The Bayesian Concept of Probability and its Application to Geologic Problems

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Abstract: The completeness of an exploration project is of crutial importance for making decision to start or to give up a mining investment, or to continue the exploration to get complementary information. The authors discuss this problem on the example of the Halimba bauxite deposit, Hungary. Resource calculations were carried out in 12 subsequent stages by fuzzy arithmetic with the aim to quantify the uncertainties of ore tonnage and grade. Prior information and prior probabilities were applied to complete the exploration data. Ranges of influence for the main variables were calculated by variograms. Spatial variability and spatial continuity of the ore bodies were mathematically evaluated. The authors found that the main geological, mining and economic factors must be evaluated separately and ranked according to their importance.

Keywords: resource assessment, fuzzy arithmetic, prior information, Bayesian probabilities

1 Introduction

Exploration of solid mineral deposits is generally an expensive task. Even more expensive and risky is the successive mining investment. It is of paramount importance therefore to optimize the exploration expenses and to minimize the risks of the mining investment. This double task was considered so far as a purely geological and mining- engineering problem, however, in our opinion, the application of some new mathematical methods may considerably improve the results. The aim of this paper is to show the application of these new methods by a case study. The Halimba bauxite deposit in Hungary has been chosen as example.

2 Basic Concepts

The completeness of an exploration campaign is generally expressed by the resource assessment (tonnage and grade) and its overall reliability. The spatial distribution and spatial variability of ore grade and the spatial continuity of ore within the deposit are further important aspects (Henley 2000, Wellmer 1989, Yamamoto 1999). However the traditional methods of resource assessment are not able to quantify the reliability of the estimation results. The fuzzy set theory has been applied by the authors for this purpose on some solid mineral deposits with success (Bárdossy, Fodor 2004). Fuzzy sets have been applied for the resource assessment of skarn tin deposits by Luo and Dimitrakopoulos (2003).

A further improvement can be achieved by applying the concept of Bayesian probabilities. If we toss a fair coin *n* times, then something is different in the tosses because otherwise the coin would always land heads or always land tails. But we are not aware of these differences. Our knowledge concerning the conditions of the experiment is always the same. Von Mises argued that, in such repeated experiments, the relative frequency of each outcome approaches a limit and he called that limit the probability of the outcome. We call such a probability a *relative frequency*. Proponents of this approach to probability are sometimes called *frequentists*.

Frequentist probability is used in most scientific work, because it is objective. It can (in principle) be determined to any desired accuracy and is the same for all observers. The definition of frequentist probability is a conceptual definition which communicates clearly its meaning and can in principle be used to evaluate it, but in practice one seldom has to resort to such a primitive procedure and go experimentally to a limit. However, even though one does not usually have to repeat experiments in order to evaluate probabilities, the definition does imply a serious limitation: it can only be applied to phenomena that are in principle exactly repeatable. This implies also that the phenomena must be random, that is, initial conditions which are experimentally indistinguishable yield results which are unpredictably different.

On the other hand, subjective (or Bayesian) probability is more general, since it can apply also to unrepeatable phenomena (for example, the probability that it will rain *tomorrow*). However, it depends not only on the *phenomenon* itself, but also on the *state of knowledge* and *beliefs* of the observer. Therefore, Bayesian probability will in general change with time. The probability that it will rain at 12:00 on Friday will change as we get closer to that date – getting more and more information – until it becomes either zero or one on Friday at 12:00.

We cannot verify if the Bayesian probability *P(E)* is ``correct" by observing the frequency with which event *E* occurs, since this is not the way probability is defined. The operational definition is based on ``the coherent bet" method. It says that an individual should liken the uncertain outcome to a game of chance by considering an urn containing white and black balls. The individual should determine for what fraction of white balls the individual would be indifferent between receiving a small prize if the uncertain outcome happened (or turned out to be true) and receiving the same small prize if a white ball was drawn from the urn. That fraction is the individual's probability of the outcome.

Subjective probabilities are unlike relative frequencies in that they do not have objective values upon which we all must agree. Indeed, that is why they are called subjective. When we are able to compute relative frequencies, the probabilities obtained agree with most individuals' beliefs.

The subjective probability approach is called `Bayesian' because its proponents use Bayes' Theorem to infer unknown probabilities from known ones. Before recalling that famous result we consider conditional probability. It is one of the key notions in probability theory. It is also important, through Bayes' theorem, in subjective probability.

Let E and F be events such that $P(F) \neq 0$. Then the *conditional probability of E given F*, denoted $P(E \mid F)$, is defined by $P(E \mid F) = \dfrac{P(E \cap F)}{P(F)}$.

That is, $P(E \mid F)$ means the probability of E occurring, given that we know F has occurred.

For decades conditional probabilities of events of interest have been computed from known probabilities using Bayes' theorem. We formulate this famous result in the simplest possible way.

Bayes' Theorem. Let E and F be two events such that $P(E) \neq 0$ and $P(F) \neq 0$. Then we have

$$P(F \mid E) = \frac{P(E \mid F) \cdot P(F)}{P(E)} = \frac{P(E \mid F) \cdot P(F)}{P(E \mid F) \cdot P(F) + P(E \mid not\, F) \cdot P(not\, F)}.$$

As an example of Bayes' theorem, suppose we have a test for influenza, such that if a person has flu, the probability of a positive result is 90%, and is only 1% if he does not have it:

$P(T^+ \mid flu) = 0.9$ (10% false negatives), and $P(T^+ \mid not\, flu) = 0.01$ (1% false positives).

Now the patient's test is positive. What is the probability that he has the flu? The answer can be given by Bayes' theorem as follows:

$$P(flu \mid T^+) = \frac{P(T^+ \mid flu) \cdot P(flu)}{P(T^+ \mid flu) \cdot P(flu) + P(T^+ \mid not\, flu) \cdot P(not\, flu)}.$$

So the answer depends on the *prior probability* of the person having flu (that is, on *P(flu)*). For frequentists, it is the frequency of occurrence of flu in the general population. For Bayesians, it is the prior belief that the person has the flu, before we know the outcome of any tests.

