The Pseudooperators in Second Order Control Problems

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Abstract: In the paper an approach to the tuning of the control gains of a PD controller has been outlined. The proposed fuzzy logic controller uses the functional relation between the rule premises and consequences, and the special class of pseudo-operators in the compositional rule of inference.

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1 Introduction

The input variables of a dynamic system to be controlled can be the error (*e*), which is the difference between the desired and the actual output of the system and the errorchange ($\dot{e}$). In a typical PD controller using these variables the *y* output is determined by the control law given by the equation

$$K_p e + K_d \dot{e} = y \tag{1}$$

where the control gains K_p, K_d also could be modified during the operation in order to bring the system to be controlled into a desired state. Some soft computing based techniques have been published for the on-line determination of these gains[1].

In further explanation a possible way for tuning these parameters is given, to achieve an efficient system-performance. The architecture of the proposed controller can be seen in Fig. 1. The conventional PD controller and the Fuzzy Logic Controller (FLC) use the same $e, \dot{e}$ input variables and the FLC also uses the output *y* of the PD controller (this is required because of the linear relationship in $K_p e + K_d \dot{e} = y$). The FLC gives two crisp outputs, the gains K_p, K_d, to the PD controller that calculates the new *y* by using these gains and $e, \dot{e}$ as inputs. The rules of the FLC are given in the the form of:

$$\text{if } \left(e \text{ is E} \quad \text{and} \quad \dot{e} \text{ is } \dot{\text{E}} \quad \text{and} \quad y \text{ is Y} \right) \text{ then } \left(K_p \text{ is } \text{K}_\text{p} \quad \text{and} \quad K_d \text{ is } \text{K}_\text{d} \right) \qquad (2)$$

where $\text{E}, \dot{\text{E}}, \text{Y}, \text{K}_\text{p}, \text{K}_\text{d}$ are linguistic terms, which can be for example N(negative), Z(zero), P(positive). Fuzzy membership functions cover linguistic terms. The scaling and normalization of parameters domains are made by experts.

The performance of the propose self tuning controller has been evaluated. For this purpose a second order differential equation has been chosen. The results serves to show the effects of the operators used in the rules of inference as well as the effects of the generalized t-norms.

The theoretical background of the membership functions of the linguistic terms is given, and the applied generalized t-norms and their generator functions are summarized, based on the general theoretical publication [3]. A theoretical interpretation of possibility measure of the rule realization is given using the same generator function as by definition of the membership functions and applied t-norm. The functional dependence used to determine the possibility of the rule plays a very important role. It can be used for the rule base construction, and for the inference mechanism as well by narrowing the linguistic rule consequence. With these narrowing rule consequences a modified FLC model can be constructed, where the rule consequences are surfaces above the K_p,K_d plane.

2 Background

2.1 Special Types of the Fuzzy Numbers

A fuzzy subset A of a universe of discourse X is defined as $A = \{(x, \mu(x)) | x \in X, \mu_A : X \rightarrow [0,1]\}$. Denote FX the set of all fuzzy subsets of X. The characteristic function of A will be denoted by χ_A. If the universe is $X = \Re$, and we have a membership function

$$A(x) = \begin{cases} g^{(-1)}\left(\dfrac{|x - \alpha|}{\delta} \right), & \text{if } \delta \neq 0 \\ \chi_\alpha(x), & \text{if } \delta = 0 \end{cases} \qquad (3)$$

$\alpha \in \Re, \delta \geq 0$, then the fuzzy set given by $A(x)$ will be called quasitriangular fuzzy number with the center α and width δ, and we will recall for it by QTFN(α,δ).

2.2 Pseudo-Operators

Generally details about pseudo-analysis and pseudo-operators we can read in [3], [4] and [5].

