The Effect of the Static Striebeck Friction in the Robust VS/Sliding Mode Control of a Ball-Beam System

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Abstract: *In this paper robust Variable Structure / Sliding Mode control of a 2 Degrees Of Freedom (DOF) Classical Mechanical System, a ball-beam system is considered. The control task has the interesting feature that only one of the DOFs of the system, i.e. the position of the ball is controlled via controlling the other axis, the tilting angle of the beam. This system is at least a 4th order one because it is the 4th time-derivative of the ball's position that can directly be influenced by the torque rotating the beam if the internal dynamics of the drive is neglected. For modeling the friction effect a static Striebeck Model describing stick-slip effects and viscous terms was applied. To enhance the significance of adhesion simple software block was applied when the Striebeck formula is ambiguous. The here applied robust control is based on the traditional concept of ‘error metrics’. It was found that the effect of friction can be compensated to some extent only at the costs of drastic chattering, and that the traditional chattering-elimination technique considerably degrades the precision of trajectory tracking. It is expedient to seek alternative approaches to reduce chattering especially in the present of static friction.*

Keywords: *Robust Control; Variable Structure / Sliding Mode Control; Chattering reduction; Static Friction; Striebeck Model*

I INTRODUCTION

The presence of friction is a very significant problem in the control of mechanical or mechatronic devices especially when high precision trajectory tracking is needed in the low velocity regime [1]. Precise approach of a static final position typically is such a task. Developing control approaches in which friction is treated by feedforward compensation is a plausible approach in our days. For this purpose the development of various friction models are needed [2],

[3]. However, the identification of the parameters of the various friction models is not an easy task. The situation is to some extent easier in the case of single variable (i.e. 1 degree of freedom) systems [4]. In his PhD Thesis Lőrinc Márton among other results developed a simplified 'cubistic' approximation of the static Striebeck model that can be identified offline. Márton and Lantos successfully applied this model for friction compensation [6].

In general the Variable Structure / Sliding Mode (VS/SM) controller is simple, useful practical means to realize robust controllers for very imprecisely modeled nonlinear physical systems. Normally a very approximate system model is satisfactory for meeting the requirement of approaching the switching surface during finite time. This mainly is true for continuous nonlinearities. As it will be seen by the use of a ball-beam system as a paradigm the discontinuous nonlinearities as the static friction mean serious problem for a VS/SM controller, too. In the sequel at first the system to be controlled is described in details.

II THE BALL-BEAM SYSTEM WITH STATIC FRICTION

In the 'ball-beam system' a ball can roll on the surface of a beam the tilting angle of which is driven by some actuator. The motion of the ball essentially is determined by the tilting angle and the force of gravitation. This means that even if we are in the possession of a very strong actuator, the acceleration of the ball along the beam is limited by the above two factors. Since the directly controllable quantity is the torque determining the 2nd time-derivative of the angle tilting the beam, this system acts as a 4th order system in the sense that the 4th time-derivative of the ball's position along the beam is determined by the tilting torque. The parameters of the particular system used as paradigm are as follows: the momentum of the beam Θ_{Beam}=2 kg×m^2, the mass of the ball m_{Ball}=2 kg, the radius of the ball r=0.05 m, and the gravitational acceleration is g=9.81 m/s^2. Via introducing the quantities $A=\Theta_{Beam}$, and $B=\Theta_{Ball}/r^2+m_{Ball}$, the following equations of motion are obtained:

$$\begin{aligned} A\ddot{\varphi} + m_{Ball}\, x\cos\varphi - m_{Ball}\, r\sin\varphi = 0 \\ B\ddot{x} + m_{Ball}\, g\sin\varphi = 0 \end{aligned} \qquad (1)$$

in which variable φ describes the rotation of the beam counter-clockwisely with respect to the horizontal position in *rad* units, and x in m units denotes the distance of the ball from the center of the beam where it is supported. From (1) d^2x/dt^2 can be expressed as a function of φ. Since this angle cannot be made abruptly vary, following two derivations by time d^4x/dt^4 can be expressed with $d^2\varphi/dt^2$ as follows:

$$x^{(4)} = \frac{m_{Ball}\, g}{B}\left(\sin\varphi\dot{\varphi}^2 - \cos\varphi\ddot{\varphi}\right) \qquad (2)$$