If we are in winter in Hungary, the prior probability *P(flu)* might be 0.01, while in some other country flu is a very rare disease, and $P(flu)=10^{-6}$ only. If we apply the same diagnostic test in each of these two places, we would get the following probabilities:

	$P(flu)=0.01$	$P(flu)=10^{-6}$
$P\left(flu \mid T^{+}\right)$	0.48	10^{-4}
$P\left(flu \mid T^{-}\right)$	0.001	10^{-7}

So this test would be useful for diagnosing the flu in Hungary, but in another country where it was a rare disease it would always lead to the conclusion that the person probably does not have the flu even if the test is positive.

Note that, as long as all the probabilities are meaningful in the context of a given methodology, Bayes' Theorem can be used as well by frequentists as by Bayesians. The use of Bayes' Theorem does not imply that a method is Bayesian, however the converse is true: all Bayesian methods make use (at least implicitly) of Bayes' Theorem.

A probability like *P(flu)* is called *a prior probability* because, in a particular model, it is the probability of some event prior to updating the probability of that event, within the framework of that model, using new information. A probability like $P(flu \mid T^{+})$ is called *a posterior probability* because it is the probability of an event after its prior probability has been updated, within the framework of a model, based on new information.

It is well known that the so called frequentist approach requires repeated identical experiments. However, this requirement can be fulfilled only rarely at the geologic investigations. The Bayesian approach, on the other hand, is able to evaluate unrepeatable phenomena as well. Bayesian probability depends only on the state of knowledge about the given problem and it changes with time as new pieces of information are acquired (Bárdossy, Fodor 2004). Mineral exploration and other applied investigations have also this changing character as new pieces of information are obtained about the given deposit by drilling new boreholes etc. For this reason,it is reasonable to apply also prior information and Bayesian probabilities for the evaluation of exploration results and other geoscientific problems (Wood and Curtis 2004).

According to our experiences, most geological investigations are characterized by a relatively large amount of prior information. The investigations produce more and more new data from boreholes, pits and laboratory analyses. Thus, when applying Bayes' theorem, posterior probabilities can be calculated.

If these calculations are repeated every time when new information is acquired, a sequence of posterior probabilities can be obtained. It is in our opinion highly useful to study the subsequent posterior probabilities, as it furnishes valuable information on the reliability of the given model. The smaller is the difference between the subsequent posterior probabilities, the higher is the reliability of the last model. This procedure is in our opinion an efficient tool to establish the completeness of the given geological investigation.

3 Initial Data

The bauxite deposit of Halimba, selected for this case study has been explored since 1943 and up to the present more than 2650 core boreholes have been performed. Underground mining started in 1950 and is still running. Computerised relational databases have been established (AutoCad) for the main data obtained about the deposit, particularly for the chemical composition of the ore. The sector of the test calculations – called Halimba II east - has been intensively explored during the last three years. It covers an area of 15 hectars with 250 borehole sites and it is situated int he southern part of the deposit.(Figure 1). A 10 to 40 m thick bed consisting of bauxite, clayey bauxite and bauxitic clay covers the karstified surface of Upper Triassic dolomite and limestone. The overburden is of Middle Eocene age. The entire deposit is of fluvial origin. The area of the studied sector is of flood-plain facies. The bauxite accumulated during short inundation phases, forming very irregular ore bodies within a continuous clayey bauxitic layer. Underground mining operations started in the western part of the study area in 2003. They confirmed the above outlined deposit model.

First the spatial variability of the main variables has been evaluated by us applying the well known methods of geostatistics (Goovaerts 1997). Ranges of influence have been calculated by the Variowin program for the thickness of the bauxitic bed, for the bauxite ore and for the Al_2O_3, SiO_2,Fe_2O_3,CaO and MgO contents of the bauxite. More than 3100 drilling cores have been analysed for 5 to 7 chemical components.

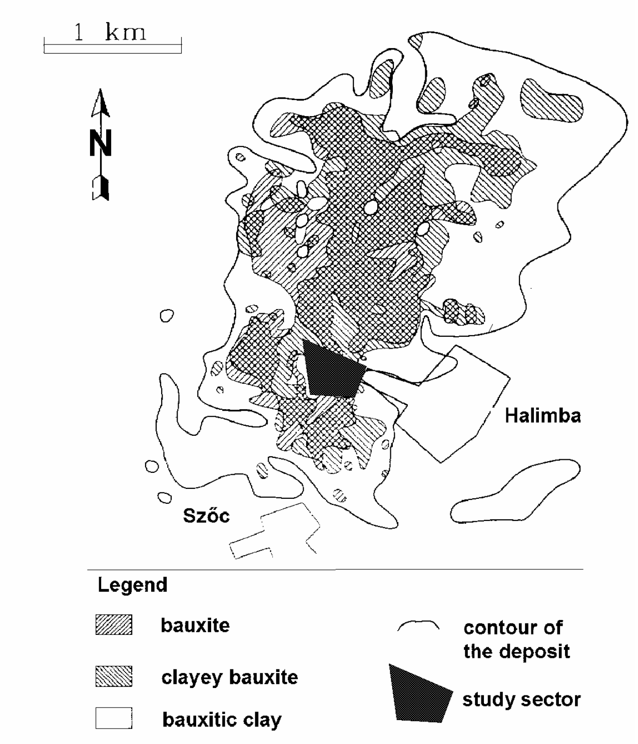


Figure 1
The Halimba bauxite deposit, Hungary

Our basic idea was to follow the changes that occurred as the exploration progressed. For this reason resource assessments were carried out by us after every 20 new boreholes finished. Thus a growing number of boreholes served as a base (prior information) of the successive resource assessments. Alltogether 12 resource assessments have been performed.

As several deposits have been detected in the area, their comparison is required for both practical and scientific purposes. Traditionally, this has been done so far by comparing the (weighted) averages of different variables, such as ore thickness, chemical components, etc. However, the averages being single valued, they cannot express the *degree of transition* between the deposits. We found that membership functions are highly suitable for these purposes. This is presented in Figure 2a and 2b, where the average values are represented in traditional (crisp) form and by fuzzy membership functions, the latter ones expressing the transitions as well. Note that neighbouring deposits can be highly different or may be connected by close transitions, depending on facies conditions.

