Extension of the Modified Renormalization Transformation for the Adaptive Control of Negative Definite SISO Systems

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Abstract: A novel branch of Computational Cybernetics was formerly developed for the adaptive control of approximately and partially modeled Classical Mechanical, Electro-mechanical and Hydro-mechanical systems. It essentially was based on the Modified Renormalization Transformation for the convergence of which the positive definite nature of the inertia matrix was needed. In this paper the extension of this method is formulated for negative definite, Single Input, Single Output (SISO) systems that may also occur in Classical Mechanics when a multiple Degree Of Freedom (DOF) system has only one driven axis and this drive is used to control a different axis utilizing the nonlinear dynamic coupling between the axes. As an example the inverted pendulum-cart system is considered in which the drive of the cart's translational DOF is used for controlling the rotational axis of the pendulum. The extension results in a whole family of parametric transformations in which two parameters, a "multiplicative" and a "shift" parameter are present. It is shown that the fact of the convergence is robust against the variation of these parameters and their actual value mainly concerns only the speed of convergence. Simulation results illustrate these statements.

Keywords: Adaptive Control; Robust Control; Computational Cybernetics; Soft Computing; Renormalization Transformation; Modified Renormalization Transformation; Extended Modified Renormalization Transformation.

1 Introduction

A new approach for the adaptive control of imprecisely known SISO dynamic systems under unmodeled dynamic interaction with their environment was initiated in [1]. Instead of tuning the supposed analytical model parameters a fast algorithm that finds a certain linear transformation to map the observed system-behavior to the expected one calculated on the basis of a very primitive initial model is applied. The so obtained „amended model" is step by step updated to trace changes by repeating this corrective mapping in each control cycle. The method essence of the method was a modified version of the Renormalization Transformation that generally is used for transforming the fixed points of nonlinear mappings. Since no any effort is exerted to identify the possible reasons of the difference between the expected and the observed response, it is also referred to as the idea of "Partial System Identification" that is very similar to the main point of the approach applied in various contexts [e.g. 2, 3]. This anticipates the possibility for real-time applications. Later the method was extended to Multiple Input, Multiple Output (MIMO) systems by constructing appropriate linear transformations with quadratic matrices. Several algebraic possibilities were investigated and successfully applied. For instance, the "Minimum Operation Symplectic Transformations" [4], „Generalized Lorentz Group" [5], and a special family of the „Symplectic Transformations" [6] can be mentioned. The conditions of convergence of this approach were investigated in a wider context in [7] on the basis of Perturbation Calculus. The key element of this proof was the positive definite nature of the inertia matrix of the Classical Mechanical Systems. This requirement meant a considerable restriction in the applicability of this method.

In the present paper we wish to increase the applicability of our method for negative definite SISO systems by introducing a whole family of parametric transformations in which two parameters, a "multiplicative" and a "shift" parameter are present I transformation that can be recognized as further generalization of the Modified Renormalization Transformation. Such systems also occur in Classical Mechanics when a multiple Degree Of Freedom (DOF) system has only one driven axis and this drive is used to control a different axis utilizing the nonlinear dynamic coupling between the axes. As an example the inverted pendulum-cart system is considered in which the drive of the cart's translational DOF is used for controlling the rotational axis of the pendulum. It is shown that the fact of the convergence is robust against the variation of these parameters and their actual value mainly concerns only the speed of convergence. Simulation results illustrate these statements.

2 The Proposed Transformation

The forthcoming considerations pertain to physical systems for which the controller tries to obtain a *desired response* x^d by applying an imprecise and incomplete model to calculate the estimated necessary excitation $e=\varphi(x^d)$ that according to the actual dynamics of the system results in the realized response $x^r=\psi(\varphi(x^d))\equiv f(x^d)$. It is supposed that the desired response is known, the realized response is measurable, and though the exact form of $f(x^d)$ is not known at least its increasing or decreasing nature can be deduced from the physics of the system to be controlled. In the ideal situation the realized response is equal to the desired