In the possession of the desired d^4x/dt^4 value, the dynamic model of the system, by the use of (2) and the 1st equation of the group (1) the necessary torque Q could be computed. Without going into details it is evident that if $\varphi\in(-\pi,\pi)$ then $\partial Q/\partial x^{(4)}<0$ that corresponds to a well defined control action according to which to increase $x^{(4)}$ Q has to be decreased, and vice versa. Normally it can be supposed that the parameters of the actual system are not precisely known.

Instead of the actual parameters some *model values* are used as $\widetilde{A}$, and $\widetilde{B}$ constructed of the model values of the other parameters. On the basis of this *rough model* at first the desired rotational acceleration of the beam is estimated as

$$\ddot{\varphi}^{Des} = \frac{\widetilde{B}x^{(4)}}{\widetilde{m}_{Ball}\, g \cos\varphi} + \tan\varphi\dot{\varphi}^2 . \qquad (3)$$

This value is then can be substituted into the 'model variant' of the 1st equation of the group (1) to calculate the necessary *actuation torque*, Q^{Act} $[N \times m]$. In the present approach the static and viscous friction is built in into the value of *actually exerted torque* Q as follows. For $\dot{\varphi} \neq 0$ this friction model yields the following torque:

$$Q = Q^{Act} \text{-} f_{vi}\dot{\varphi} \text{-} Q_c \text{-} F_s \exp\left(\text{-}\left|\dot{\varphi}\right|/c_s\right) \qquad (4)$$

in which f_{vi}=10 $[Nm/(m/s)]$ is the *Viscous Friction Coefficient*, F_c=10 [Nm] denotes the *Coulomb Friction Torque*, F_s=20 $[Nm]$ represents the *Static Friction Torque*, and c_s=0.05 $[rad/s]$ denotes the typical angular velocity value that describes the transmission between the 0 velocity and the 'high' velocity region. For $\dot{\varphi} = 0$ this model does not yield any reliable force. According to the rough model of adhesion at zero velocity the rotary axis sticks in, that is arbitrary torque smaller in absolute value than $|Q_c+Q_s|$ can be compensated without letting the axis rotate. Any motor torque exceeding this limit brings the shaft into rotation with a friction force according to (4). While more sophisticated *dynamic friction models* as the *LuGre Model* resolve this ambiguity by introducing a kind of a 'pre-sliding' (elastic deformation) model for the solid to solid contact as well as the relaxation of his deformation by a 'brush' model in which the bended bristles can be regain their original straight shape by sliding and relaxation [2], [3], in the forthcoming simulations the following approximation was applied to describe adhesion: whenever the rotational velocity of the beam's shaft has zero transition or its absolute value decreases under $v_{abslim}=10^{-4}$ *rad/s* $\dot{\varphi}$ takes the value of *0* if for the actuating torque it holds that $|Q^{Act}|<|Q_c+Q_s|$ and Q takes the value of zero. This means that the full actuating torque is consumed up by the friction during the phase of sticking. This approximate model is evidently describes some qualitative features of the stick-slip phenomena and it can be used for studying the behavior of a VS/SM controller in the presence of static friction.

III THE VS/SM CONTROLLER

In the case of *robust Variable Structure / Sliding Mode Controllers* it is a popular choice to introduce the operator $(d/dt+\lambda)^{m-1}$ and apply it to the trajectory tracking error if the order of the set of differential equations determining the state propagation is m (in our case m=4) [7]:

$$S := \left(\frac{d}{dt} + \lambda\right)^{m-1} \left[x^{Nom} - x\right], \lambda > 0 \qquad (5)$$

If S=0 then $(d/dt+\lambda)^{m-2} \to 0$ exponentially. Roughly speaking it can be stated that during the period of $\approx 2\lambda$ this quantity practically becomes 0. If this situation is achieved the term $(d/dt+\lambda)^{m-3} \to 0$ exponentially, etc. Via following this argumentation it can be expected that after finite time the tracking error $[x^{Nom}-x]$ starts to converge to zero exponentially.

