Dynamics and Control of Two Planar Robot Manipulators Handling a Flexible Beam

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Abstract: *In this paper dynamics and control of two cooperating planar rigid robots handling a flexible beam are presented. Each robot has three revolute joints. The boundary conditions of the beam are considered as clamped-clamped model. First, kinematics and dynamics of the system and the relation between different forces acting on the object using different Jacobians are derived. To obtain the dynamic equations of motion of the object, its Lagrangian has been developed and then Lagrange's equations are derived. Second, an I-type impedance control is elaborated that causes the position and orientation of the mass center of the beam converge to their desired values while suppressing the vibration of the beam. The simulation results show the efficiency of the considered control scheme for this type of boundary conditions.*

Keywords: *Cooperating Robots, Flexible Beam, Impedance Control*

I INTRODUCTION

Dual arm manipulation of flexible objects is a complex and challenging problem and has recently attracted a lot of attention due to its potential applications in industry. Chen and Zheng studied coordinating of two grippers to handle deformable object [1]. Svinin *et a1*. [2] applied the geometrical analysis to perform the position control and vibration suppression of the flexible object. In their research, the flexible object was consisted of lumped masses and springs. Zheng *et al.* [3] examined the position control of flexible objects. Their purpose was to insert the flexible object's one end into a hole in concrete while holding the other end. Yukawa and Uchiyama dealt with the problem of handling an end of the flexible object by a robot while the other end was fixed in the wall [4]. Sun and Liu studied a more general case: handling a flexible object with an arbitrary shape [5]. Tanner and Kyriakopoulos viewed a manipulated deformable object as an underactuated mechanical system [6]. They discussed controllability and constraints issues of an important class of deformable objects being modeled by finite element. Jiang and Kohno dealt with the issues of vibration measurement and control design in order to establish flexible objects manipulating system using industrial robot arms [7]. Doulgeri and Peltekis considered a rectangular object grasped by two robot fingers with spherical end effectors that are allowed to roll along the object surface [8].

In this paper, two planar robots, each with three revolute joints, grasping and handling a flexible beam by clamped-clamped model are considered (Fig. 1). The motion of the beam is combined of its rigid body motion and vibration of the beam. The vibration of the beam is taken into account with respect to the rigid body motion and modeled by mode summation procedure where we should consider the mode shapes and natural frequencies of the clamped-clamped beam.

The proposed control for this purpose is I-type impedance that does not require any information about the vibration of the beam. The underlying idea of impedance control is to assign a prescribed dynamic behavior for a robot manipulator while its end effector is interacting with the environment. The desired performance is specified by a generalized dynamic impedance, i.e., by a complete set of linear or nonlinear second-order differential equations representing a mass-spring-damper system. Using programmable stiffness and damping matrices in the impedance model, a compromise is reached between contact force and position accuracy as a result of unexpected interaction with the handled object. Also, to achieve a desired dynamic characteristic between the manipulator and the object at the contact the inverse dynamics control is combined with impedance control, that is, the imposition of desired impedance at the end-effector level will be obtained by an inverse dynamic scheme [9].

At the end the simulation results show that the proposed dynamics and control for clamped-clamped model is a convenient choice.

II KINEMATICS

A Coordinate Frames

To analyze motion of beam elements, five principal coordinate frames are considered: $F_1$and F_2, the inertial coordinate frames of robot manipulator bases, F_{g1} and F_{g2}, the frames of grippers at the contact points with the object, and F_0, the mobile frame that is attached to rigid object and its origin is at the mass center of the beam. We will write all of the equations with respect to the reference frame F_1. The deformation of the flexible beam is modeled as relative displacement with respect to the mobile frame.