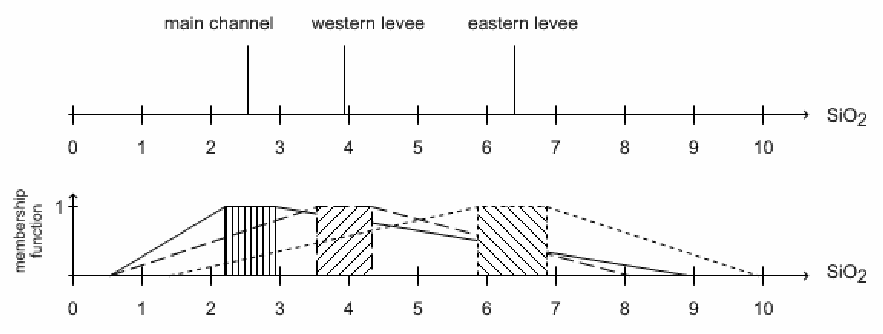


Figure 2a

Average SiO_2 content of the bauxite deposits (analytical error 0.3%), Halimba II/SW

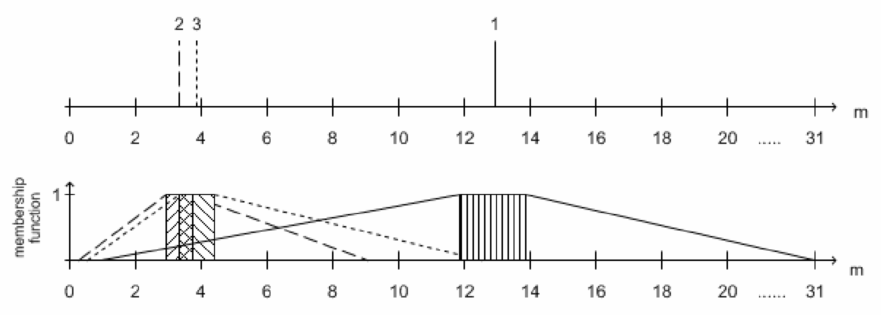


Figure 2b

Average bauxite thickness (measurement error 0.1 m);

Number of boreholes: 70 for 1, 58 for 2, 37 for 3.

The three basic components of any *resource assessment* of solid mineral deposits are the area of the deposit, the thickness of the ore and its bulk density. Fuzzy numbers have been constructed for all the three components. The „support" of the fuzzy number extends from the minimum to the maximum possible value. In the case of the deposit area the minimum value is determined by straight lines connecting the extreme productive boreholes. The maximum possible area is obtained by connecting the closest improductive boreholes around the productive area. The „core" of the fuzzy number represents the geologically most possible area, determined by the deposit model and its contour line.

This is a relatively simple and unambiguous task in the case of well explored deposits. However, in the early stages of exploration the number of boreholes is often not sufficient for the above outlined constructions. In that cases we extrapolated from the given productive borehole the range of influence of the bauxite thickness in all directions, obtaining this way the minimum possible area. The maximum possible area was obtained by taking in all directions twice the range of influence. In this case an interval has been chosen also for the core of the fuzzy number, expressing the larger uncertainty of the deposit area. The extrapolated resource boundaries are replaced gradually by straight lines connecting the neighbouring boreholes, as new boreholes are drilled. According to our experience, exploration should not be finished before replacing all the extrapolated boundaries by the connecting straight lines. Thus the trapesoidal fuzzy numbers are gradually replaced by triangular ones. As an example the area of the resource assessment at the end of the third stage is shown in Figure 3, and that at the end of the last (12th) stage in Figure 4.

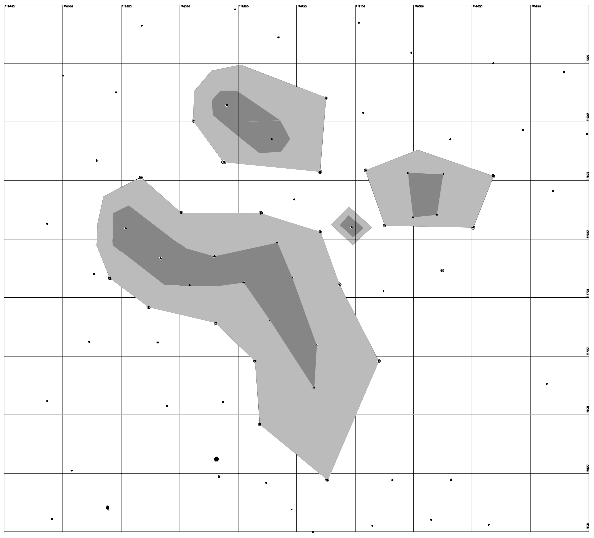


Figure 3
The area of the resource assessment at the end of the third stage

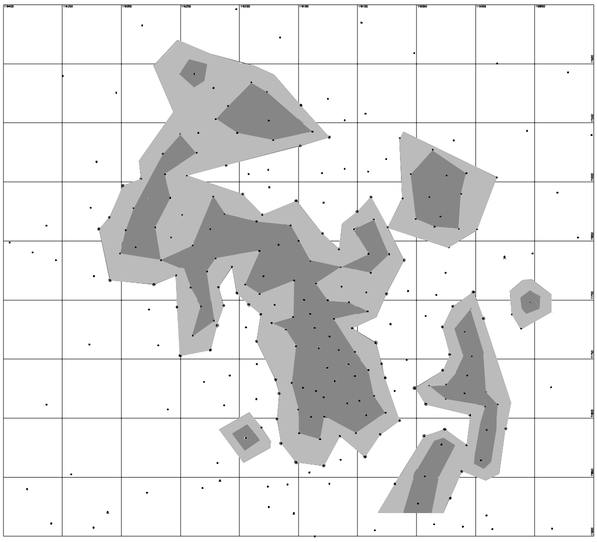


Figure 4

The area of the resource assessment at the end of the last stage

The fuzzy numbers representing the ore thickness correspond to the averages of the borehole results. Before calculating the averages, the main statistics of ore thickness have been calculated by us, applying the 12.0 version of the SPSS program. The histograms and the „skewness” values indicated a strong right-asymmetrical distribution. To eliminate the corresponding bias, „maximum likelihood” estimators have been calculated instead of the common averages. Tukey’s biweight estimator was found to correspond best to an unbiased average. It has been applied in all cases when the skewness statistic exceeded 1. The minimum and the maximum values of the support of the fuzzy numbers correspond to the endpoints of the corresponding confidence interval, at 95% level of confidence. The core of the fuzzy number is an interval determined by the standard error of the mean.

