A Novel Index to Compare the Representation Quality of Objects Approximated with Spheres

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Abstract: *Practical collision detection problems require very fast algorithms. The quality of the object representation plays a key role in the efficiency of these algorithms. In this paper a new index is presented which makes it possible to compare in terms of quality of representation different algorithms that approximate an object by means of spheres. Comparative results with other indexes are provided.*

Keywords: *Representation Quality, Collision Detection, Sphere-Tree, Quality Index*

I INTRODUCTION

Virtual environment simulation involves both steady and moving objects. If these are solid objects, they do not interpenetrate. *Collision Detection* (CD) is the process that automatically determines when and where the object intersections occur. The effects caused by the intersections are known as *Collision Response*. CD is fundamental within the fields of computer graphics, computational geometry, virtual prototyping and robotics.

In recent years many CD algorithms have been proposed, as described in the survey [1]. These algorithms have long sought to exploit the trade-off between object representation quality and computational time, the CD being a major bottleneck in interactive simulations.

The key of any CD scheme is to restrict as much as possible the number of intersection tests thereby improving its performance. In the literature this problem is tackled by multi-phase (*hybrid*) algorithms which decompose it in various phases, as specified in [2].

To restrict the surface of the object part that may collide, the strategies widely used rely upon a hierarchy of bounding volumes, defined as *Bounding Volume Hierarchies* (BVH), that better and better successively approximate the objects.

In the literature there are many different geometric surfaces (primitives) that have been used for BVH construction. The choice of the primitive type depends on the object shape to approximate and strongly influences the computational total cost, as described in [3]. Frequently a tree-like hierarchy is adopted where the most commonly primitives used to partition the object representation are spheres.

Each level of the sphere-tree can be used to generate an approximate response.
In the literature there are many different algorithms to create a hierarchical representation by spherical primitives. Among the most widely used algorithms there is *Octree* [2] that is a data structure based on a recursive partition of cubes into octants. The sphere-tree is achieved by placing the centre of each sphere in the centre of each cube. The radius of the sphere equals the distance from the cube centre to a cube vertex. Other algorithms use the *medial axis* method as a guide for the initial sphere placement [4]. This allows the spheres to be placed along the skeleton of the object, thus obtaining a tighter fitting set of spheres. The performances of the algorithms can be estimated in terms of quality of representation and computational time. Their efficiency depends on the object representation method.
In this paper a new index which evaluates the approximation quality of representation of the objects with spheres is presented. The index makes it possible to compare the sphere-tree construction algorithms in terms of quality of representation.
The index will be compared with other ones from the literature. Finally it will be used, as an example of application, to select the sphere-tree algorithm that best approximates a human ankle bone.

II RELATED WORK

In the literature many indices endeavour to define a plausible measure of the representation quality.
In [5] an index, δ, is proposed which is based on a coverage criterion, *i.e.* the representation can be described as a covering by spheres of the surface of a given object. The index δ is obtained from the ratio between the volume measurement of the outer part of the spheres (outer with respect to the object surface) used for approximating the surface itself and the object boundary surface area covered by the spheres. The index δ describes the quality of the object shape approximation. The index δ tries to capture the average error distance between the object represented by the spheres and the real object.
In [4] the *Hausdorff distance* (*Hd*) is used for a rapid and accurate measurement of the upper bound of the separation distance between two objects. The *Hd* is defined as:

$$Hd(A,B) = \max(h(A,B), h(B,A)) \quad (1)$$

where A and B are two sets of sample points and

$$h(A,B) = \max_{a\in A} \min_{b\in B} \|a-b\|$$

$$h(B,A) = \max_{b\in B} \min_{a\in A} \|a-b\|$$

where $\|\cdot\|$ is the Euclidean norm.
One of the main applications of the *Hd* is in the field of *Computer Vision*.

III INDEX *K*

The definition of the new index proposed in this study, that will be called *K*, is based on the maximum Euclidean distance between the spheres and the boundary surface of the object. The mean value of this distance defines the index of approximation quality of the object when approximated by spheres.
Some definitions and preliminary considerations will be reported in the following before finally defining the new index.
Let an object be described with a

triangular mesh and let the normal unit vector **n** to the triangular surface be directed outward the object. The object must be closed but not necessarily convex. The spheres can be created by any of the previously quoted algorithms.

For each sphere, the signed distance d from the centre of the sphere to the triangular plane that intersects the sphere itself is considered; d is positive or negative according to whether the sphere centre is outside or inside the plane.

For a better understanding of how the new index K is defined (without losing generality), the intersection between a sphere, with centre C and radius r, and the corner built from the meeting of two triangles is considered.

The assumptions are made that the corner ends cannot be in the sphere and the configurations with more than two triangles in the same corner are excluded.