B Robots Kinematics

The kinematics of each robot can be easily developed by using rotation matrices and translation vectors of the links. We consider $r_1$and r_2 as the position and orientation vectors from reference frame to the robots grippers. If we consider J_1 and J_2 as the Jacobian matrices and q_1 and q_2 as the joint variables vectors of robot 1 and robot 2, respectively, we can write

$$\dot{r}_1 = J_1\dot{q}_1,\ \dot{r}_2 = J_2\dot{q}_2 \qquad (1)$$

Assembling equations (1) we can write

$$\dot{r} = J\dot{q} \qquad (2)$$

C Beam Kinematics

We consider a beam with length l, moment inertia I_o and mass $m_o = \rho A l$ where ρ and A denote the mass density and area of cross section of the beam respectively.

For any point on the beam, the transverse deflection $w(s,t)$ and the slope $w'(s,t)$ are approximated by finite series of assumed modes,

$$w(s,t) = \sum_{k=1}^{m} \Phi_k(s)\xi_k(t)$$

$$w'(s,t) = \sum_{k=1}^{m} \Phi'_k(s)\xi_k(t) \quad (3)$$

where s denotes the normalized curvilinear coordinate of the rigid beam in the range of 0 to l. $\Phi_k(s)$ is the *k*th mode shape of the flexible beam corresponding to its boundary conditions and $\Phi'_k(s)$ is its derivative with respect to s. $\xi_k(t)$ is the *k*th generalized coordinate representing the contribution of the *k*th mode shape in the flexible motion. We consider w_1 and w_2 as the deformation of the beam at the two ends (s=0, l) and w'_1 and w'_2 the corresponding slops. For the clamped-clamped model we have

$w_1 = w_2 = 0$ and $w'_1 = w'_2 = 0$

The position vector from reference frame to a point on the beam is considered as $p = [x \quad y]^T$. The kinematics equations of a beam point position x and y, can be written as

$$x = x_o + v\cos(\theta) - w\sin(\theta) \quad (4)$$

$$y = y_o + vs_\theta + wc_\theta \quad (5)$$

where $p_o = [x_o \quad y_o]^T$ shows the position of rigid body motion of the mass center of the beam and $v = ls - l/2$.

Considering $r_o = [p_o \quad \theta]^T$ as the position and orientation of the rigid body motion of mass center of the object, under clamped-clamped boundary conditions we can write

$$r_1 = r_o - [\frac{l}{2}c_\theta \quad \frac{l}{2}s_\theta \quad 0]^T \quad (6)$$

$$r_2 = r_o + [\frac{l}{2}c_\theta \quad \frac{l}{2}s_\theta \quad \pi]^T,$$

where

$c_\theta = \cos(\theta)$ and $s_\theta = \sin(\theta)$.

Differentiating equation (6) we can write

$$\dot{r} = \begin{bmatrix} R'(\theta) & 0 \end{bmatrix} \begin{bmatrix} \dot{r}_o \\ \dot{\xi} \end{bmatrix} = R(\theta) \begin{bmatrix} \dot{r}_o \\ \dot{\xi} \end{bmatrix}, \quad (7)$$

where $R'(\theta)$ is a 6×3 matrix and $R(\theta)$ is a $6 \times (3+m)$ one.

III DYNAMICS

A Robots Dynamics

The dynamic equation of robot *i* can be written as

$$M_i(q_i)\ddot{q}_i + C(q_i, \dot{q}_i)\dot{q}_i + G_i(q_i) = u_i + J_i^T f_i \quad (8)$$

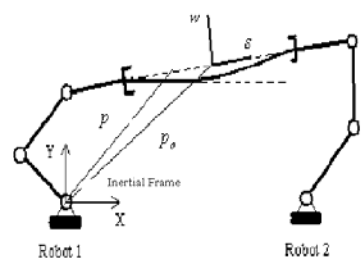

Figure 1
Two robot manipulators handling a flexible beam

where $M_i(q_i)$, $C(q_i, \dot{q}_i)$ and $G_i(q_i)$ denote the inertia matrix, Coriolis effects matrix and gravitational vector of robot *i*. u_i and f_i are the applied torque vector at joints of robot i and interaction force between robot i and the beam.

