A New Worst Case Lower Bound for the Width of Single Row Routing in the Unconstrained Two-Layer Model

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Abstract: We show that, for a single row routing specification with maximum density d, the difference between [d/2] and the real lower bound of the minimum width in the unconstrained two-layer routing can be as high as a positive percentage of d.

One of the simplest questions in the theory of the detailed routing of very large scale integrated circuits (VLSI) is to find the minimum width required to realize a single row routing problem. (For definitions and previous results the reader is referred to the survey [3].)

In the two-layer Manhattan model the minimum width w is known to equal the maximum density d of the specification [1] and a linear time algorithm is available to find a routing with this width.

On the other hand, in the two-layer unconstrained model even the complexity of finding the minimum width is unknown (several people believe that it is **NP**-hard). The definition of the models clearly imply $[d/2] \leq w \leq d$, where square brackets indicate upper integer part.

Simple examples like

$$1\ 2\ 3 \ldots d{-}1\ \ d \,|\, d\ \ d{-}1\ \ldots 3\ 2\ 1$$

show that the lower bound can be attained. (The vertical bar in the middle indicates that the maximum density is really d for this specification and the optimal routing is shown as the left hand side of Figure 1.)

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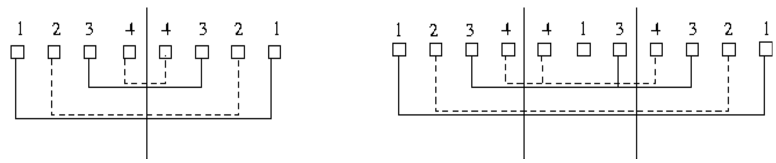

Figure 1

Csaba Megyeri observed [2] that certain examples like

$$1\ 2\ 3 \ldots d{-}1\ \ d \mid d\ \ 1\ \ d{-}1 \mid d\ \ d{-}1\ \ldots 3\ 2\ 1$$

cannot be routed with width $[d/2]$ if $d \geq 4$ (see an incomplete routing at the right hand side of Figure 1 and observe that the maximum density is attained in every column between the two vertical bars).

This construction can be extended in a straightforward way to show that, for any fixed positive integer *t,* one can find a (sufficiently long) specification so that its minimum width is at least $[d/2] + t$. However, this would not imply that the difference between the real lower bound and $[d/2]$ can be as high as a positive percentage of *d.* In this note we show that this is indeed the case.

THEOREM: Let, for simplicity, *d* be even and let $k \leq d/2 - 1$ be an arbitrary integer. Then the specification

$$1\ 2\ 3 \ldots k{+}1\ \ k{+}2\ \ldots d{-}1\ \ d \mid 1\ 2\ 3\ \ldots\ \ k{+}1 \mid k{+}2 \ldots d{-}1\ \ d\ \ 1\ 2\ 3\ \ldots k{+}1$$

cannot be routed with width $d/2 + k/2$. This shows that if c' is the best possible lower bound of form $cd \leq w$ then c' cannot be smaller than ¾.

Proof: For simplicity, let us call the area consisting of the first *d* columns (left to the first vertical bar) as the *leftmost* part and that with the last *d* columns (right to the second vertical bar) as the *rightmost* part (see Figure 2). Observe that *d* is the maximum density of the specification and it is attained in every column of the *central* part, i. e. between the two vertical bars.