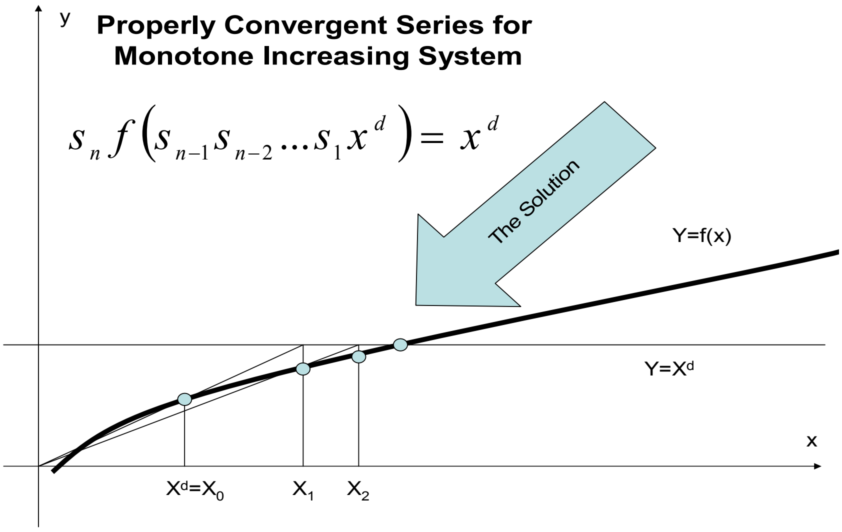


Figure 1
Proper convergence of the originally Modified Renormaliaztion Transformation

one that corresponds to finding the fixed point of f as $f(x^d)=x^d$.

As is well known the Renormalization Transformation can transform a function $f(x)$ by a scalar parameter γ as $f_\gamma(x)\equiv\gamma^{-1}f(\gamma x)$ that transforms the fixed point $f(x)=x$ since if $z=f_\gamma(z)\equiv\gamma^{-1}f(\gamma z)$ then $f(\gamma z)=\gamma z=x$. It was plausible to try to use this transformation for the adaptive control that can also be formulated as a fixed point problem. However, due to the fact that in the control just x^d is needed as the output, the modified algorithm defined as

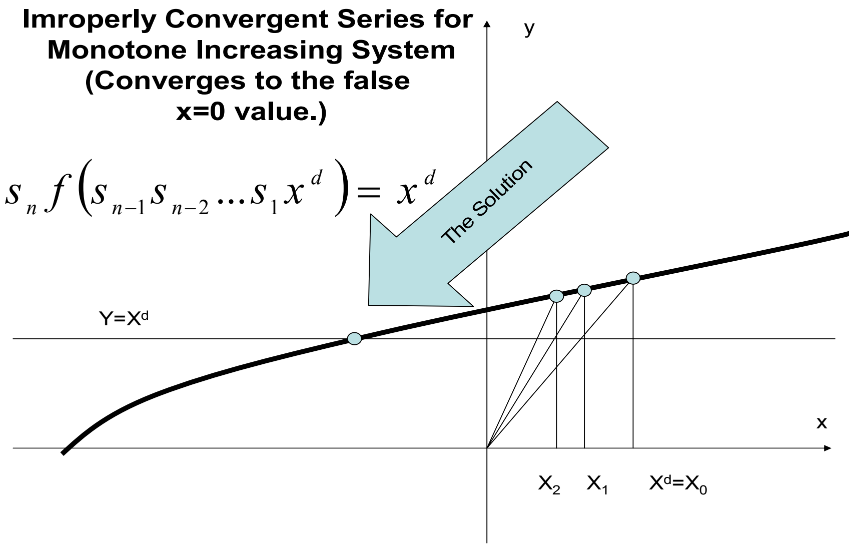
$$s_n f\left(s_{n-1}s_{n-2}\ldots s_1 x^d\right)= x^d \qquad (1)$$

was introduced in [1]. As it qualitatively is illustrated in Figs. 1 and 2 for monotone increasing system this series can properly ($s_n\to 1$) and improperly ($s_n\to k<1$) convergent i.e. if the solution of the $f(sx^d)=x^d$ equation is properly or improperly situated. (Divergent solutions can also be constructed).