Let T: $I^2 \rightarrow I$, (I=[0,1]) be a a t-norm. The t-norm is Archimedian if and only if it admits the representation $T(a,b) = g^{-1}(g(a) + g(b))$, where the generator function $g\colon I \rightarrow \Re^+$ is continuous, strictly decreasing function, with the boundary conditions, $g(0) = 1, g(1) = 0$ and let

$$g^{(-1)}(x) = \begin{cases} g^{-1}(x) & x \in I \\ 0 & x \notin I \end{cases} \tag{4}$$

the pseudoinverse of the function g. The generalization of this representation is

$$T_{gp}(a,b) = g^{-1}\left(g^p(a) + g^p(b)\right)^{\frac{1}{p}}, \tag{5}$$

and it can be said, that the T_{gp} function is an Archimedian t-norm given by generator function g^p, $p \in [1,\infty)$.

2.3 FLC Systems

One rule in a FLC system has form: if x is $A(x)$ then y is $B(y)$, where x is the system input, y is the system output, x is $A(x)$ is the rule-premise, y is $B(y)$ is the rule-consequence. $A(x)$ and $B(y)$ are linguistic terms and they can be described by QTFN-s.

For a given input fuzzy set $A'(x)$, in a mathematical-logical sense, the output fuzzy set $B'(y)$, the model of the compositional rule of inference in the Mamdani type controller will be generated with a Generalized Modus Ponens (GMP):

$$B'(y) = \sup_{x \in X} T(T(A(x), A'(x)), B(y)) = T\left(\underbrace{\sup_{x \in X} T(A(x), A'(x))}_{DOF}, B(y)\right) \tag{6}$$

where DOF is the degree of firing value for the rule.

2.4 The Fuzzification of the Linear Equalities

The (g,p,δ) fuzzification of a linear equality $\alpha_1 x_1 + \alpha_2 x_2 + \ldots + \alpha_n x_n = \alpha_0$ by the fuzzy vector parameter $\mu_a = (\mu_1, \mu_2, \ldots \mu_n)$ (where the coefficients α_i are uncertainly parameters, and replaced by $\mu_i(\alpha_i, \delta_i)$ QTFN-s, and the fuzzification of function will be defined by T_{gp} norm), is

$$\sigma(x) = g^{(-1)}\left(\frac{|l(\alpha, x)|}{\|diag(\delta)\cdot x\|_q}\right) = g^{(-1)}\left(\frac{|l(\alpha_1 x_1 + \alpha_2 x_2 + \ldots + \alpha_n x_n - \alpha_0)|}{\|diag(\delta)\cdot x\|_q}\right) \quad (7)$$

where

$$\|(v_1, v_2, \ldots v_n)\|_p = \begin{cases} \left(\sum_{j=1}^{p} |v_j|^p\right)^{1/p} & 1 \le p < \infty \\ \max_j |v_j| & p = \infty \end{cases}, \qquad q = \begin{cases} 1 & \text{if } p = \infty \\ \infty & \text{if } p = 1 \\ \dfrac{p}{p-1} & \text{otherwise} \end{cases}$$

and diag(δ) is a diagonal matrix from elements δ_i. $\sigma(x)$ will be called *possibility measure* of equality [2].

3 FLC for the Investigated System

Let $K_p e + K_d \dot{e} = y$ the model of a dynamic problem. The model of the rule for the obtaining of the sufficient gain parameters K_p, and K_d is described in the form (2):

$$\text{if } \left(e \text{ is E and } \dot{e} \text{ is } \dot{\mathrm{E}} \text{ and } y \text{ is Y}\right) \text{ then } \left(K_p \text{ is } \mathrm{K_p} \text{ and } K_d \text{ is } \mathrm{K_d}\right).$$

From the rule (2) $e, \dot{e}, y$ quantities are fuzzified, and constitute the FLC rule-inputs. K_p,K_d are also uncertain, fuzzified but they comprise the outputs. The form of the rule is

$$\text{if } \mathrm{E} \cap \dot{\mathrm{E}} \cap \mathrm{Y}(\underline{e}) \text{ then } \mathrm{K_p} \cap \mathrm{K_d}(\underline{k}) \text{ or shortly if } \mathrm{E} \cap \dot{\mathrm{E}} \cap \mathrm{Y}(\underline{e}) \text{ then } \mathrm{K}(\underline{k}) \quad (8)$$

where the intersection of the fuzzy sets will be calculated by t-norms, and $\underline{e}, \underline{k}$ mean the vektors of the error and errorchange, and variables K_p, and K_d.

