Time-optimal motion planning for robots

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A new aspect of robot using

Robots will never replace humans
Cannibal:

• „Robots will never replace humans”

• A new research topic: Is a robot with time-optimal motion tastier?
Content

• Introduction
• Robot Motion Planning
• Time-Optimal Cruising Trajectory Planning
• Time-Optimal PTP Motions
• Realization
• Applications
• Conclusions
Introduction

The Lagrange equation of the robot motion

\[ H(\mathbf{q}) \dot{\mathbf{q}} + h(\mathbf{q}, \mathbf{q}) = \tau \]
Introduction

- Path planning
- Trajectory planning
- Trajectory tracking
Robot Motion Planning

State-of-the-Art
Robot Motion Planning

State-of-the –Art

[Google search results for "robot motion planning khatib"]

Springer Handbook of Robotics - Google Books eredmény
Bruno Siciliano, Oussama Khatib - 2000 - Computers - 1611 oldal
These systems were based on seminal ideas which have been very fruitful in the development of robot motion planning algorithms. For instance, in 1983, ... books.google.hu/books?id=3y4023g67X...

Főfórmátum: PDF/Adobe Acrobat - Gyorsnézet
previously used for robot path-planning (Khatib, O., ... problem of multi-robot motion planning is tested through simulation tests ... intechweb.org/downoadpdf.php?id=4282 - Hazúzd

CiteSeerX — Decomposition-based motion planning: Towards real-time ... - [Oldal lefordítása]
It is O Brock - 2000 - időzetei száma: 4 - Kapcsolódó cikkek
1655, Robot Motion Planning – Lotombe – 1991, 856, Real-time obstacle avoidance for manipulators and mobile robots; 'Int – Khatib – 1986, ...
citeseerx.ist.psu.edu/viewdoc/summary?doi=10.1.1... - Tájékoztató

Oliver Brock - Homepage - [Oldal lefordítása]
@ Nov 2001 ... Brock, Oliver and Oussama Khatib. Integrated Planning and Execution: ... Generation of Robot Motion: Integrating Planning and Execution.

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Robot Motion Planning

State-of-the –Art
Book review


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ABSTRACT

The textbook on motion planning “Principles of Robot Motion: Theory, Algorithms, and Implementations”, by H. Choset et al., MIT Press, which appeared in June 2005, is reviewed and compared with two other textbooks on the same subject, from 1991 and 2006, respectively. The ground-breaking developments over the last decade justify the necessity of the newer textbooks, which appear to be complementary, despite some overlap in the contents.

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1. Overview

**Note:** The text continues with more details and analysis related to robot motion planning and the comparison of textbooks.
Robot Motion Planning

- H. Choset, K.M. Lynch, S. Hutchinson, G.A. Kantor,


Time-Optimal Cruising Trajectory Planning
Trajectory planning (methods used)


Trajectory planning


- At this approach, the coordinates of a series of points in Cartesian coordinate system are given. The corresponding joint coordinates values are determined by inverse transformation. If the joint positions, speeds and possibly accelerations (deceleration) are known in the given points (and, also, the desired time of motion from point to point), the paths for joints satisfying the given conditions can be determined using proper-order splines.
Trajectory planning

- For example, if in two points the joint coordinates and speed values are given,
  \[ q_i(t_i), \dot{q}_i(t_i), q_i(t_{i+1}), \dot{q}_i(t_{i+1}) \]
- a third-order spline
  \[ q_i(t) = a_{0i} + a_{1i}t + a_{2i}t^2 + a_{3i}t^3 \]
- may be used for path determination of the motion. Because
  the parameter values can be determined from the 4 equations obtained at
  \( t=t_i \) and \( t=t_{i+1} \).

- For Path Motions:
  - Interpolating Polinomials with Velocity Constraints at Path Points
  - are proposed
Time-Optimal Trajectory Planning


Using PMP (Pontryagin Maximum Principle)
The results are very nice but not very simple
Time-Optimal Trajectory Planning
I am not an aeroplane or a spacecraft
Indeed, who I am!!!!!!
O.K.

• You are a good boy !!!!!

• You should behave well !!!!!
Time-Optimal Cruising Trajectory Planning

Motion on a path

- **A₀** to **A**: Run in
- **B₀** to **B**: Run out
- **A₀** to **B**: Cruising part
- **A₀** to Initial point
- **B** to Final point
- **A₀** to **B₀**: Transient part

Symbols:
- **v_A**: Velocity of point A
- **v_B**: Velocity of point B
Time-Optimal Cruising Trajectory Planning


- Features:

  - The use of Parametric Method (Shin, McKay 1983-86)
  - Use of the limit velocities of the joints
Time-Optimal Cruising Trajectory Planning

Polar manipulator kinematics

\[ \Delta \lambda_i = \sqrt{(x_{i+1} - x_i)^2 + (y_{i+1} - y_i)^2} \]

\[ \alpha_i = \arctan \left( \frac{y_{i+1} - y_i}{x_{i+1} - x_i} \right) \]
Time-Optimal Cruising Trajectory Planning

Direct and Inverse Geometry

\[ x = q_2 \cos q_1 \]
\[ y = q_2 \sin q_1 \]
\[ q_1 = \arctan \frac{y}{x} \]
\[ q_2 = \sqrt{x^2 + y^2} \]
Time-Optimal Cruising Trajectory Planning

Parametric Method and Projections of \( \|v\| \)

\[
\dot{x} = |v| \cos \alpha_i \\
\dot{y} = |v| \sin \alpha_i \\
x = \lambda \cos \alpha_i \\
y = \lambda \sin \alpha_i
\]
Time-Optimal Cruising Trajectory Planning