For monotone decreasing SISO systems it is a plausible idea to extend the above given transformation by a parameter ζ that can either be positive or negative, and that for the special case of $\zeta=0$ corresponds to the original transformation [Fig. 3].

$$s_n f\left(x_{n-1}\right)= f\left(x_{n-1}\right)+\zeta\left(f\left(x_{n-1}\right)-x^d\right),\ x_n = s_n x_{n-1}, \zeta > 0 \qquad (2)$$

To give a satisfactory condition for the convergence of the proposed method consider a flat differentiable function $g(x)$, for which the following estimations can be done according to which if the derivative of g is small enough in a region it



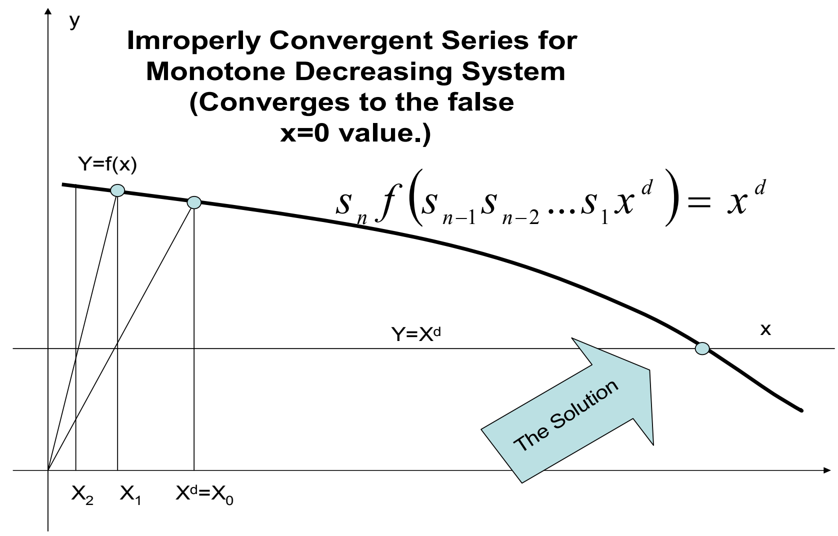
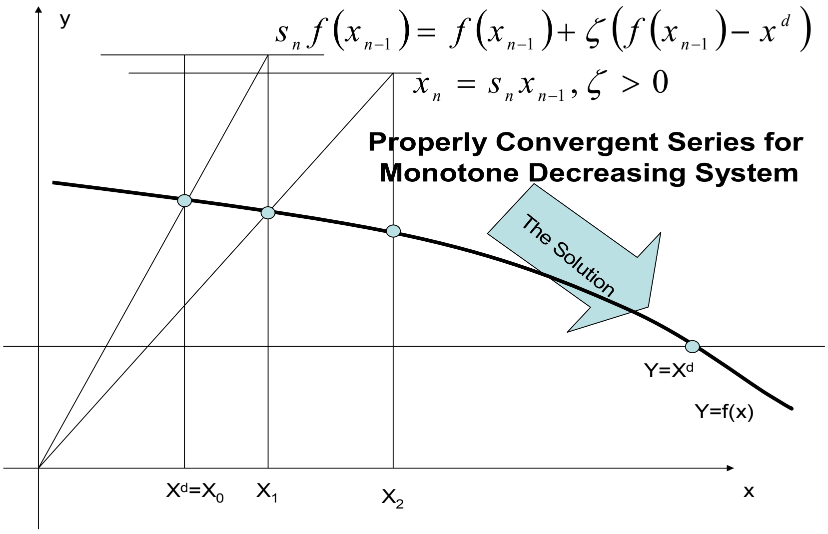


Figure 2

Improper convergence of the originally Modified Renormaliaztion Transformation

realizes a contractive mapping.