Experts can provide those $[-L_e, L_e], [-L_{\dot{e}}, L_{\dot{e}}]$ intervals where $e, \dot{e}$ quantities exist, and for the simplification and generalization of the problem these

$[-L_e, L_e],[-L_{\dot{e}}, L_{\dot{e}}]$ intervals are normalized and transformed into interval [-1,1]. During this transformation $e, \dot{e}$ receive 3-3 linguistic terms determined by (3).

These 9 possibilities would increase three times if the y quantity would be normalized and transformed likewise. It should be noted, however, that $e, \dot{e}, y$ quantities are not independent from each other. The relationship generally used by experts in such controllers was applied for selecting the rule premises. As a result of this 9 different rule-premises are obtained.

For the rule outputs also linguistic terms are defined which are obtained within the domain of K_p, K_d by transforming. The $\left[-L_{K_p}, L_{K_p}\right],\left[-L_{K_d}, L_{K_d}\right]$ intervals and the scaling are determined by experts. For the given $E, \dot{E}, Y$ the suitable K_p, K_d rule consequence are chosen based on experience meta-rules or tiresome experimental work.

In our case the K_p, K_d output fuzzy sets will be so determined that the possibility of law (1) will be the greatest.

First let us assign linguistic terms to K_p, K_d (similarly to $e, \dot{e}, y$) on the interval $\left[-L_{K_p}, L_{K_p}\right],\left[-L_{K_d}, L_{K_d}\right]$ The number of the possible K_p is K_p and K_d is K_d (i.e. $K_p \cap K_d$) rule consequence is 9.

Define the possibility measure based on (7):

$$\sigma(K_p, K_d) = g^{(-1)}\left(\frac{\left|K_p e_c + K_d \dot{e}_c - y_c\right|}{\left\|diag(\delta)\cdot[K_p, K_d, 1]^T\right\|_q}\right) \tag{9}$$

for each rule-premise. The efficient possibility measure based rule, associated with the rule (8) is defined as follows:

$$\text{if } E \cap \dot{E} \cap Y(\underline{e}) \text{ then } \sigma(K_p, K_d) \text{ or shortly if } E \cap \dot{E} \cap Y(\underline{e}) \text{ then } \sigma(\underline{k}) \tag{10}$$

In principle, any $K_p \cap K_d$ intersection can be assigned as output to the rule-premise, but in our case it is one with the greatest possibility, i.e. where

$$poss(i\,max, j\,max) = \max_{i,j}\left(\min_{\underline{k}}\left(\sigma(\underline{k}), K_{ij}(\underline{k})\right)\right), \quad i, j = 1,2,3. \tag{11}$$

is the the greatest. The suitable output is $K_{i\,max, j\,max}(\underline{k})$.
$\left(K_p \cap K_d \in \{K_{ij}, i, j = 1,2,3\}\right)$

So finally the obtained rule-base is:

if e is N & $\dot{e}$ is Z & y is N then kp is Z & kd is P

if e is N & $\dot{e}$ is P & y is Z then kp is Z & kd is Z

if e is Z & $\dot{e}$ is N & y is N then kp is P & kd is Z

if e is Z & $\dot{e}$ is Z & y is Z then kp is Z & kd is Z

if e is Z & $\dot{e}$ is P & y is P then kp is N & kd is Z

if e is P & $\dot{e}$ is N & y is N then kp is Z & kd is Z

if e is P & $\dot{e}$ is Z & y is P then kp is Z & kd is N

if e is P & $\dot{e}$ is P & y is P then kp is Z & kd is P

The inference mechanism is the generalized MP.