Polar manipulator kinematics

\[
\begin{align*}
    \dot{x} &= -q_2 \sin(q_1) \dot{q}_1 + \cos(q_1) \dot{q}_2 \\
    \dot{y} &= q_2 \cos(q_1) \dot{q}_1 + \sin(q_1) \dot{q}_2 \\
    \dot{q}_1 &= \frac{xx + yy}{\sqrt{x^2 + y^2}} \\
    \dot{q}_2 &= \frac{xy - yx}{x^2 + y^2}
\end{align*}
\]
Time-Optimal Cruising Trajectory Planning

Time-optimal cruising trajectory planning (The dominant joint conception)

\[
\begin{align*}
\dot{q}_{1\text{max}} &= S_1(...) |v|_{1\text{max}} \\
\dot{q}_{2\text{max}} &= S_2(...) |v|_{2\text{max}} \\
|v|_{opt} &= \text{Min}(|v|_{1\text{max}}, |v|_{2\text{max}})
\end{align*}
\]
Time-Optimal Cruising Trajectory Planning

The velocity (length) diagram

\[ |v| = \dot{\lambda} \]
Time-Optimal Cruising Trajectory Planning

- We get the joint coordinates required values as the function of parameter (path length)

\[ q_i = f_i(\lambda) \]

\[ i = 1, 2 \]

- We need them as time functions. So we apply

\[ t = \int_{0}^{L} \lambda dt \]
Time-Optimal Cruising Trajectory Planning

We get the inputs for the drives realizing the required motion

\[ q_i = g_i(t) \]

\[ i = 1, 2 \]
Example

Figure A1
Polar manipulator and given path

\[ [AB] = 0.65 \text{ m} \]
\[ R = 0.5 \text{ m} \]
\[ \alpha = 157.5^\circ \]
\[ \beta = 100^\circ \]
\[ \gamma = 100^\circ \]

\[ \lambda = -0.3, -0.2, -0.1, 0, 0.1, 0.2, 0.3, 0.4 \]
\[ t_B = 4.69 [s]; t_C = 11.69 [s] \]
\[ [v]_{\text{max}} = 0.349 [\text{m/s}] \]

Time-optimal cruising velocity

1st joint velocity
2nd joint velocity

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Time-Optimal Cruising Trajectory Planning

General Method

\[ q_i = f_i(\lambda) \]

\[
\frac{dq_i}{dt} = \frac{\partial f_i}{\partial \lambda} \frac{d\lambda}{dt}
\]

\[
\left( \frac{d\lambda}{dt} \right)_{i_{\text{max}}} = \left| \frac{\partial f_i}{\partial \lambda} \right|_{i_{\text{max}}} = \frac{\dot{q}_{i_{\text{max}}}}{\partial f_i} \frac{\partial \lambda}{\partial \lambda}
\]

\[
\left| \nu \right|_{\text{opt}} = \text{Min} \left\{ \left| \nu \right|_{i_{\text{max}}} \right\}
\]
Time-Optimal PTP Motions

It is not the shortest way
Time-Optimal PTP Motions

Computing the minimum time \((i=1,2)\)

\[
t_{1\text{min}} = \frac{\Delta q_1}{q_{1\text{max}}} \quad \text{and} \quad t_{2\text{min}} = \frac{\Delta q_2}{q_{2\text{max}}}
\]

\[
t_{\text{min}} = \text{Max}(t_{i\text{min}})
\]
Time-Optimal PTP Motions

• Every realizable trajectory in the „optimal domain“ results minimum time motion

• The method is easily applicable to higher D problems
Realization

• It is possible to realize the optimal cruising trajectory planning results at many application task. The constraints of robot control constraint the effectiveness. But it still may be very high.

• The idea can be applied to full extent by using open system architecture robot control devices (see: Sokolov A.G.,”Optimal Computer Control of Redundant Robots” PhD Dissertation. Budapest University of Technology. 2000.
The structure of closed-loop position control.
Application example; the Corsettes
DSF (Dieless Sheet Forming)

Images showing examples of DSF process:

a) Equipment setup for DSF

b) Diagram showing moving forming tool and supporting tools

c) Example of DSF process in action

d) Final product formed using DSF
3D human scanning

Serial communication

Ethernet connection

Wireless (blue tooth) connection
KUKA KR6 Robot
Experimental system

• Fixture

• Tool

• Process
Tool

3 D head-motion pressing tool
Fixture
Process
Results of the experiments 1
Results of the experiments 2 (Trough making)
DSF

- The key problem to be solved is: to use small depths, very high speeds
Conclusions 1.

• Proposals in the paper give a general and simple to-realize method for time-optimal cruising trajectory planning for industrial robots. The proposed approach is based on the parametric method. All the parameters which are needed for the application (for example, joint velocities limit values) are easily available. The basic relations reflecting the essence of the approach are given by Equations (3.29) (3.32) of the paper.
Conclusions 2.

• Then determining the $\dot{i}(\lambda)$ function and from that the $t = t(\lambda)$ relations, the joint drives inputs may readily be determined and consequently the time-optimal motion may be realized. A slightly different but in the spirit close method can be used for free paths.

• The time-optimal trajectory planning method provides plenty of information for application planning. Existing applications’ time needs may be shortened, and new applications may be developed with outstanding characteristics.
Conclusions 3

• Topics for Future Research
  1. New applications and details
  2. Velocity-length diagrams tayloring
  3. Comparison with time-optimal sequential motions
  4. Etc.