The bulk density of the ore has been measured in the laboratory on borehole cores and in the mine on large samples, several hundred times. The distribution of the results is symmetrical. The mean value is 2,29 tons/m3. The analytical error is less than 10 relative percents. The variability of the bulk density is very limited over the test area. For this reason the same fuzzy number has been applied for all the twelve resource assessments. In the same way as for the ore thickness, the support corresponds to the confidence interval at 95% level of confidence and the core to the standard error of the mean, plus the analytical error.

The tonnage of the resource is the product of the above discussed three components. Fuzzy multiplication was applied for the three corresponding fuzzy numbers. The uncertainty of the resource assessment is expressed in tons by the length of the support and the core. Additionally relative deviations from the average values – expressed as percentages – were also calculated.

The average grade of the ore has been calculated in a similar way constructing average fuzzy numbers for all the listed chemical components. To avoid biases due to asymmetrical distribution histograms and skewness values were calculated and robust M-estimators were applied whenever the skewness exceeded the value 1,0. As in the case of tonnage, absolute and relative uncertainties have been calculated for all the evaluated chemical components.

As mentioned above, all the above listed calculations have been repeated 12 times, adding every time 20 new boreholes. Fuzzy numbers for selected stages are presented in Figure 5.

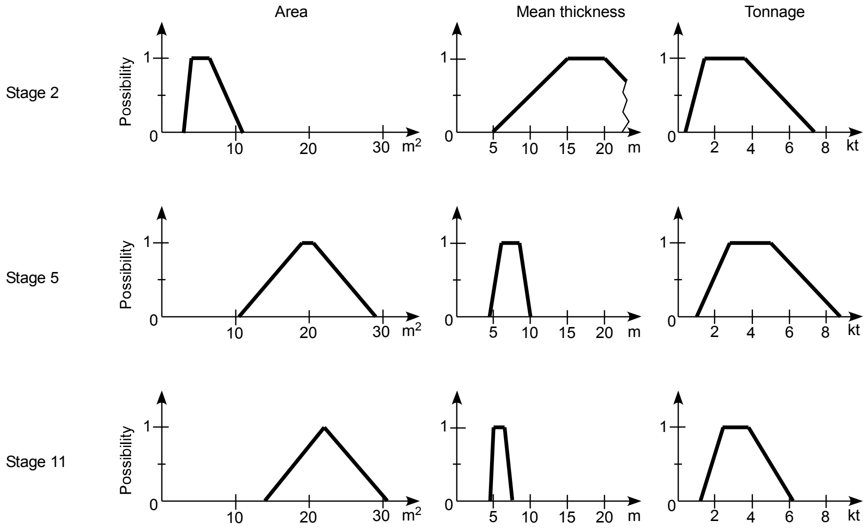


Figure 5

Fuzzy numbers expressing the area, mean thickness, and tonnage for stages 2, 5, 11

4 Evaluation of the Completeness of the Exploration

For the starting situation, that is before the drilling of the first bore hole in the sector, the following prior probabilities have been assumed, based on the experiences of the neighbouring explored and mined sectors:

- the bauxite – clayey bauxite bed is continuous over the exploration sector *0.8 probability*
- the bed is not continuous over the exploration sector *0.2 probability*

Second item of the prior probabilities:

- - commercial bauxite ore bodies are situated within the above bed *0.6 probability*
- - no commercial bauxite ore bodies occur within the bed *0.4 probability*

The end situation- after the 12th stage - confirmed both larger prior probabilities.

For the first two stages of exploration the number of productive boreholes was not sufficient to calculate reliable variograms. For this reason, the already calculated ranges of influence of the neighbouring sectors were applied, supposing similar values in the study area. At the end of the third stage variograms could be calculated for the ore thickness. By applying different „lags” and variogram models ranges of influence from 10 to 20 m length were obtained. With growing number of boreholes the variograms became more accurate and after the 12th stage 15 m range of influence was accepted for the entire exploration area. However, locally even smaller ranges of influence exist, as confirmed ba the latest mining operations As outlined later, these changes significantly influenced the results of the successive resource assessments.

At the end of each exploration stage circles were constructed around each borehole, expressing the range of influence. The boreholes were placed in a „random-stratified” grid, with the aim to optimize the contouring of the very irregularly shaped ore bodies. For this reason „unknown” slices remained between some neighbouring boreholes. Prior probabilities have been calculated for these slices separately and if they exceeded the 0.5 value, they have been included into the resource assessment. This procedure ameliorated considerably the fitting of the resource contours to the real boundaries of the ore bodies.

Different variables have been chosen for the quantitative evaluation of the completeness of exploration,first of all the tonnage of the resources. In Figure 6 the successive changes of the minimum and maximum values of the support are represented. The minimum value of the tonnage steeply increases in the first four stages of exploration, followed by much smaller increase in the later stages. The fluctuation of the diagram reflects the randomness of the results at some stages. The possible maximum tonnage also increases steeply in the first stages, but it is followed by an unexpected gradual decrease untill the eighth stage. The last stages show a slight increase. The peculiar form of this diagram can be explained by the higher uncertainty of the maximum tonnage, influenced by the position of the closest improductive boreholes and by the extrapolation of the contour line in the first stages of exploration. The peak between the third and fourth stages is clearly a random effect, that may occur in the first stages of any exploration campaign. As exploration progresses, the difference between the two diagrams diminishes, as the area between the manimum and maximum contours becomes narrower.