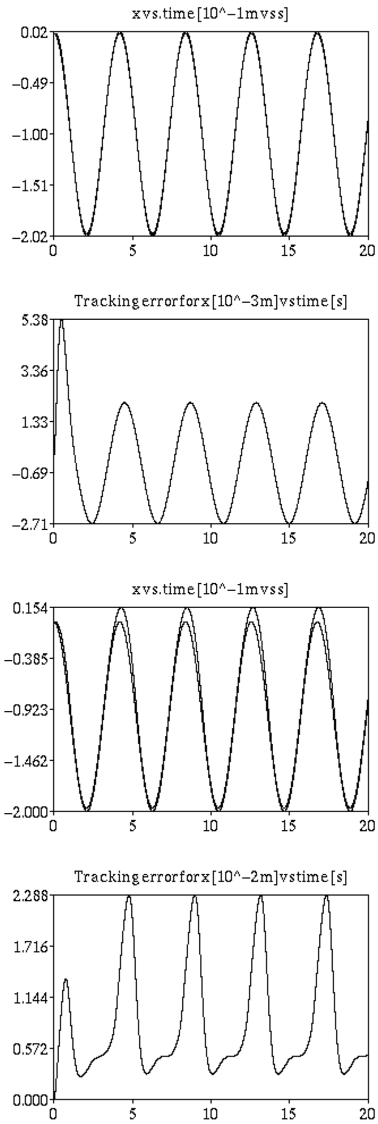

Figure 1
The trajectory tracking and tracking errors without (upper graphs) and with (lower graphs) friction (K=50 m/s^4)

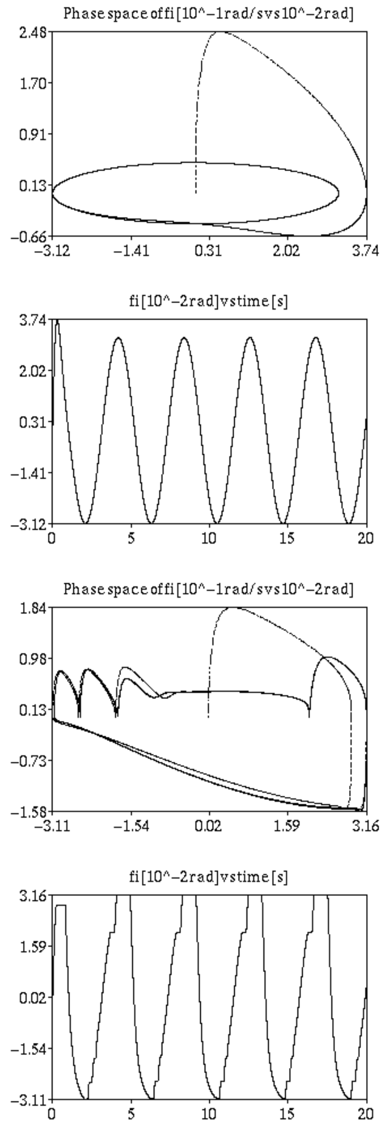

Figure 2
The phase space of the beam and the rotation angle of the beam vs. time without (upper graphs) and with (lower graphs) friction (K=50 m/s^4)

In the simulations λ=6 [s^{-1}] was used. Since by calculating the time-derivative of S in (5) $x^{(m)Des}$ can be determined, in the typical case of *Robust Controllers* an *approximate* *system model* used to be satisfactory to drive S into the vicinity of 0 during finite time. For this purpose various strategies can be described.