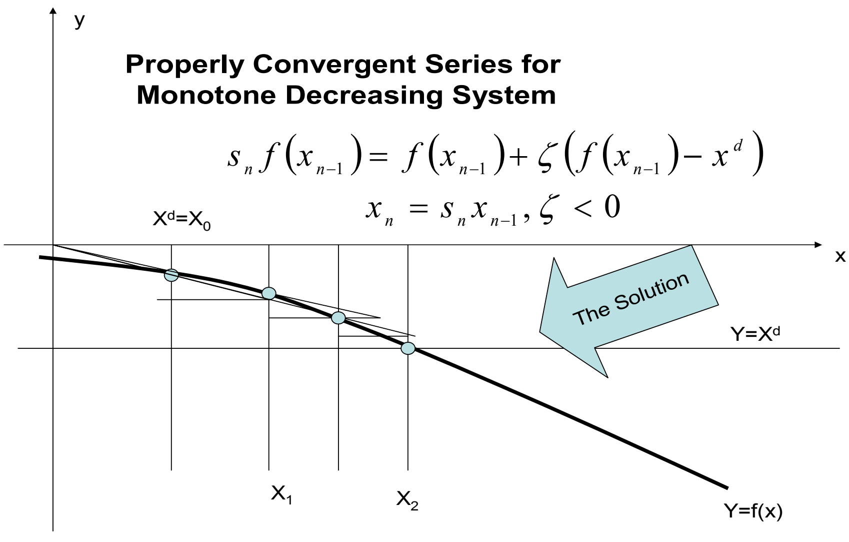


Figure 3

Proper convergence of the originally Modified Renormaliaztion Transformation

$$|g'(x)| \le K < 1,\ g(b) - g(a) = \int_a^b g'(t)dt,\ |g(b) - g(a)| \le \int_a^b |g'(t)|dt \le K|b - a| \tag{3}$$

For a contractive mapping the $x_n = g(x_{n-1})$ series is a Cauchy series since

$$\begin{aligned}|x_{n+1}-x_n| &= |g(x_n)-g(x_{n-1})| \le K|x_n-x_{n-1}| = K|g(x_{n-1})-g(x_{n-2})| \le \ldots \le \\ &\le K^n|x_1-x_0| \xrightarrow[n\to\infty]{} 0\end{aligned} \quad . \tag{4}$$

In a complete metric space that converges to a well defined value u that must be the fixed point $u=g(u)$ since

$$\begin{aligned}|g(u)-u| &\le |g(u)-x_n+x_n-u| \le |g(u)-x_n|+|x_n-u| = |g(u)-g(x_{n-1})|+|x_n-u| \le \\ &\le K|u-x_{n-1}|+|x_n-u| \xrightarrow[n\to\infty]{} 0\end{aligned} \tag{5}$$

The series defined in (2) corresponds to seeking the solution of the following fixed point problem in which $g_\zeta(x)$ has to be contractive:

$$x_n = \frac{f(x_{n-1})+\zeta\left(f(x_{n-1})-x^d\right)}{f(x_{n-1})}x_{n-1} \Rightarrow x = g_\zeta(x) := \frac{f(x)+\zeta\left(f(x)-x^d\right)}{f(x)}x \tag{6}$$

In the above expression parameter ζ corresponds to the "multiplicative" factor. In order to obtain more "treatable" behavior when the fixed point is zero, it is expedient to introduce a "shift" parameter D in the formula determining the multiplication factor. If $f(x)\to 0$ then $f(x)+D\to D$ and the division in (6) will not become critical:

$$x = g_{\zeta,D}(x) := \frac{f(x)+D+\zeta\left(f(x)+D-\left[x^d+D\right]\right)}{f(x)+D}x \tag{7}$$

As Fig. 3 intuitively shows it, in many cases this iteration can be convergent.

As an illustration in the sequel the cart+inverted pendulum system is considered in which the pendulum's shaft is not actuated but the linear translation of the cart has a drive. This drive can be used for controlling the motion of the pendulum due to the nonlinear coupling of the axes in the Euler-Lagrange Equations of motion of this system.

3 The Mathematical Model of the Cart – Inverted Pendulum System and Simulation Results

The Euler-Lagrange Equations of motion of the system is

$$\begin{bmatrix} M+m & mL\cos\varphi \\ mL\cos\varphi & I+mL^2 \end{bmatrix}\begin{bmatrix}\ddot{x} \\ \ddot{\varphi}\end{bmatrix} = \begin{bmatrix} -b\dot{x}-mL\sin\varphi\dot{\varphi}^2+Q_1 \\ -f\dot{\varphi}+mgL\sin\varphi+Q_2\end{bmatrix}, \mathrm{Q}_2 \equiv 0 \tag{8}$$