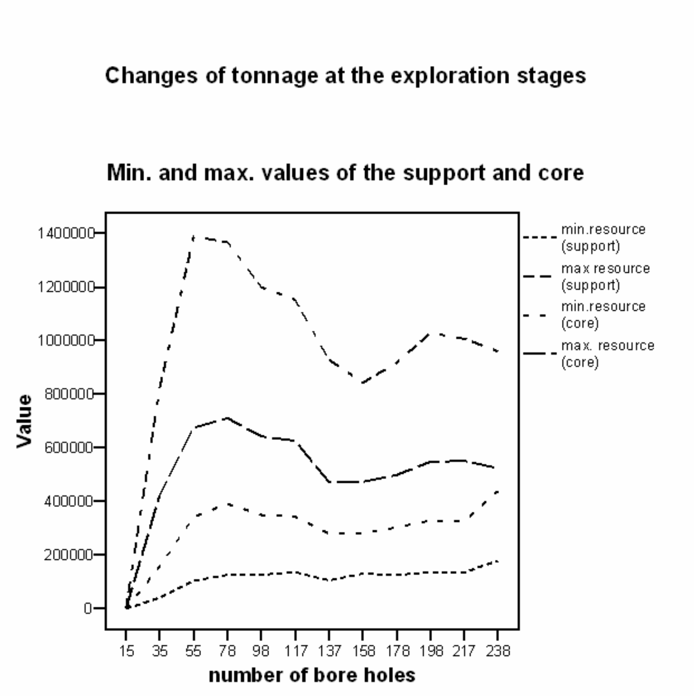


Figure 6

Quantitative evaluation of tonnage: successive changes of the maximum and minimum of the support and the core, respectively

The tonnage expressed by the core of the fuzzy numbers has a much shorter uncertainty interval, presented also in Figure 6. The random overestimation of the tonnage between the third and fourth stages is clearly visible on both diagrams, but it is gradually equalized in the later stages without reaching a constant value. Theoretically, the exploration is still not complete, but the changes of the tonnage are insignificant. Thus the tonnage of the resources alone is in favour of finishing the exploration drilling.

A further aspect of the evaluation is the relative uncertainty of the tonnage, expressed as a percentage of the mean (crisp) tonnage. We calculated it separately for the support and for the core of the corresponding fuzzy numbers. The results are presented in Figure 7. It is obvious that the uncertainty of the tonnage expressed by the support is much larger than that of the core. It decreases in the successive stages untill the eighth stage – from ±91% to ±73%. This is followed in the later stages by a fluctuation and a final value of ±69%. On the other hand, the relative uncertainty of the tonnages expressed by the core are much smaller. The starting ±46% relative uncertainty diminishes to ±9%. This indicates a near complete exploration result.

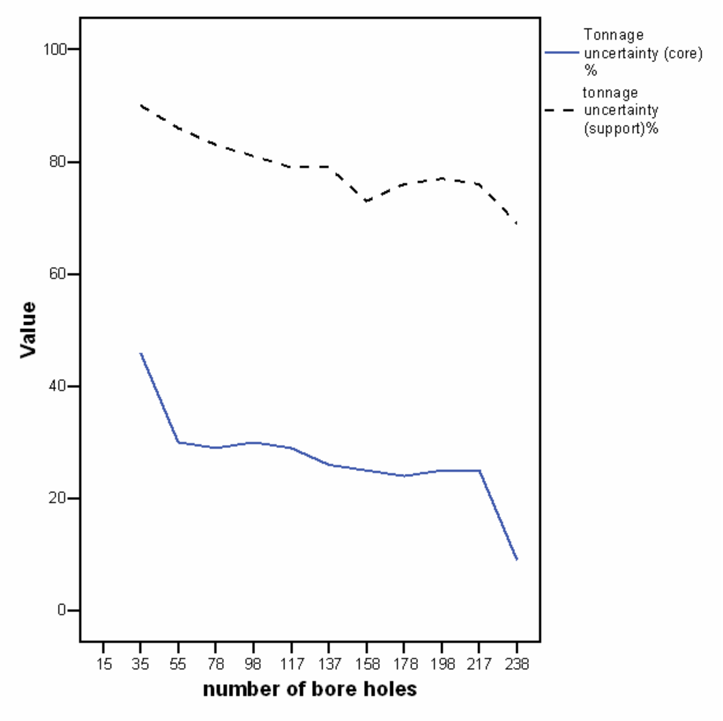


Figure 7

Relative uncertainty of the tonnage for the core and the support, respectively

The tonnage values of the fuzzy numbers and their relative uncertainties are presented in Table 1. Let us stress that these data represent a significant complement to the single-valued traditional resource estimation results. But even these data are insufficient in our opinion to make a reliable decision on the completenes of an exploration campaign. The main chemical components have been evaluated by us too, in function of the successive exploration stages.The resulting main statistics, calculated by the SPSS program are presented in Table 2.

The Al_2O_3 content has the smallest relative variance, ±7%. The distribution of this component is almost normal, thus the mean value is unbiased. It diminished from the second to the latest exploration stage from 52.8 to 51.2%, considered by us as a very small change. In the same time, the standard error of the mean diminished from ±1.0 to ±0.3%, indicating a high reliability of the results. It can be concluded that regarding the alumina content the exploration has been complete since the early stages.

The Fe_2O_3 content follows with ±14-16% relative variance. The distribution is symmetrical and the mean decreased from 25.3 to 24.6% as exploration progressed, close to the range of the analytical error.