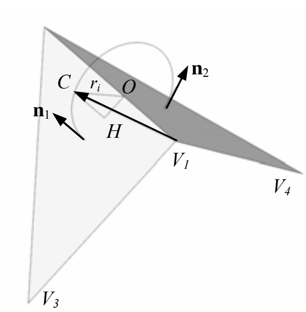


Figure 1
Corner

With reference to Fig. 1 consider all the circumferences γ_i, having radius r_i with $0 \le r_i \le r$, which belong to a plane orthogonal to the corner, and are centred at the intersection of the corner with the plane. Let π_1 and π_2 be the triangular planes, defined by the vertices V_1, V_2, V_3 and V_1, V_2, V_4 that generate the corner and $\mathbf{n}_1$, $\mathbf{n}_2$ the normal to these planes.

Imagine moving the centre C of the sphere on a generic circumference γ_i with radius r_i. In a generic position of point C, an index k_1, for the sphere with radius r, is defined as:

$$k_1 = (r + d_1) + (r + d_2) \\ = 2r + (C - V) \cdot (\overline{n}_1 + \overline{n}_2) \qquad (2)$$

where d_1 and d_2 are respectively the distance from the planes π_1 and π_2, and V is one of the two vertices that define the corner.

The k_1 index, for each value of r_i gives a clear indication of the sphere position with respect to the corner. For a distance r_i from the corner, the minimum value of k_1 represents the optimal sphere position. Figure 2 shows the value of k_1 for six increasing values $r_1, \ldots, r_6$, of r_i as a function of the angle $\alpha \equiv COH$ which defines the position of C on γ_i.

A careful inspection of Fig. 2 shows that a reduction of r_i increases the minimum value of k_1 thus defining a worse approximation of the planes π_1 and π_2.

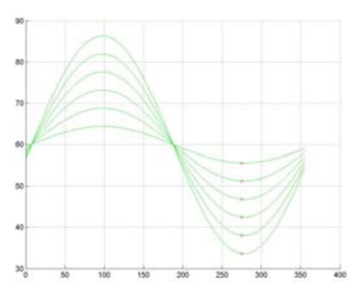


Figure 2
Parameter k_1 versus α

Equation (2) shows that another sphere exists with different radius and different centre position C but with the same value of k_1.

Hence the parameter that describes the approximation quality of the corner may be defined as:

$$k_2 = k_1 + (r - d_s) \quad (3)$$

where $d_s = \|C - O\|$.

This index depends on the position and on the dimension of the sphere, as shown in Figure 3.

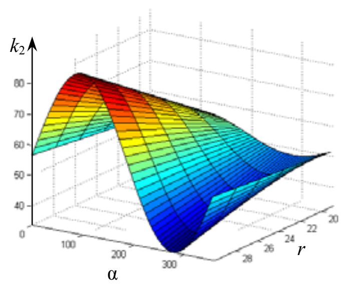


Figure 3
Parameter k_2 versus r and α

The parameter k_2 can be generalized to the case in which a vertex V is inside the sphere, *i.e.* $\|C - V\| \le r$. Since the object is closed, then the vertex V is a common point for at least three triangles. In general, if V is composed of N triangles, for a sphere with centre C and radius r, the following parameter can be defined:

$$k_3 = \frac{\sum_{i=1}^{N_{tri}} \left(r + (C - V) \cdot \bar{n}_i\right) + \left(r - \|C - V\|\right)}{N_{tri}} \quad (4)$$

where N_{tri} is the number of triangles.

In particular, in the case of intersection between a sphere and a corner, Eq. (4) becomes:

$$k_3 = \frac{\sum_{i=1}^{2} \left(r + (C - V) \cdot \bar{n}_i\right) + \left(r - \|C - O\|\right)}{2}$$

or better $k_3 = k_2 / 2$, where V, in this case, is one of the two corner extremities, and O is the point belonging to the corner so that $\|C - O\|$ is minimum. If there is intersection between the sphere and a triangular plane π then Eq. (4) becomes simpler:

$$k_3 = r + (C - V) \cdot \bar{n} = r + d_p$$

where d_p is the distance of the centre C from the plane π.

Hence, we obtain as many values of k_3 as there are intersections (Fig. 4).

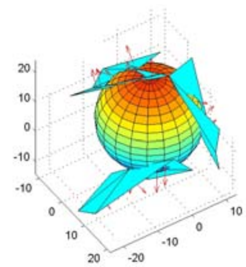


Figure 4
Multiple intersections with a sphere

Finally, as a result of the previously reported considerations, an index, K, that globally describes the quality of approximation of a generic object represented by means of spheres can then be defined as follows:

$$K = \frac{\sum_{j=1}^{N_{int}} k_{3,j}}{N_{int}} \quad (5)$$

where N_{int} is the number of intersections between the spheres and the triangular primitives and $k_{3,j}$, the index k_3 for the j-th intersection, is given by Eq. (4).

