# **Error Correction of Co-ordinate Measuring Machines**

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**Abstract:** The paper presents a rigid body kinematic model of the co-ordinate measuring machin. The individual error components are described by means of transformation matrices and the resulting global error is given in the form of twisted space curves. A new techniques for measuring these error components is described. Using these parameters, an error compensation program has been implemented, which has been applied to a suitable three coordinate measuring machine. The results of this research are presented in the subsequent sections.

*Keywords: Coordinate Measuring Machines, volumetric error compensation, volumetric measuring accuracy, polynomial fitting* 

## **1** Introduction

Co-ordinate metrology has now become a firmly established technique in industry. The universal applicability and high degree of automation accounts for the succes of co-ordinate metrology in the last 20 years. The measuring of complex freeform surface would be unthinkable without coordinate measuring machines. The workpiece geometrical feaures (forms and sizes) are determined from the relative co-ordinates of characteristic surface points. The accuracy of these measurements depends on the mechanical properties of the co-ordinate measuring machine, the measuring system, the probe, the drive and control system, and the software used to determine the geometric features. The environmental conditions and the selected measuring strategy also largly influence the accuracy of the results.

Fortunaly, we now have the capability to electronically compensate for a large portion of the measuring deviations. After manufacture, the co-ordinate measuring machine can be "mapped" for deviations with standardized instruments. These errors are then accounted for supplying a computer intern mathematical model for that particular machine. The software runs in real time to compensate for the deviations. However error compensation still depends on the stability of the structure. In this paper we will present a general model describing the error components and the resulting volumetric error of the co-ordinate measuring machine.

#### 2 The mathematical model

A linear stage of precision machinery is expected to travel along a straight line and stop at a predefined position. However in the practiced the actual path deviate from the straight line due to the geometric errors of the guideways and it results also in angular errors as it is given in Fig. 1.



Fig.1. Representation of the error components of a linear stage (abstract and actual carriage)

For each axis a transformation matrix can be used to describe in homogenious coordinates the the deviations from the ideal motion. Neglecting the second order angular error terms we get for the x-axis:

$$dT(x) = \begin{bmatrix} 1 & -\theta_{z}(x) & \theta_{y}(x) & d_{x}(x) \\ \theta_{z}(x) & 1 & -\theta_{x}(x) & d_{y}(x) \\ -\theta_{y}(x) & \theta_{x}(x) & 1 & d_{z}(x) \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Analog results can be derived for the y and z axis. Note that only the translational kinematic components are described in the form of homogenious transformations, but the rotational components can presented on a similar way.

A traditional co-ordinate measuring machine consists of three translational components x, y and z, and a probe is attached to the end of the z component. Usually the probe can be considered as a constant translational transformation.

Performing distance measurement in the measuring volume of the co-ordinate measuring machine we observe the relative change of the probe tip, therefore only the relative deviations are of interest.

They are listed below:

$$\begin{split} \Delta x &= d_x(x) + d_x(y) + d_x(z) - y \cdot \delta \theta_z(x) + z [\delta \theta_y(x) + \delta \theta_y(y)] \\ \Delta y &= d_y(x) + d_y(y) + d_y(z) + \alpha \cdot x + z [\delta \theta_x(x) + \delta \theta_x(y)] \\ \Delta z &= d_z(x) + d_z(y) + d_z(z) + \beta \cdot x + y [\gamma + \delta \theta_x(x)] \end{split}$$

The resulting 21 geometrical parameters are now defined relative to the measuring frame. Also the actual out of straightness  $\delta$  and the out of squareness parameters  $\alpha$ ,  $\beta$ ,  $\gamma$  are introduced.

Currently the application of the single plane stages gain more and more importance. In that case a modified version of the above model should be used. This model and corresponding error correction algorithm will be presented in a separate paper.

#### **3** Measurement methode and techniques

In Fig. 2. one can see the setup for the measurement of the six error components of a moving stage.

The stationary part consists of a laser head with three parallel laser sources, a beam splitter and two quadrant photo detectors. The moving part consists of three retroreflectors. As laser heads single and double beam interferometers are used. In order to minimize the cosine errors the beams are adjusted accuratly before the measurement. From the displacement of the three retrereflectors the position, pitch and yaw error could be calculated. Two of the three reflected laser beams are split by a large area beam splitter before entering into the laser head. These beams are received by the two four quadrant detectors. From signal induced by the deviation of the laser spots error of two directional straightness and the roll error can be determinded.

The displacements is the avaredge of two simultaniously measured displacements

$$L = (L_1 + L_2)/2$$

as provided by the two beam laser interferometer.

The size of yaw error is also delivered by the the two beam interferometer:

Yaw error  $(\theta_z) = \arctan(L_2 - L_1)/D_h \approx (L_2 - L_1)/D_h$ 

where D<sub>h</sub> is the distance between the two laser beams (h stands for horizontal)



Fig. 2. The setup for a six-degree-of-freedom

measuring system

Similarly

Pitch error 
$$(\theta_v) = argtg (L_3 - L_1)/D_v \approx (L_3 - L_1)/D_v$$

where  $D_v$  is the distance between the two laser beams (v stands for vertical)

and the rool error can be calculated from the relative vertical deviation of the two fou

Roll error 
$$(\theta_x) = (v_2 - v_1)/2S$$

The four quadrant photo detectors are used to determine the out of straightness error by sensing the position of the laser spot centroid deviated form the center. As given in Fig. 3.



Fig. 3. Measuring pricipal for the out of straightness error

When the stage is moved along the the axis of motion, with a certain lateral displacement  $\Delta$ , the laser beam reflected by the retroreflector and split by the beam splitter will be shifted on the corresponding detector by a distance  $2\Delta$ , this means a sensitivity improvement by a factor 2. The out of strightness errors can be calculated from the horizontal and vertical positions of the laser spot on the detector as given below:

$$\Delta = (h_1 + h_2)/4$$
  $\Delta = (v_1 + v_2)/4$ 

Where  $(h_1, v_1)$  and  $(h_2, v_2)$  are the outputs of the detector 1 and 2 respectively.

### **4** Numerical error compensation

A numerical error compensation means the correction of the measurement values of the co-ordinate measuring machine for the relative three dimensional translational deviation at the specific co-ordinates. In order to obtain continuous correction for the whole measuring volume an appropriate interpolation of the obtained data is calculated. As interpolating function piecewise polynomials are chosen because they are computed easely during the correction process. As the data points are equally spaced and are "smooth" the selection of cubic splines seems to be obvious. In order to compute the de Boor point from the set of data point the following system of linear equations should solved:



where the  $X_1,...,X_{L-2}$  are the data points and the  $d_0,...,d_L$  are the de Boor points of the interpolating cubic uniform B-spline. The de Boor points are stored in the computer and used to calculate correction values for any arbitrary point in space.

#### Conclusions

In this paper the volumetric error components of co-ordinate measuring machines was described using homogenous co-ordinate transformation. A measuring methode was developed for simultaniously obtaining the corresponding error components. Volumetric error description has been obtained by the presented functions, which are calculated by special polynomial fitting procedures. As a result a simple and fast digital error correction methode has been developed.

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