Table 1
Main results of the resource calculations

Stages	Number of boreholes	Deposit area				Tonnage				Length of the core interval	Relative uncertainty of the tonnage (%)	Length of the support interval	Relative uncertainty of the tonnage (%)
		a	b	c	d	a	b	c	d				
1	15	0	0	0	0	0	0	0	0	0	0	0	0
2	35	5300	8100	12000	17400	41700	158200	425400	826400	267200	46	784700	90
3	55	11800	26200	29800	46400	104400	342400	675200	1388400	332800	30	1284000	86
4	78	12610	29100	30800	47960	126100	391400	710900	1367200	319500	29	1241100	83
5	98	14090	29300	31000	46600	126600	350000	643000	1200000	293000	30	1073400	81
6	117	15630	30400	32100	47700	136500	344500	626700	1152500	282200	29	1016000	79
7	137	14090	29700	30000	45630	106000	279600	472600	929000	193000	26	823000	79
8	158	16980	30200	30700	44100	130900	281900	473800	843200	191900	25	712300	73
9	178	16700	32200	32400	47870	126400	303300	499100	917500	195800	24	791100	76
10	198	18940	35300	35300	52280	135900	329200	548000	1028500	218800	25	892600	77
11	217	20800	36000	36300	52200	136200	323800	549100	1002700	224100	25	870900	76
12	238	24300	41600	42000	60300	178800	439000	524200	960000	85200	9	781100	69

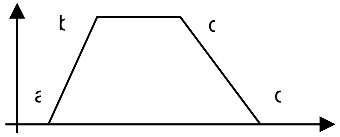
Legend:

a: lower bound of the support of trapezoidal fuzzy number [a,b,c,d]

b: lower bound of the core of trapezoidal fuzzy number [a,b,c,d]

c: upper bound of the core of trapezoidal fuzzy number [a,b,c,d]

d: upper bound of the support of trapezoidal fuzzy number [a,b,c,d]



As with the Al_2O_3, the standard error of the mean diminished from ±1.0 to ±0.3%. Thus the exploration is considered complete also in this respect.

The SiO_2 content of the ore is more variable, the relative variance ranging from ±39 to 45%. The distribution is almost symmetrical and the mean remained the same within the range of the analytical error. Only the standard error of the mean diminished from ±0.7 to ±0.2%, indicating a high reliability of the results. The exploration is complete also in this respect.

The CaO is one of the main contaminants in the bauxite. This is the most variable analysed chemical component, the relative variance ranging from ±83 to 114%. The distribution is strongly skewed, as indicated by the high positive skewness value. For this reason Tukey's M-estimator has been applied intead of the normal mean. It increased gradually from 0.6 to 1,0% to the last stage of the exploration. It cannot be predicted whether a further increase would occur with the drilling of new boreholes. The reason for this high variability is the presence of CaO in the form of secondary calcite precipitations, irregularly distributed within the ore bodies.Thus regarding the evaluated chemical components, the exploration can be considered as completed, except the CaO content.

A further aspect influencing the completeness of the exploration is the detection of the spatial distribution of the orebodies and the degree of their variability. The question is, how much increased the precision of these predictions by the exploration and can it be regarded complete after the twelfth stage? To answer these questions prior probabilities have been applied. The borehole sites have been ordered into five categories and a prior probability has been attached to each category, based on the overall exploration experiences of the entire deposit:

1	the site is within the productive arera	0.3 prior probability
2	the site is on the border of the productive area	0.05
3	the site is within the possible area	0.2
4	the site is on the outer border of the possible area	0.05
5	the site is within the improductive area	0.4
	alltogether	1.0 prior probability

The borehole sites situated beyond the range of influence of bauxite thickness have not been categorized. In the next step all existing borehole sites were categorized based on the resource assessment maps of the 12 exploration stages and the changes of categories were presented in the form of a table. Table 3 shows these changes for 20 selected borehole sites, as the limited extent of this paper does not allow the presentation of all the 237 borehole sites. (The not categorized sites are indicated by question-marks). It is obvious that the number of not categorized sites diminishes in the successive exploration stages.

Table 2
Main statistics of selected chemical components at the end of exploration stages

Chemical components	Stage 12	Stage 8	Stage 6	Stage 4	Stage 2
SiO2					
Mean (%)	5.30	5.70	5.70	5.50	5.40
Standard error of the mean (%)	± 0.20	± 0.20	± 0.30	± 0.30	± 0.70
Standard deviation (%)	± 2.10	± 2.30	± 2.30	± 2.2	± 2.40
Coefficient of variation (%)	39.00	40.00	41.00	40.00	45.00
Skewness	0.02	-0.05	-0.03	0.03	-0.16
Min (%)	1.30	1.30	1.30	1.30	1.30
Max (%)	9.90	9.50	9.50	9.50	9.40
Al2O3					
Mean (%)	51.20	51.60	51.90	51.90	52.80
Standard error of the mean (%)	± 0.30	± 0.40	± 0.40	± 0.50	± 1.00
Standard deviation (%)	± 3.60	± 3.70	± 3.50	±3.20	± 3.70
Coefficient of variation (%)	7.00	7.00	7.00	6.00	7.00
Skewness	0.24	-0.09	0.45	0.88	-0.89
Min (%)	38.70	38.70	42.90	44.00	44.00
Max (%)	64.10	63.10	63.10	63.10	59.50
Fe2O3					
Mean (%)	24.60	24.60	24.30	24.30	25.30
Standard error of the mean (%)	± 0.30	± 0.40	± 0.40	± 0.60	± 1.00
Standard deviation (%)	± 3.80	± 3.90	± 3.50	± 3.80	± 3.50
Coefficient of variation (%)	16.00	16.00	15.00	16.00	14.00
Skewness	-0.89	0.65	-0.77	-0.94	0.38
Min (%)	10.30	11.10	11.10	11.10	20.10
Max (%)	36.70	36.70	32.60	32.60	32.60
CaO					
Mean (%)	1.00	0.90	0.89	0.76	0.60
Standard error of the mean (%)	± 0.08	± 0.09	± 0.12	± 0.19	± 0.27
Standard deviation (%)	± 0.83	± 0.81	± 0.85	± 0.87	± 0.55
Coefficient of variation (%)	83.00	90.00	96.00	114.00	91.00
Skewness	1.32	1.54	1.74	2.57	1.54
Min (%)	0.11	0.11	0.11	0.11	0.20
Max (%)	3.90	3.90	3.90	3.90	1.38

Several boreholes have been drilled at such sites having no prior information. This „haphazard" approach leaded to some negative results, as illustrated by the Table 3. The categories of the borehole sites often changed in positive or negative sense indicating the incompleteness of the exploration. Theoretically, exploration should be considered complete if the site-category would not change in the final two or three exploration stages, before the drilling of the given borehole. Unfortunately, this condition was only partly fulfilled even for the last, twelfth stage. Thus, in this respect the exploration cannot be accepted as complete.