in which some "realistic" data were used, i.e. M=1.096 *kg* and m=0.109 *kg* denote the mass of the cart and the pendulum, L= 0.25 *m* and φ [*rad*] is the length and the rotational angle of the pendulum with respect to the upper vertical direction (clockwisely), x [m] denotes the horizontal translation of the cart+pendulum system in the right direction, b=0.1 $N/(m/s)$ and f=0.00218 $kg\times m^2/s$ are viscous friction coefficients, I=0.0034 $kg\times m^2$ denotes the momentum of the arm of the pendulum, and Q_1 [N] denotes the force horizontally accelerating the whole system. The local torque on pendulum's shaft is identical to zero because it is not actuated. The actual state of the system defines a constraint between the 2nd time-derivatives of x and φ according to the lower part of (8). From this constraint the 2nd derivative of x can be expressed as a function of the 2nd derivative of φ. Via substituting it into the upper part of (8) the appropriate Q_1.necessary for achieving this angular acceleration can be calculated. The result is as follows:

$$\ddot{\varphi} = \frac{mL\cos\varphi\left[Q_1 - b\dot{x} - mL\dot{\varphi}^2\sin\varphi\right] + (M+m)(f\dot{\varphi} - mgL\sin\varphi)}{(mL\cos\varphi)^2 - (I + mL^2)(M+m)} \quad (9)$$

It is evident that about φ =0 $d^2\varphi/dt^2$ is decreasing function of Q_1. In the forthcoming simulations the following primitive model was used:

$$Q_1 = -0.05\times\ddot{\varphi} + 15 \quad (10)$$

that is a very rough estimation of such a "negative definite" system. For the transformation ζ=-0.8 and D=200 were applied. (It is worth noting that the forthcoming task differs from the stabilization of the pendulum in its unstable upper position. This special task can be executed with simultaneously stabilizing the horizontal position of the cart via various fuzzy controllers. In the here considered examples x cannot be controlled at all.) The desired second time-derivative of the angle of the pendulum was given on purely kinematical basis. The cycle time of the control was 1 *ms*, but the adaptive loop was activated only in each 10th *ms* and used the averaged angular velocities for 10 *ms* to update the adaptive factor.

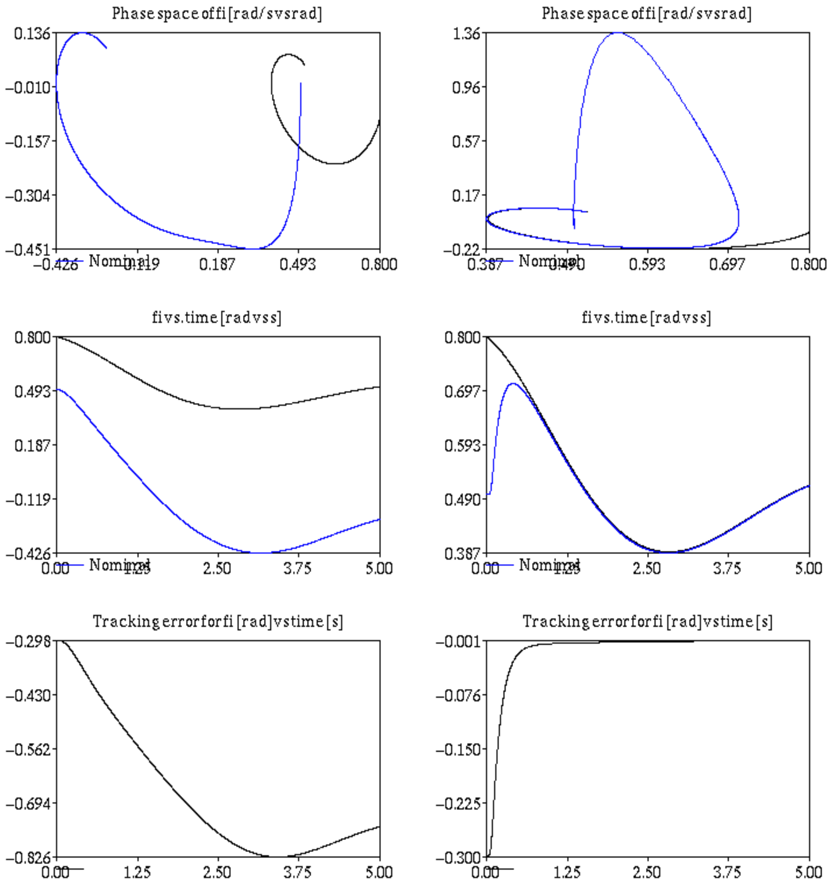


Figure 4

Phase trajectories (1st row) [*rad/s* vs. *rad*], trajectory tracking (2nd row), and trajectory tracking error (3rd row) vs. time [*rad* vs. s] for the non-adaptive (left column) and the adaptive (right column) control of the system for slow motion.