Assembling dynamic equations of two robots we can write

$$M(q)\ddot{q}+C(q,\dot{q})\dot{q}+G(q)=u+J^T f \quad (9)$$

B Beam Dynamics

To obtain beam dynamics we use Lagrange's method. For this, first, we should drive kinetic and potential energies of the beam under rigid body motion and vibration of the elements of the beam. The kinetic energy T of the beam can be written as

$$T=\frac{1}{2}\int_{-l/2}^{l/2}\rho A(\dot{x}^2+\dot{y}^2)dv \quad (10)$$

Differentiating equations (4) and (5) and substituting in equation (10), we can write

$$T=\frac{1}{2}\dot{r}_o^T M_o(\theta,\xi)\dot{r}_o \quad (11)$$
$$+\frac{1}{2}\dot{\xi}^T M_q\dot{\xi}+\dot{r}_o^T W(\theta)\dot{\xi}$$

where

$$M_o=\begin{bmatrix} m_o & 0 & -c_\theta\alpha\xi \\ 0 & m_o & -s_\theta\alpha\xi \\ -c_\theta\alpha\xi & -s_\theta\alpha\xi & I_o+\xi^T M_q\xi \end{bmatrix}$$

$$M_q=diag\left[\int_0^l \rho A[\Phi_k(s)]^2 ds\right] \quad (12)$$

$$W=[-s_\theta\alpha^T \quad c_\theta\alpha^T \quad \beta^T]^T$$

in which

$$\alpha=\int_0^l \rho A\Phi(s)ds \quad (13)$$

$$\beta=\int_0^l \rho A(s-l/2)\Phi(s)ds \quad (14)$$

The elastic potential energy of the beam is defined as

$$U=\frac{1}{2}\int_{-l/2}^{l/2} EI(\frac{\partial^2 w}{\partial s^2})^2 ds=\frac{1}{2}\xi^T K\xi\,, \quad (15)$$

where

$$K=dig\left[\int_0^l EI[\Phi_k'']^2 ds\right], \quad (16)$$

and *EI* is the beam bending stiffness.

Applying Lagrange equation and considering $X=[r_o^T \quad \xi^T]^T$, the equation of motion of the beam can be written as

$$A\ddot{X}+B\dot{X}+CX=-R^T(\theta)f\,, \quad (17)$$

where

$$A=\begin{bmatrix} M_o & W \\ W^T & M_q \end{bmatrix} \quad C=\begin{bmatrix} 0 & 0 \\ 0 & K \end{bmatrix}$$

$$B=\begin{bmatrix} \dot{M}_o-\frac{1}{2}\dot{r}_o^T\frac{\partial M_o}{\partial r_o} & \dot{W}-\dot{r}_o^T\frac{\partial W}{\partial r_o} \\ \dot{W}^T-\frac{1}{2}\dot{r}_o^T\frac{\partial M_o}{\partial \xi} & 0 \end{bmatrix} \quad (18)$$

IV IMPEDANCE CONTROL

One may decompose the forces and moments at the contact points to internal and external forces. External forces f_e are those that contribute to rigid motion of the beam and internal forces f_I contribute only to vibration of the beam. f_i should be decomposed to internal and external forces as $f_i=f_{I_i}+f_{ei}$. Since f_e has no contribution to the rigid body motion it can be written as

$$f_{e_i}=-R_i^{-T}c_i(t)\sum_{i=1}^{2}R_i^T f_i \quad (19)$$

and $f_e=[f_{e_1}^T \quad f_{e_2}^T]^T$ where $c_i(t)$ is the distribution factor.