IV COMPARISON OF INDEX K WITH THE HAUSDORFF DISTANCE

The K index was validated by computing it for some simple objects that are approximated by using *Octree,* *Hubbard* and *Burst* sphere-tree generation algorithms. Afterwards the results were compared with the corresponding *Hd* value.

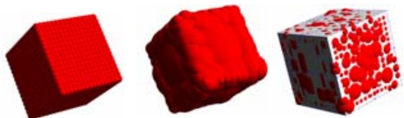


Figure 5
Sphere representation using different algorithms

Based on the algorithms previously mentioned an example of a cube approximated with spheres is shown in Fig. 5. Four different levels of approximation (from level 1 to level 4) were considered in the sphere-tree. For each level of the sphere-tree the parameter *K* was computed (Fig. 6). This parameter decreases as long as the precision of the representation increases.

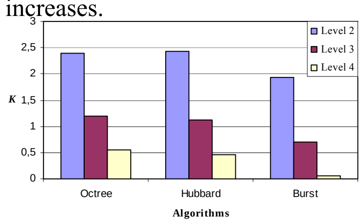


Figure 6
Parameter *K* for a cube

The particular geometry of the object shows that *Hubbard* and *Octree* algorithms are equivalent in terms of quality of representation while *Burst* provides the best results.

By using the tools proposed in [6] the *Hd*, *Min* and *Max* functions for the three considered algorithms and different levels of approximation were computed and are reported in Table 1.

Here *Min* and *Max* are respectively the minimum and maximum signed distance between the surface of the cube and the points on the spherical surface. The signed distance is negative for the points on the spherical surface inside the cube or positive for the points on the spherical surface outside the cube. A careful inspection of the results of Table 1 shows that the *Hd* computed misinterprets the approximation quality, indeed $|Min| > Max$ thus $Hd = |Min|$.

Algorithms	Level	Min	Max	Hd
Octree	2	-4,99	1,83	4,99
	3	-3,42	0,92	3,42
	4	-1,71	0,53	1,71
Hubbard	2	-4,65	3,49	4,65
	3	-3,9	2,62	3,9
	4	-4,83	1,62	4,83
Burst	2	-4,98	2,09	4,98
	3	-4,89	0,94	4,89
	4	-4,97	0,35	4,97

Table 1
Hausdorff distance *Hd* for different algorithms

Therefore the parameter to compare with *K* index is the parameter *Max* which, in fact, provides results analogous to *K* index as shown in Fig. 7. Based on the *Max* parameter the *Hubbard* algorithm would give the worst representation quality.

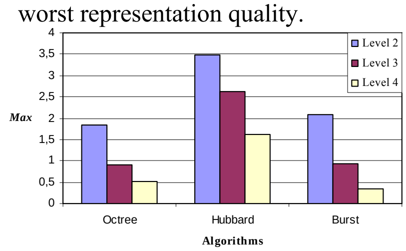
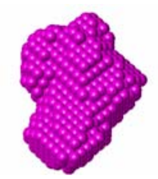


Figure 7
Parameter *Max* for a cube

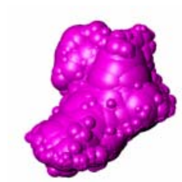
V APPLICATION

The *K* index was computed to find which algorithm among *Octree*, *Hubbard* and *Burst*, provides a better approximation of an object of complex geometry represented, for instance, by the union of the talus and the calcaneus bones of the human ankle (Fig. 8).

As shown in Fig. 9, level five of the



(a) Octree spheres



(b) Hubbard spheres

(c) Burst spheres

(d) Burst spheres with bones.

Fig. 8 – Level five of the sphere-tree algorithms

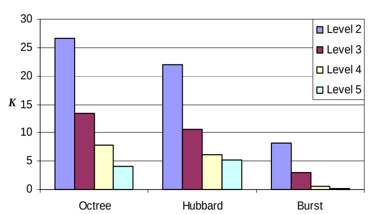


Fig. 9 – Parameter *K* for ankle-heel bone

Burst algorithm provided the best quality of representation.

The same result can be achieved using the parameter *Max* previously defined.

Conclusions

A new index, *K*, for the representation quality of objects has been presented. The proposed index *K* makes it possible to choose the sphere-tree algorithm that best approximates an object represented by means of spheres.

Vice versa, knowing the algorithm for the construction of the spheres-tree to apply to the object under study and the level of detail required by the approximation, the coefficient *K* also makes it possible to define the level of the hierarchy tree that ensures the desired representation quality.

In the light of numerous examples presented in [7], the *K* coefficient gave excellent results always confirmed by direct visual inspection. The *Hausdorff distance* is a general method but the results it provides, in case of sphere representation, need interpretation, unlike those of the simple method proposed in this paper which provide a straightforward interpretation.

Acknowledgement

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