According to Fig. 4 the adaptive control considerably improved the tracking of the phase trajectories.

Similar results are obtained for fast desired motion according to Fig. 5.

To demonstrate the "robustness" of the method the counterpart of Fig. 5 vas calculated for ζ=-0.2 and D=150. The results in Fig. 6 in comparison with Fig. 5 reveal that the method is really robust not only against the variation of the speed of the desired pendulum motion, but also against the modification of the adaptive parameters. It can be seen, too, that the quality of trajectory and phase trajectory tracking was concerned by these parameters but the method remained convergent.

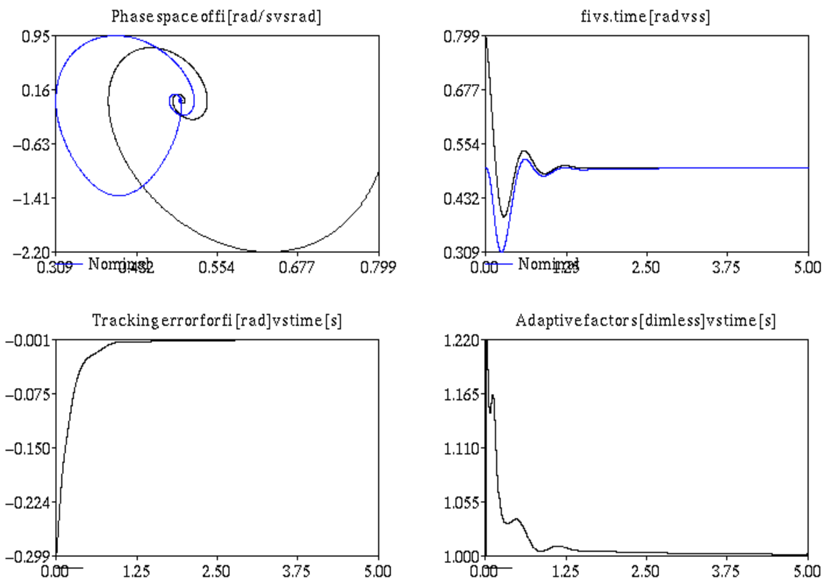


Figure 5

Phase trajectories [*rad/s* vs. *rad*] (upper left), trajectory tracking (upper right), trajectory tracking error [*rad* vs. s] (lower left), and the adaptive parameter vs. time [*dimensionless* vs. *s*](lower right) for the non-adaptive (left column) and the adaptive (right column) control of the system for fast motion.

It is worth noting, too, that according to the expectations the series of the adaptive parameter s_n in each case converged to 1.

Conclusions

In this paper the extension of a Modified Renormalization Transformation based method was introduced and investigated via simulations. While the original modification was designed for positive definite systems, the extension aims at the control of negative definite, Single Input, Single Output (SISO) systems. Such systems occur in Classical Mechanics when a multiple Degree Of Freedom (DOF) system has only one driven axis and this drive is used to control a different axis utilizing the nonlinear dynamic coupling between the axes. As an example the inverted pendulum-cart system was considered in which the drive of the cart's translational DOF was used for controlling the rotational axis of the pendulum.

The example investigated well demonstrates that the method can be properly convergent for realistic physical systems. Of course improper convergence can also occur but with the variation of the very primitive model parameters and the adaptive parameters proper convergence can also be achieved.