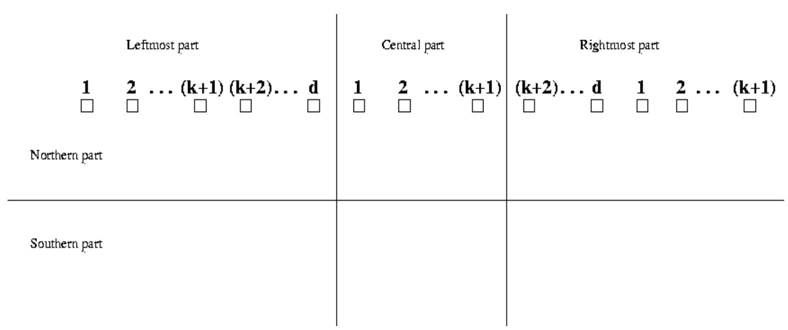

Figure 2

All the d nets have terminals both in the leftmost and in the rightmost parts. Hence, even in the „best" case (that is, if every net is routed on a single horizontal track, without „doglegs"), we need at least width $d/2$ (if every horizontal track is used by two distinct nets at the two different sides of the board, as in the left hand side of Figure 1). More generally, if a routing with width $d/2 + t$ exists then $2t$ horizontal tracks will be used by a single net only (they will be called *single* tracks) and the remaining $d/2 - t$ tracks will be used by two distinct nets (they will be called *double* tracks). If doglegs were also permitted then the number of the single tracks would be even smaller.

We may suppose, without loss of generality, that the single tracks are closer to the row of the terminals than the double tracks (see the *Northern* and the *Southern* parts of the whole area in Figure 2) since a double track l separates the tracks south of l from the terminals. Similarly, we may suppose that all the single tracks use the same side of the board, for otherwise two adjacent single tracks using different sides of the board together would lead to the same separation, only in a less area-effective way. This common side of the board will be called the *top side* and the other one the *bottom side.*

Among the d nets of the specification, $(k+1)$ consist of 3 terminals each and the remaining $d - (k+1)$ nets consist of 2 terminals each. We shall refer to them shortly as 3-*nets* and 2-*nets,* respectively. Observe that the central part of the area consists of just the columns, corresponding to the central terminals of the 3-nets. If such a central terminal is connected to its corresponding track then this track must be in the Northern part and the connection must be performed in the bottom side.

Our next observation is illustrated in Figure 3 which shows the only possible routing of the given specification

$$1 \ldots 2l \mid 1 \ldots 2l$$

with width l. The vertical line e has density $2l$ and the nets, containing its neighbouring terminals $2l$ and 1 must use the northernmost track.

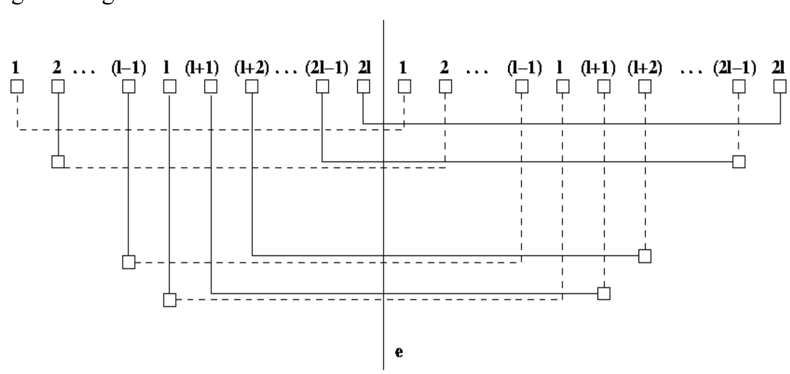

Figure 3

Since every track must be double, the northernmost one would separate terminals $2l$ and 1 from their nets in any other case. Similarly, the nets for the next two terminals $(2l-1)$ and 2 must use the second track etc.

We use indirect proof.

There are more than k ones among the 2-nets and also more than k ones among the 3-nets in this construction, hence at least one net of each group must be wired in the southern part.

We distinguish three cases according to the numbers of the terminals of those nets which are wired in the northern part.

If there are only 2-nets in this area, then the only way how the central terminals of the 3-nets can be wired is illustrated in Figure 4. In this way the wiring order of the nets is determined: we may suppose without loss of generality that we begin wiring from the leftmost part, then the numbers of the wired 2-nets are going from $(k+2)$ to $(2k+1)$ and the numbers of the 3-nets must go from $(k+1)$ to 2 (see Figure 4). Figure 4 also shows that nets 1 and $(2k+2)$ cannot be routed: They must use the northernmost track of the southern part and they can really reach it using the bottom side but there none of them can change layers.