$$\frac{\begin{array}{l}\text{if}\quad \mathrm{E}\cap\dot{\mathrm{E}}\cap\mathrm{Y}(\underline{e})\quad \text{then}\quad \mathrm{K}(\underline{k})\\ \quad \mathrm{E_i}\cap\dot{\mathrm{E}}_\mathrm{i}\cap\mathrm{Y_i}(\underline{e})\end{array}}{\mathrm{K_o}(\underline{k})} \tag{12}$$

where $\mathrm{E_i}\cap\dot{\mathrm{E}}_\mathrm{i}\cap\mathrm{Y_i}(\underline{e})$ is the real, actual FLC input.

The defuzzification can be one of the generally accepted methods. The outputs of the *j*-th rule are $\mathrm{KP}_j^o(Kp)$ and $\mathrm{KD}_j^o(Kd)$, the rule base output is obtained by summarizing all of them:

$$\mathrm{KP}^o(K_p)=\max_{j=1,2\ldots10}\ \mathrm{KP}_j^o(K_p),\mathrm{KD}^o(K_d)=\max_{j=1,2\ldots10}\ \mathrm{KD}_j^o(K_d) \tag{13}$$

The FLC outputs after defuzzification are:

$$K_p^*=\frac{\underset{Kp}{sum}\,K_p\cdot\mathrm{KP}^o(K_p)}{\underset{Kp}{sum}\,\mathrm{KP}^o(K_p)},\ K_d^*=\frac{\underset{Kd}{sum}\,K_d\cdot\mathrm{KD}^o(K_d)}{\underset{Kp}{sum}\,\mathrm{KD}^o(K_d)} \tag{14}$$

The modified system of rules consists of if $\mathrm{E}\cap\dot{\mathrm{E}}\cap\mathrm{Y}(\underline{e})$ then $\mathrm{K}(\underline{k})\cap\sigma(\underline{k})$ rules.

Thus the output in case of one rule is as follows:

$$\frac{\begin{array}{l}\text{if}\quad \mathrm{E}\cap\dot{\mathrm{E}}\cap\mathrm{Y}(\underline{e})\quad \text{then}\quad \mathrm{K}(\underline{k})\cap\sigma(\underline{k})\\ \quad \mathrm{E_i}\cap\dot{\mathrm{E}}_\mathrm{i}\cap\mathrm{Y_i}(\underline{e})\end{array}}{\mathrm{K_{poss}}(\underline{k})} \tag{15}$$

$K_{poss}(\underline{k}) = K_{poss}(K_p, K_d)$ is a surface above the K_p,K_d plaine, and it is described with a matrix. The summarized rule base output is computed with *max* too, as in (11).

The defuzzification process is:

$$K_p^* = \frac{\underset{Kp}{sum} K_p * \mathrm{KP}^o(K_p)}{\underset{Kp}{Sum} \mathrm{KP}^o(K_p)}, \quad K_d^* = \frac{\underset{Kd}{sum} \mathrm{KD}^o(K_d) * K_d^T}{\underset{Kp}{Sum} \mathrm{KD}^o(K_d)},$$ where the $*$ operation is

multiplication of matrixes KP^o, KD^o and vectors K_p,,K_d , and the *Sum* is summa of all the elements of matrixes KP^o, Kd^o.

Conclusions

In this paper a new method for on-line determination of the gains of a PD controller by using a separate new type fuzzy logic controller is given. Based on the linearity of the control law the possibility measure of the rules of the FLC were introduced. The calculation of these possibility measures offers new horizons for the rule base construction. The proposed new FLC model restricts the Mamdani type rule consequences to possibility domain. In order to verify the performance of the proposed controller simulation has been carried out. It was concluded that in case of the application of generalized t-norms and pseudo-operators in the rules and in the inference mechanism the new method provide better performance than the conventional type controller.

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