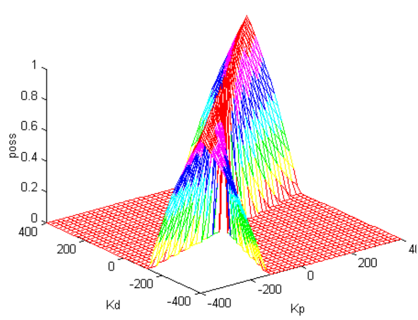

Figure 2

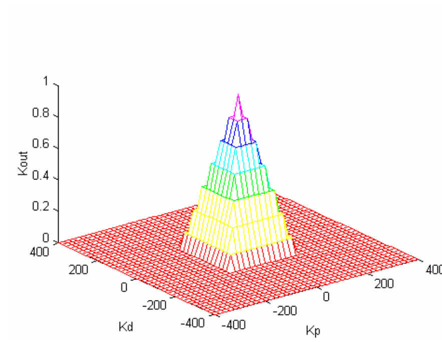

Figure 3

For rule-premise if e is PM and $\dot{e}$ is NM and y is ZE from the rule-base, and for $g(t) = 1 - t$, $q = \infty$ the possibility measure is:

$$\sigma\left(K_p, K_d\right) = g^{(-1)}\left(\frac{\left|K_p\cdot\frac{2}{3} + K_d\cdot\left(-\frac{2}{3}\right) - 0\right|}{\frac{1}{3}\cdot\left(\left|K_p\right| + \left|K_d\right| + 1\right)}\right),$$

$poss(\,i\,max, j\,max) = poss(4,4) = 1$ (see Figure 2)

Figure 3 shows the chosen rule-consequence. The complete rule is if e is PM and $\dot{e}$ is NM and y is ZE then K_p is ZE and K_d is ZE .

st4 The inference mechanism is the GMP.

$$\frac{\begin{array}{l}\text{if} \quad \mathrm{E} \cap \dot{\mathrm{E}} \cap \mathrm{Y}(\underline{e}) \quad \text{then} \quad \mathrm{K}(\underline{k}) \\ \mathrm{E_i} \cap \dot{\mathrm{E}}_\mathrm{i} \cap \mathrm{Y_i}(\underline{e})\end{array}}{\mathrm{K_o}(\underline{k})}$$

where $\mathrm{E_i} \cap \dot{\mathrm{E}}_\mathrm{i} \cap \mathrm{Y_i}(\underline{e})$ is the really, actual FLC input.

st5 The defuzzification can be one of the generally accepted methods.

3.1 Modified Mamdani Model

The modified model differs from the one described in the previous part, in which instead of using only possibility measure based or only linguistic outputs their intersection is used. Therefore, the modified system of rules consist of if $\mathrm{E} \cap \dot{\mathrm{E}} \cap \mathrm{Y}(\underline{e})$ then $\mathrm{K}(\underline{k}) \cap \sigma(\underline{k})$ rules. Thus the inference mechanism is as follows:

$$\frac{\begin{array}{l}\text{if} \quad \mathrm{E} \cap \dot{\mathrm{E}} \cap \mathrm{Y}(\underline{e}) \quad \text{then} \quad \mathrm{K}(\underline{k}) \cap \sigma(\underline{k}) \\ \mathrm{E_i} \cap \dot{\mathrm{E}}_\mathrm{i} \cap \mathrm{Y_i}(\underline{e})\end{array}}{\mathrm{K_{poss}}(\underline{k})}$$

Example 2 For the same parameter-choice from Example 1. $\mathrm{K_{poss}}(\underline{k})$ form is shown on the Figure 4. Consequently, the linear dependence of the parameters are not used only in the rule base construction andverification but in the inference mechanism as well, thus narrowing the linguistic rule consequence. Bearing in mind that the rule output is two-dimensional, geometrically the $\mathrm{K_{poss}}(\underline{k})$ forms are more complex nevertheless, a suitable defuzzification procedure can be found.

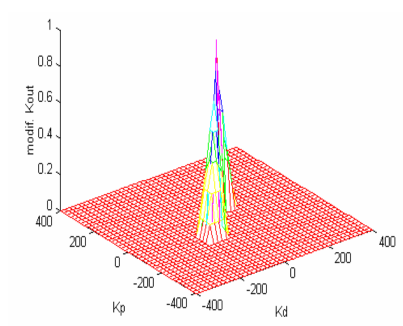

Figure 4

Conclusions

The (1) type law of the PD-type controller, as linear function relationship, was fuzzified using function-fuzzification theory. The calculation of possibility measure offers new horizons for the rule base construction and verification not only in the case of linear function relationship but also in any general function relationships. Out of the values $poss(\, i\, max,\, j\, max)$, the greatest that determined the $\mathrm{K}(\underline{k})$ domain, is in interval [0,1], and as realization measure of the given rule, it is a rule-weighing. So we obtain a narrowing linguistic rule-consequence.

Acknowledgement

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References

[1] R. R. Yager, D. P. Filev, *Essential of FuzzyModeling and Control,* Book, New York /John Wiles and Sons Inc./, 1994

[2] M. Kovács, *An optimum concept for fuzzified mathematical programming problems*, Interactive Fuzzy Optimization, Berlin-Heidelberg-New York /Springer Verlag/, 1991, pp. 36-44

[3] M. Takács, *Fuzzy control of dynamic systems based on possibility and necessity measures,* Proc., RAAD'96, 5th International Workshop on ROBOTICS IN ALPE-ADRIA-DANUBE REGION, Budapest, June,1996

[4] M. Takacs, *The Pseudooperators in Secound Oreder Control Problems*, 6th International Symposium of Hungarian Researchers on Computional Intelligence, Budapest, 2005