Contribution to Transfer Functions of Variable Structure Systems

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Abstract: The paper contains a detailed analysis of a symmetrical resonant buck (SRB) converter, the topology, typical wave forms, and simulation results are also presented. Operation boundaries are analyzed using energetic considerations as well. The mathematical model (transfer function) of SRB was determined. The results obtained reveal that the same transfer function belongs to both the continuous, and the discontinuous conduction modes.

Keywords: symmetrical resonant buck resonant DC-DC converter, injected-absorbed current method

1 Introduction

Symmetrical resonant buck, boost and buck-boost converters are known from [1, 2, 3, 4, 9]. A transformer type one belonging to this converter family has been developed in laboratories recently [6, 7, 8, 10, 12].

The most important advantage of the resonant topology is the fact, that the switch on time of the switching elements (transistors) is determined by the operation of the main circuit part of the converter and not by the control circuit. The main task of the control circuit is the alternative switching of the two switches in the primary part with a duty factor of ca. 50%.

It is typical for the circuit the resonant L and C elements.

P=const. operation is also characteristic for the circuit at a given frequency which means that the power transferred is not depending on the value of resistor R_0.

The results obtained reveal that the same transfer function belongs to both the continuous, and the discontinuous conduction modes.

2 Circuit Analysis

The full bridge resonant converter contains a well-known full bridge circuit with a supply (V_i=V1+V2) and 2 switching elements (T1, T2) as well as 2 output diodes D1, D2 and two currents choke (L=L1=L2).

The capacitor C=C1 is characteristic for the circuit which gives a resonant operation for the circuit (Fig. 1).

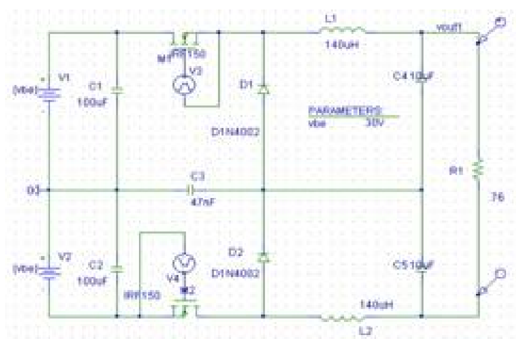

Figure 1

Symmetrical resonant buck converter

3 Operation of the Converter

This type of converters also has 2 operation modes: the Discontinuous Conduction Mode (DCM) and the Continuous Conduction Mode (CCM) depending on the currentless condition of the current choke (L).

4 DCM Operation

Investigating the steady state condition it can be supposed that the capacitor C is charged on the voltage ($-V_i$) and the inductor L has no energy.

Switching on T2 and T22, a sinusoidal current will flow through the elements $(+)V_i$ – T2–•L1 – C1 – (-)V1.

Voltage source of this resonant circuit is the $V_i/2$=V1 voltage source and the capacitor voltage (v_C=-V_i).

The maximum value of this current is:

$$I_{p\max} = \frac{V_i - V_0}{\omega_0 L} \tag{1}$$

where $\omega_0 = \frac{1}{\sqrt{LC}}$ is the natural resonant frequency of the LC circuit without transformer.

The started current decreases the voltage of the capacitor C then the capacitor charges to opposite polarity it to $+V_i$.

The natural radian frequency of the sinusoidal current (with transformer) is as follows:

The instantaneous primary current

$$i_p = \frac{V_i - V_0}{\omega_0 L} \sin(\omega_0)t \tag{2}$$

The instantaneous voltage across the current choke is

$$v_L = (V_i - V_0)\cos(\omega_0)t \tag{3}$$

The process of charging to opposite polarity will be cut short at ($t=t_\alpha$), when the voltage of the capacitor C is equal to $+V_i$.

When $t = t_\alpha$, then $v_C = V_i$, $v_p = v_{sec} = 0$, $v_L = -V_0$ by using (3) we obtain

$$\cos(\omega_0)t_\alpha = \frac{-V_0}{V_i - V_0} \tag{4}$$

and

$$\cos\alpha = \frac{V_0}{V_0 - 2V_i} \tag{5}$$

where

$$\alpha = \omega_0 t_\alpha \tag{6}$$

The energy stored in the inductor L $(W = \frac{LI_\alpha^2}{2})$ can freely come to the output. The current $i_s = i_L$ will be decreased linearly, where the slope of it is V_0/L.