The extension results in a whole family of parametric transformations in which

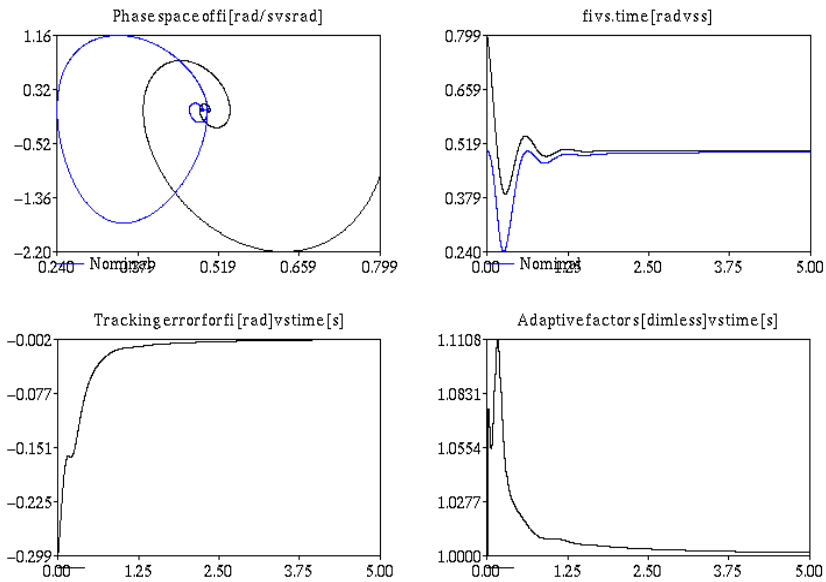


Figure 6

Phase trajectories [*rad/s* vs. *rad*] (upper left), trajectory tracking (upper right), trajectory tracking error [*rad* vs. s] (lower left), and the adaptive parameter vs. time [*dimensionless* vs. *s*](lower right) for the non-adaptive (left column) and the adaptive (right column) control of the system for fast motion and modified control parameters.

two parameters, a “multiplicative” and a “shift” parameter are present. It as shown via simulations that the fact of the convergence is robust against the variation of these parameters and their actual value mainly concerns only the speed of convergence.

Further investigations should aim at the control of negative definite MIMO systems.

References

[1] J. K. Tar, M. Rontó: "Adaptive Control Based on the Application of Simplified Uniform Structures and Learning Procedures", Zbornik Radova, Vol. 24 No. 2, 2000, pp. 174-194, (ISSN: 0351-1804)

[2] Madarász, L.: Methodology of Situational Control and its Applications in Robotics, RAAD '96, 5th International Workshop on Robotics, in Alpe - Adria - Danube Region. Budapest, Hungary, June 10 - 13, 1996, pp. 113-117

[3] Madarász, L.: Situational Control of Electric Power Systems. Bulletins for Applied Mathematics, Technical University of Budapest, April 1984, pp. 65-76

[4] J. K. Tar, I. J. Rudas, Touré Balladji: "Minimum Operation Algebraic Adaptive Control for Robot Based on a Novel Branch of Soft Computing", in the Proc. of EUREL '2000, Vol. 2, Session 3, (European Advancded Robotics Systems - Masterclass and Conference - Robotics 2000), 12-14 April 2000, The University of Salford, Manchester, UK

[5] J. K. Tar, J. F. Bitó, I. J. Rudas, K. Jezernik, Seppo J. Torvinen: "A Lorentz-Group Based Adaptive Control for Electro-Mechanical Systems", Proc. of the 2001 IEEE/RSJ International Conference on Intelligent Robots and Systems, October 29-November 3 2001, Maui, Hawaii, USA., pp. 2045-2050, ISBN 0-7803-6614-X, IEEE Catalog No.: 01CH37180C

[6] József K. Tar, Imre J. Rudas, J. F. Bitó, Krzysztof Kozlowski: “Special Symplectic Transformations Used in Nonlinear System’s Control”, International Journal of Mechanics and Control, Vol. 04 No 02 – 2003, ISSN 1590-8844, pp. 3-8

[7] József K. Tar, Imre J. Rudas, János F. Bitó, and Krzysztof R. Kozłowski: “A Modified Renormalization Algorithm Based Adaptive Control Guaranteeing Complete Stability for a Wide Class of Physical Systems”, Intelligent Systems at the Service of Mankind, (Editors: Wilfried Elmenreich, J. Tenreiro Machado, Imre J. Rudas), UBOOKS, Germany, 2004, Volume 1, pp. 3-13, ISBN 3-935-789-25-3