Table 3
Prior categorization of selected borehole sites

Borehole	Exploration stages										
	1	2	3	4	5	6	7	8	9	10	11
H-2564	?	5	3	3	3	3	3	3	3	3	**2**
H-2557	5	5	5	5	5	5	5	5	5	5	**1**
H-2556	?	?	?	?	?	?	?	?	?	?	**5**
H-2555	?	?	?	?	?	?	?	?	?	?	**5**
H-2554	?	(1)	(1)	3	3	3	3	3	3	3	**2**
H-2553	?	(2)	(1)	3	3	3	3	3	3	3	**2**
H-2552	?	(3)	3	3	3	3	3	3	3	3	**2**
H-2551	?	(1)	(1)	1	1	1	1	1	1	1	**3**
H-2550	5	5	3	3	3	3	3	3	3	3	**2**
H-2549	?	5	5	5	5	5	5	3	3	3	**4**
H-2548	?	?	?	?	?	?	?	?	?	?	**5**
H-2547	?	5	3	3	3	3	3	3	3	3	**3**
H-2546	?	5	(1)	1	1	1	1	1	1	1	**1**
H-2545	?	5	(4)	(4)	(4)	(4)	(4)	(4)	4	**4**	n.a.
H-2544	?	?	5	?	?	?	?	5	3	**2**	n.a.
H-2543	5	5	5	?	?	?	?	?	?	**5**	n.a.
H-2542	5	5	5	?	?	?	?	?	?	**4**	n.a.
H-2541	?	?	?	?	?	?	?	?	5	**2**	n.a.
H-2540	?	?	?	?	?	?	?	?	?	**2**	n.a.
H-2539	?	?	?	?	?	?	?	?	?	**4**	n.a.

Legend:

1. Site within the productive area.
2. Site on the border of the productive area
3. Site within the possible area.
4. Site on the outer border of the possible area.
5. Site within improductive area (clayey bauxite and bauxitic clay).

( ) Site categorized by extrapolation.

? Not categorized site, outside the ranges of influence.

Bold numbers: categories after drilling the corresponding borehole site.

Table 4

Summary results of the prior categorization of the first seven exploration stages

Categories	Productive	Possible	Improductive	Sum of row
Productive	20	16	14	50
Possible	10	17	11	38
Improductive	0	2	12	14
Sum of column	30	35	37	102

A more complete evaluation can be obtained if several exploration stages are considered together. Table 4 shows the results of the first seven stages.(Obviously, the first stage can not be evaluated). Even more interesting results were obtained, when evaluating all stages together, as presented in Table 5. From the 203 prior probabilities 92 were confirmed by the drilling of the corresponding bore-holes. Even more important is that in 97 cases the prior probabilities were changed positively and only in 14 cases negatively. These result underline the effectiveness of the exploration campaign.

Table 5

Summary results of the prior categorization of all the 12 exploration stages

Categories	Productive	Possible	Improductive	Sum of row
Productive	28	35	37	100
Possible	12	32	25	69
Improductive	0	2	32	34
Sum of column	40	69	94	203

A further aspect, important for the planning of a mining investment, is the completeness of the contouring of the orebodies. In our case this means that the orebodies should be surrounded from all sides by improductive boreholes. The evaluation is simple: the exploration is incomplete at all places where the contour of the orebody is determined only by extrapolation. In the study area four places remained incomplete in this respect after ending the 12th stage. An overall relative index can be computed when comparing the length of the completely contoured borders with the length of the extrapolated ones.

A further aspect is the rate of lateral changes in the thickness and altitude of the orebodies. This aspect is very important in the case of underground mining, as it can be a limiting factor for the choice of the excavation and production systems. We evaluated this aspect by calculating the specific rates of lateral changes for the bauxite thickness of neighbouring boreholes. An example of this evaluation is presented in Figure 8. In the ore bodies of our test sector these specific rates of lateral changes are often very strong and they may vary quickly in the different directions, making difficulties in the choice of the mining methods. Note that the boreholes beyond the range of influence were excluded from this evaluation. The entire productive sector has been evaluated in this way. The exploration is complete in this respect.

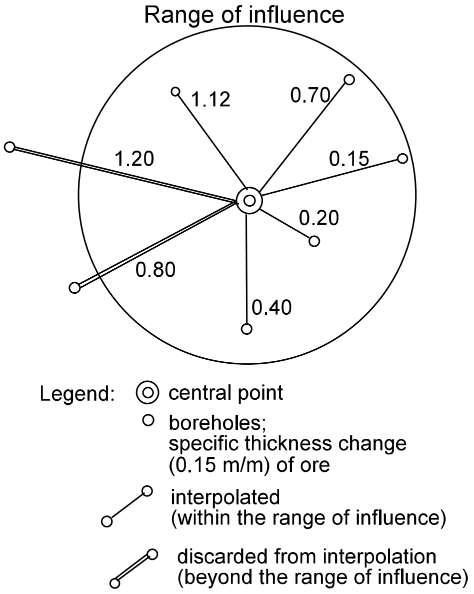


Figure 8

Evaluation of the specific rate of changes for ore thickness in an underground mine around a measured central point

It is mathematically possible to aggregate all the discussed aspects into one fuzzy completeness index of the exploration, following the methodology of Luo and Dimitrakopoulos (2003) for their fuzzy mineral favourability index. This is a useful estimator for the stakeholder, but for the mining engineer, planning and starting the mining operations, it is more useful to evaluate and to compare all the discussed aspects separately. We recommend therefore the stepwise evaluation of each aspect after every exploration stage and making decisions after ranking them in both respects of completeness (reliability) and the additional costs of the drilling of further boreholes.