The i_L current of inductor will commutate to the diode D1 and the resonant process will be cut. The magnetizing current of the current choke flows through the diodes D1 to the output (via inductor L1). The current value i_L decreasing linearly will be zero before the transistor T1 is switched on.

The current components a SRB's of the L bobbins in $T_s/2$ period can be seen in the Fig. 2.

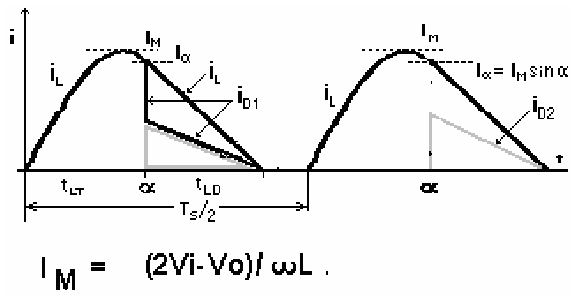

Figure 2

The currents of the L bobbin (DCM)

In the second half period of the switching period Ts, the transistor T1 will be switched ON and the whole process will be repeated in the circuit containing the diode D2.

5 Energetic Considerations

In the first half period of the switching period Ts (in the interval $0 \ldots t_\alpha$), the Vi voltage source transfers an amount of energy (energy parcel) with i_C current

$$W_{i\,T_s/2} = V_i \int_0^{t\alpha} i_C\, dt \tag{7}$$

The voltage level on the C capacitor is changing from

$-V_i$ to $+V_i$

$$\Delta V_C = \frac{1}{C}\int_0^{t\alpha} i_C\, dt = 2V_i \tag{8}$$

$$W_{i\,T_s/2} = 2CV_i^2 \tag{9}$$

In the next half period the same amount of energy comes to the output, thus

$$W_{iTs} = 4CV_i^2 \tag{10}$$

which shows, that the energy drawn by the converter is not depending on the value of the load resistor R_0.

Supposing the output voltage V_0 being constant during the period Ts (with the help of capacitor C_0), the output energy during the period Ts is the following:

$$W_{0Ts} = T_s \frac{V_0^2}{R_0} = \frac{1}{f_s} \frac{V_0^2}{R_0} \tag{11}$$

Relations (10) show that the converter provides constant power at a given frequency and the output voltage can be determined using (10) and (11).

$$V_0 = V_i \sqrt{C R_0 f_s} \tag{12}$$

The output power is

$$P_0 = 4 C V_i^2 f_s \tag{13}$$

Substituting (12) into (1) the maximum current will be

$$I_{p\,\max} = \frac{2V_i}{\omega_0 L}\left(1 - \sqrt{C R_0 f_s}\right) \tag{14}$$

Sinusoidal current (energy transfer) can only be developed, when $I_{p\,\max} > 0$

$$1 \geq \sqrt{C R_0 f_s} \tag{15}$$

Thus for $R_{0,\max}$

$$R_{0,\max} = \frac{1}{C f_s} \tag{16}$$

Using (12) and (16)

$$V_0 = V_i \sqrt{\frac{R_0}{R_{0,\max}}} \tag{17}$$

Obtain

$$V_{0,\max} = V_i \tag{18}$$

On the fs switching frequency if the parameter is R_0 we'll obtain Fig. 3.

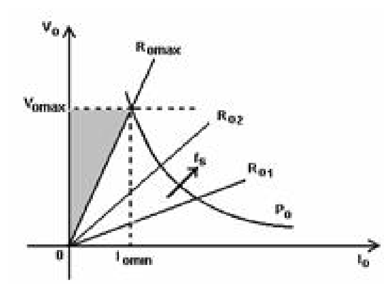

Figure 3

V_0-I_0 characteristic

6 CCM Operation

It has been proved *P=const.* in DCM (13). Though in CCM the charge current of the capacitor C has two components (15), the transferred energy amount will be constant during a switching period. This holds true because the voltage change on the capacitor C will be the same in both cases (V_i).

In Fig. 4. are shown see the i_L current and one (i_{D2}) component of it.