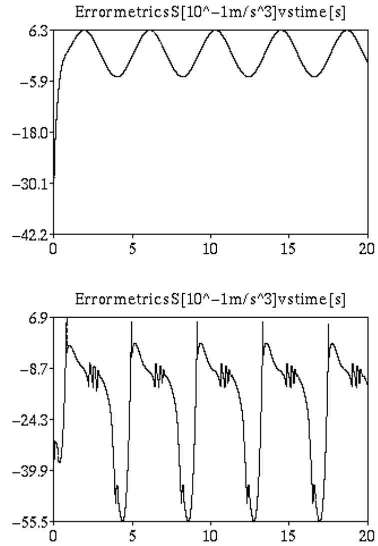

Figure 3
Variation of the Error Metrics S vs. time without (upper graph) and with (lower graph) friction (K=50 m/s^4)

In this paper we try to prescribe the error metrics relaxation

$$\dot{S} = -K \tanh(2S / w) \qquad (6)$$

in which the switching width w=3 [rad/s^3] is introduced to reduce chattering, and K=50 [m/s^4] was chosen for fast tracking error relaxation. As it can be seen from Fig. 1 in spite of the 'robustness' of the VS/SM controller the effect of the friction is quite significant. To reveal details Fig. 2 was created that reveals that the friction-free motion is accurate and free of chattering, while in the case of the presence of the friction sticking of the driving shaft as well as typical jumps in the velocity characteristic to chattering can be revealed. The effect of friction and chattering is more obvious in the variation of the Error Metrics as its is given in Fig. 3.

To increase the tracking precision

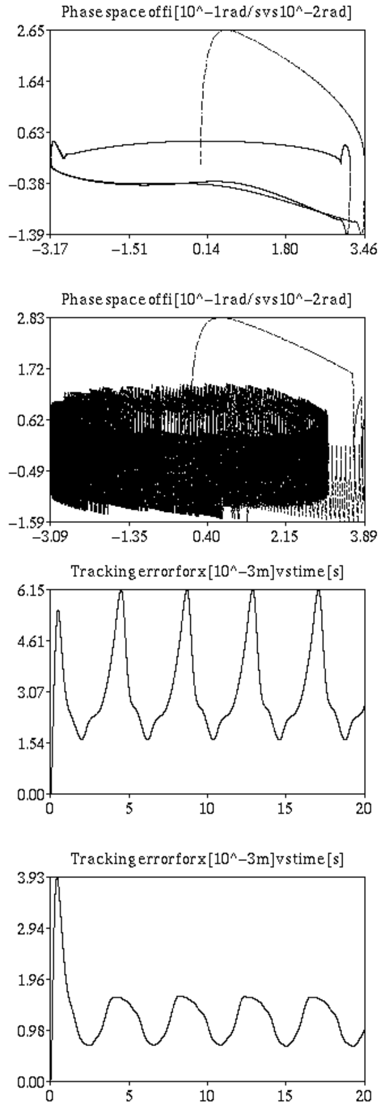

Figure 4
The phase space of the beam with and without smoothing the switching rule (upper graphs), and the trajectory tracking error (lower graphs) in the case of fast prescribed error metrics relaxation (K=80 m/s^4)

more drastic error metrics relaxation was prescribed (K=80 m/s^4). The results for a 'sharp' controller according to (7) and for the above described one with w=3 [rad/s^3] were compared to each other.

$$\dot{S} = -K \operatorname{sgn}(S) \quad (7)$$

It is evident that the traditional smoothing of the switching law is efficient against chattering but it considerably degrades the tracking efficiency (Fig. 4).

Conclusion

In this paper robust Variable Structure / Sliding Mode control of a 2 DOF Classical Mechanical System, a ball-beam system was considered for the case of considerable static and viscous friction at the driving axle. It was found that the effect of friction can be compensated to some extent only at the costs of drastic chattering, and that the traditional chattering-elimination technique considerably degrades the precision of trajectory tracking. It is expedient to seek alternative approaches to reduce chattering especially in the present of static friction.

Acknowledgement

The authors gratefully acknowledge the support by the Intergovernmental Hungarian-Romanian- S&T Co-operation Project (No. Ro-51/05 in Hungary, No. 35 ID 17 in Romania), conducted by the Research and Technology Fund in Hungary. The support by the Hungarian National Research Fund OTKA No. K063405 is kindly acknowledged, too.

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