5 Verification of the Exploration Results

The underground mining operations quickly followed the above outlined exploration, offering us a possibility to check the validity of our evaluations. Boreholes were drilled from the galleries at 5 meter intervals vertically up and down and also laterally. The bauxite has been sampled and analysed at every one meter interval. The bauxite ore of more than 2 meters thickness have been excavated.

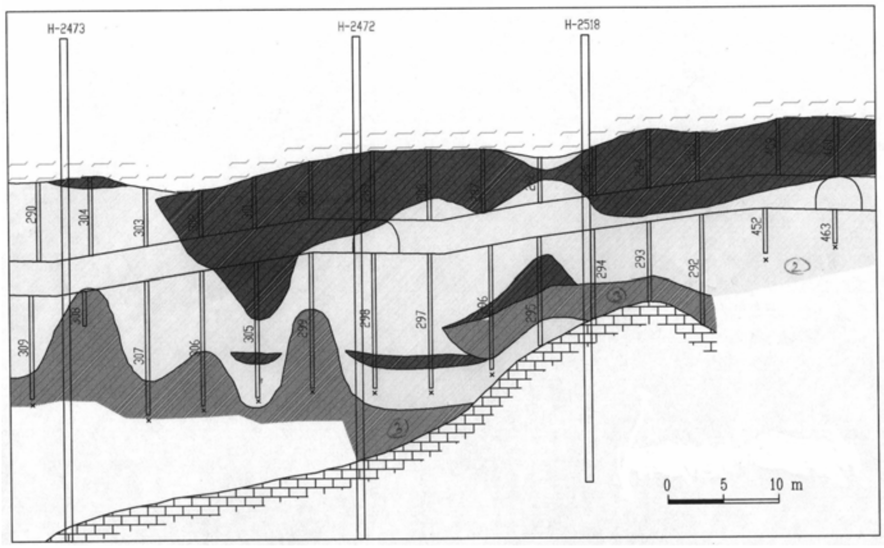


Figure 9

Bauxite (1), clayey bauxite (2), bauxitic clay (3)

Figure 9 presents the section along a main gallery, taking into account both the surface and the underground boreholes. It shows the very high variability of bauxite, clayey-bauxite and bauxitic clay thickness.

All these data have been evaluated by us by applying the AutoCAD program and the resulting 2 meters contour has been constructed. This line has been compared with our last (12th) resource assessment map – for the selected part of the deposit (Figure 10). The productive area of our resource assessment is completely confirmed by this contour line. It runs generally within the possible area, and at some places it even extends beyond it. There is no positive or negative bias (over- or under estimation) in this respect. Thus our deposit model, applied to our resource assesment has been confirmed by the mining operations.

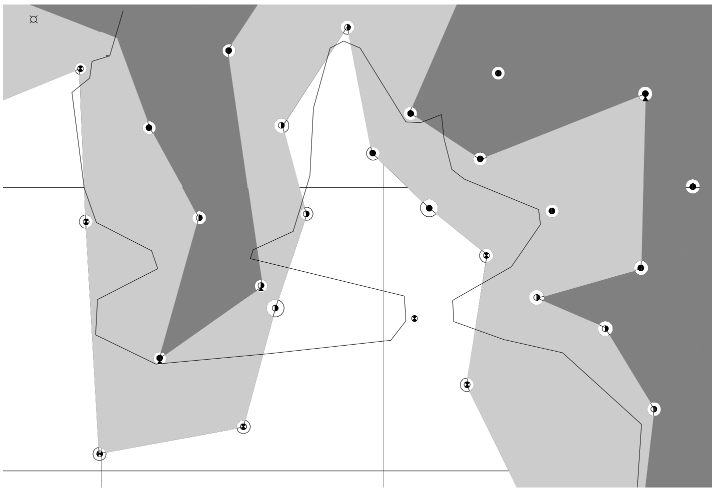


Figure 10

Comparison of estimation and reality

Conclusions

The completeness of a mineral exploration can be best evaluated by a joint application of the fuzzy set theory and Bayesian probabilities. The establishment of appropriate computerized databases is indispensable for these tasks.

The method consists of the stepwise evaluation of successive exploration stages (contouring the productive and possible areas and calculating the resources) and a comparison of the prior and posterior information.

According to our experiences, completeness of exploration is achieved at different stages of exploration regarding the different evaluated variables. The criterion for completeness should be the decrease or complete equalization of the given variable.

Even in the case of best planned and evaluated exploration random effects (over- or under-estimation of the given variable) cannot be excluded, mainly in the early stages of the exploration campaign.

A reliable deposit model is the precondition of any evaluation in this respect. The model can be verified by the evaluation of the successive mining operations.

This methodology can be applied to other types of solid mineral deposits as well, taking into account their specific deposit models.

References

[1] Bárdossy, Gy., R. Szabó, J., Varga, G. 2003: A new method of resource estimation for bauxite and other solid mineral deposits. – BHM, 148. Jg. pp. 57-64. Leoben

[2] Bárdossy, Gy., Fodor, J. 2004: Evaluation of Uncertainties and Risks in Geology. – Springer Verlag. Berlin, Heidelberg, London, New York. 221 pages

[3] Henley, S. 2000: Resources,reserves and reality. – Earth Sciences Computer Applications. Vol. 15. No. 10. pp1-2

[4] Luo, X., Dimitrakopoulos, R. 2003: Data-driven fuzzy analysis in quantitative mineral resource assessment. – Computers and Geosciences. 29. pp. 3-13

[5] Wellmer, F. W. 1989: Economic Evaluations in Exploration. – Springer Verlag. Berlin, Heidelberg, London, New York. 150 pages

[6] Wood, R. Curtis, A. 2004: Geological prior information, and its applications in solving geo-scientific problems. – In: „Geological Prior Information" Eds: A. Cueris and R. Wood. – Geol. Society of London Special Publication. 239. (in press)

[7] Yamamoto, K. 1999: Quantification of uncertainty in ore reserve estimation. Applications to Chapada Copper Deposit,Brazil. – Natural Resources Research. 8. pp. 153-163