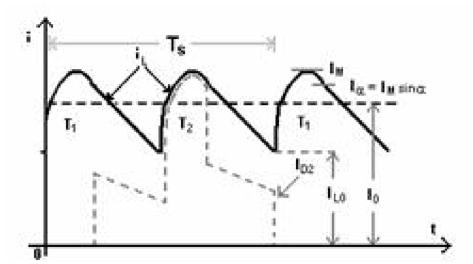

Figure 4

Current i_L in CCM

In CCM the primary current can be given by (19), where I_{L0} is the value of the i_L when the transistor switches ON.

$$i_p = \frac{V_i - V_0}{\omega_0 L} \sin\omega_0 t + I_{L0} \cos\omega_0 t \tag{19}$$

7 Mathematical Model

The injected-absorbed current method [11] will be used to the identification to the transfer function of SRB.

For this we have to determine the average current which is injected to the output.

<u>The general case (DCM or CCM)</u>:

In case of the lossless converter we can write

$$W_{iTs} = W_0 = V_0 Q_0 \tag{20}$$

where Q_0 is the output charge on the V_0 output voltage. By using (8) it is true that

$$Q_0 = \frac{4CV_i^2}{V_0} = I_0 T_s \tag{21}$$

I_0 is the average current in general case. It is

$$I_0 = 4Cf_s \frac{V_i^2}{V_0} \tag{22}$$

The average current in DCM

The average current of i_L (Fig. 2) can be written as follows:

$$I_0 = \frac{2}{T_s}\int_0^{\alpha} i_l dt + \frac{2}{T_s}\frac{1}{2} I_{\alpha} t_{LD} \tag{23}$$

where t_{LD} is the period when only D1 or D2 conducts.

$$t_{LD} = \frac{L}{V_0} I_M \sin\alpha \tag{24}$$

$$I_0 = \frac{2}{T_s \omega n} I_{LM} (1 - \cos\alpha) + \frac{2}{T_s}\frac{t_{LD}}{2} I_{LM} \sin\alpha \tag{25}$$

If we apply (23). (24), (6), (4) the result is

$$I_0 = 4Cf_s \frac{V_i^2}{V_0} \tag{26}$$

The result is in the DCM same as in the general case CCM (25, 29). Injected currents are the same the transfer function will be common in both of the two causes.

The partial differentiation of the injected current is:

$$\delta I_0 = \frac{\delta I_0}{\delta v_i}\delta v_i + \frac{\delta I_0}{v}\delta v_0 + \frac{\delta I_0}{\delta f_s} f_s \tag{27}$$

$$\frac{\delta I_0}{\delta v_i} = 8Cf_s \frac{V_i}{V_0} \tag{28}$$

$$\frac{\delta I_0}{\delta v_0} = -4Cf_s \frac{V^2{}_i}{V_0{}^2} \tag{29}$$

$$\frac{\delta I_0}{\delta f_s} = 4C \frac{V_i^2}{V_0} \tag{30}$$

The total differential of the injected current dI_0 is then

$$\delta I_0 = 8Cf_s \frac{V_i}{V_0}\delta v_i - 4Cf_s \frac{V^2{}_i}{V_0{}^2}\delta v_0 + 4C\frac{V_i^2}{V_0}\delta f_s \tag{31}$$

After taking the Laplace transform of (31), we obtain an equation for the injected current [11].

The *s* model of the R_0-C_0 output is

$$\frac{V_0(s)}{I_0(s)} = \frac{R_0}{1 + sR_0C_0} \tag{32}$$

The transfer function model of SRB is shown in Fig. 5.

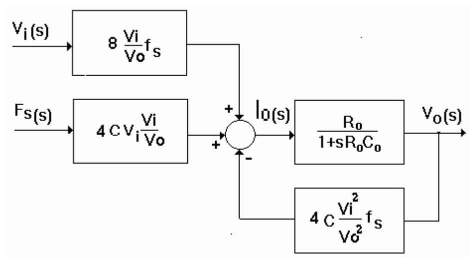

Figure 5

The transfer function model of a FBRC

8 Converter with Closed Loop Control System

Fig. 6 illustrates the block diagram for closed loop control. (where: VFO - Variable Frequency Oscillator, contr = controller)

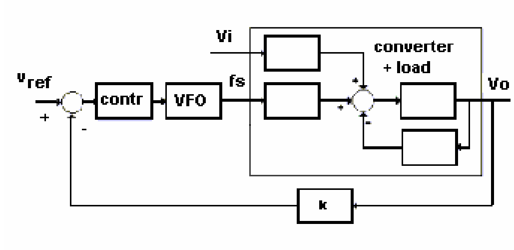

Figure 6

Block diagram of converter control system

About the control

The strategy of the way to obtain a constant output voltage (Vo=const) is pointed by Fig. 7.