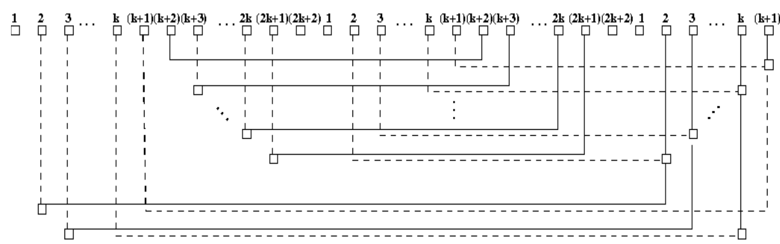

Figure 4

In the second case we use only 3-nets in the northern part, see Figure 5. The wires of these nets must change layer, otherwise the 2-nets cannot be wired. And if the northenmost track of the southern part is wired, the remaining 3-net also has to change layer, but it blocks a southern track of another net.

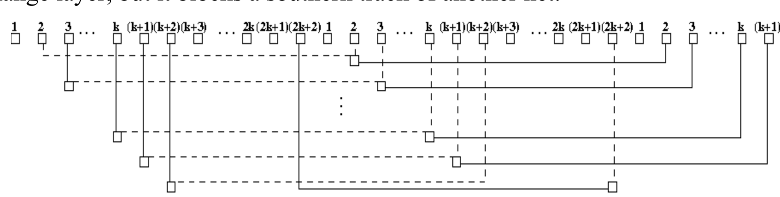

Figure 5

In the last case both 2-nets and 3-nets are allowed in the northern part, see Figure 6, but this case is reduced to the others because of the binding order of the nets.

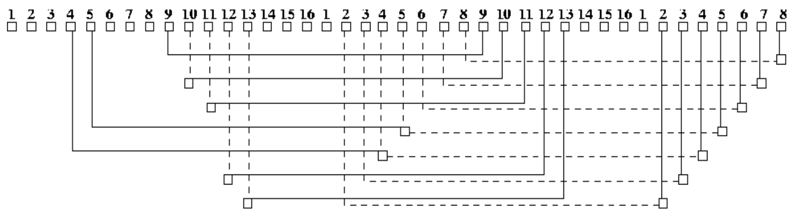

Figure 6

As a last remark let us mention that if the numberof the 3-nets and/or that of the 2-nets is less than (*k*+1) then the routing can be performed with a smaller width since every wire can be placed to the Northern part (see Figure 7).

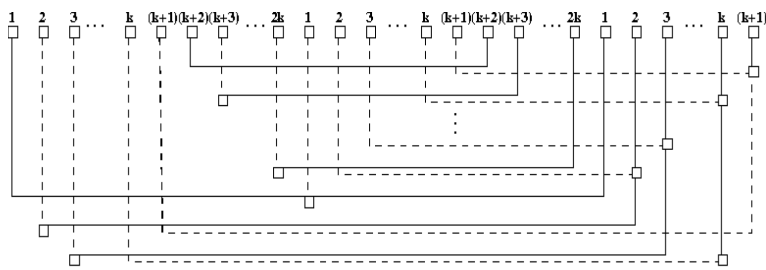

Figure 7

References

[1] T. Gallai, His unpublished results have been announced in A. Hajnal, J. Surányi: Über die Auflösung von Graphen in vollständige Teilgraphen, Ann. Univ. Sci. Budapest Eötvös Sect. Math. 1 (1958), pp. 115-123

[2] B. Golda, B. Laczay, Z. Á. Mann, Cs. Megyeri, A. Recski: Implementation of VLSI Routing Algorithms, INES 2002, Opatija, Croatia, May 26-28

[3] A. Recski: Some polynomially solvable subcases of the detailed routing problem in VLSI design, Discrete Applied Math. 115 (2001), pp. 199-208