To achieve a desired dynamic characteristic for the interaction between the manipulator and the object at the contact points, we should consider two steps. The first step decouples and linearizes the closed-loop dynamics in task space coordinates using inverse dynamics algorithm [9]. In the second step, the desired impedance model that dynamically balances contact forces at the manipulator end effector is chosen. For the second step,

$$f = f_e + f_I \qquad (20)$$

We let f_e satisfy

$$f_e = M_e \ddot{r} + K_v \dot{r} + K_p \Delta r \qquad (21)$$

where M_e, C_e and K_e are the desired inertia, damping and stiffness matrices, and $\Delta r = r - r_d$ where r_d is the desired position of the end-effectors corresponding to the desired position of the mass center of the beam. We let f_I be an I-type force feedback as:

$$f_I = k\lambda_d - kK_f \int_0^t \Delta\lambda dt \qquad (22)$$

where λ_d is the desired internal force magnitude, K_f is positive definite matrix, $\Delta\lambda = \lambda - \lambda_d$ and $\lambda = k\left(f - \left[f_{e_1}^T \quad f_{e_2}^T\right]^T\right)$ in which $k = \left[c_\theta \quad -s_\theta \quad 0 \quad -c_\theta \quad -c_\theta \quad 0\right]$ and f_{e_i} is derived from (19).

Considering equations (21) and (22), f is written as

$$f = f_e + f_I = M_e \ddot{r} + K_v \dot{r} + K_p \Delta r + k\lambda_d - kK_f \int_0^t \Delta\lambda dt \qquad (23)$$

Solving equation (23) for $\ddot{r}$ and substituting into equation (9), the control low is derived as

$$u = M(q) M_e^{-1} \left(f - k\lambda_d + kK_f \int_0^t \Delta\lambda dt - K_v \dot{r} - K_p \Delta r \right) + C(q, \dot{q})\dot{q} + G(q) - J^T f \qquad (24)$$

V SIMULATION RESULTS

In the simulation, we have considered two planar robots, each with three revolute joints. These two robots are handling a flexible beam that moves from initial position and orientation of $r_o = [2.2 \quad 1.7 \quad 0]$ to the desired position and orientation of $r_d = [2.32 \quad 1.35 \quad .2]$. The area cross section and length of the beam are (.001 m^2) and (1.5 *m*), respectively. The elastic modulus and density of the beam are (100 *GPa*) and (30000 *kg/m*). Each link of the robots has weight of (1.5 *kg*) and length of (1 *m*). The initial conditions of the deflection of the beam and its time derivative have been considered to be zero. The controller gains are considered as $M_e = I_{6\times6}$, $K_v = 48 I_{6\times6}$, $K_p = 200 I_{6\times6}$.

Figure 2 shows the errors of the rigid motion vs. time where first two curves represent the position error and the right curve shows the orientation error. Also, Figure 3 shows the flexible coordinates of the beam vs. time. It can be seen from Figures 2 and 3 that the rigid body motion errors as well as the flexible coordinates converge to zero.

Conclusion

This paper presented an approach for transporting a flexible beam by two manipulators to a desired position/ orientation while suppressing its vibration and controlling the internal forces. The model of clamped-clamped boundary conditions was discussed. The vibration of the beam was modeled by using 6 assumed modes. A hybrid impedance control algorithm was utilized by combining impedance control and an I-type force feedback into one scheme. Simulations results demonstrate the usefulness and efficiency of the proposed method.

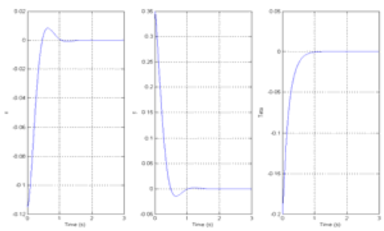

Figure 2
Errors of the rigid body coordinates vs. time

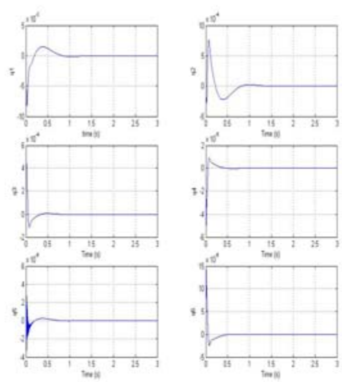

Figure 3
The flexible coordinates vs.time

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