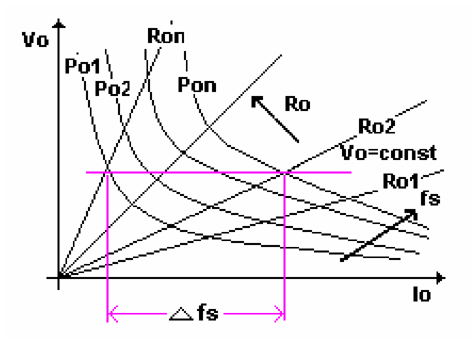

Figure 7
The strategy of the control of the output voltage

If we know the transfer function of the converter we will design the parameter of the controller. The (variable) input voltage V_i is the disturbances of this control circuit.

The presented closed loop control requires knowing the transfer function of the converter. In control circuit the complex and relative expensive VFO has to be used.

After construction of the converter, simulation was performed using SPICE and MATLAB simulation programs. The simulation results were very close to the ones which could be obtained using converter equivalent circuit.

The MATLAB realization for the transfer function of the converter (Fig. 5) can be seen on Fig. 8.

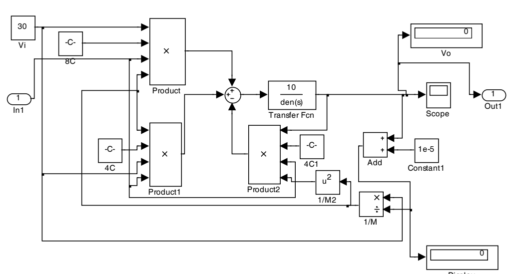

Figure 8
MATLAB-realization for transfer function

During run of the MATLAB transfer function realization, the frequency was the input signal (In1) and output voltage Vo was the output signal (Out1).

Taking into consideration the real converter data (R=10 Ω) the transfer function is as follows:

$$\frac{1.762e006}{s+5321}$$

During run the load resistor R of the converter was varied between 10 and 150 Ω and the resultant transfer values were calculated, for example in case of R=100 Ω the resultant value is as follows:

$$\frac{1.762e007}{s+5321}.$$

As results the transfer characteristics of the model can be seen in the time and frequency domain with different load resistors (R= 10, 50, 100 Ω) on Fig. 9:

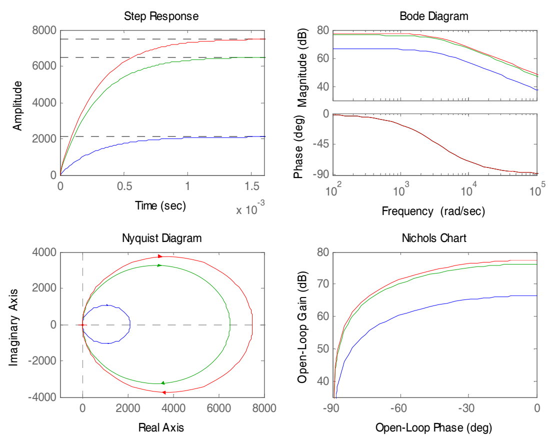

Figure 9

Transfer characteristics of the model

The operation of the converter is stable also on the frequency of some hundred of KHz which is very advantageous.

Fig. 10 shows the converter according to Fig. 6 in a closed loop control system.

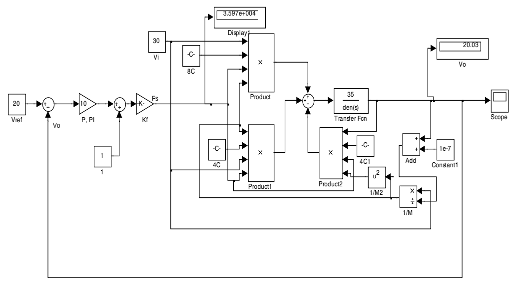

Figure 10

Converter with closed loop control system

Conclusions

A lot of SRB equations were determined. The possible modes of operation of the converter were identified. Boundary conditions for critical mode ($R_{0,max}$) were found. The converter was constructed. The designed and the simulated parameters are rather close to each other.

P=const. operation is also characteristic for the circuit at a given frequency which means the transferred power is not depend on the value of the load (resistor R_0).

Because the resonant process cuts automatically the switches can be gated with a 50% duty cycle pulse, regardless of the on-time of them.

It has been determined the transfer function model of the converter that is identical in the DCM and CCM. The closed-loop control system was blocked in.

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