Colour – balanced Networks

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Abstract: This paper is a specific application of the combinatorial optimization methods for a problem in occuring in practice. We have created a mathematical model which describes clearly and in details real mathematical situation.

Keywords: Spanning tree, optimization problem, algorithm, mathematical model.

1 Introduction

Let us have ***P*** a combinatorial optimization problem by $\boldsymbol{P} = (\mathcal{D}(G), f)$ on the graph $G=(V,E)$, where V is set of vertices, E is set of edges, $\mathcal{D}(G)$ is a finite set of all feasible solutions of problem ***P*** and f is an objective function. We suppose that feasible solutions are spanning trees of the graph G. We denote the weight of the edge $e \in E$ by $w(e) \in \langle 0, \infty)$. What will happen with standard problems if we change values of edges in unique?

If the objective function f is a *sum function* in the form

$$f_1 = \sum_{e \in D} w(e), \text{ where } D \in \mathcal{D}(G),$$

then the *minimal spanning tree problem* $f_1(D) \xrightarrow[D \in \mathcal{D}(G)]{} \min$ is changing to find the number of edges which create spanning tree of the graph G.

If the objective function f is a *bottleneck function* in the form

$$f_2 = \max_{e \in D} w(e), \text{ where } D \in \mathcal{D}(G),$$

then the *bottleneck problem of spanning tree* $f_2(D) \xrightarrow[D \in \mathcal{D}(G)]{} \min$ is changing to trivial problem.

If the objective function f is in the form $f_3 = \max_{e\in D} w(e) - \min_{e\in D} w(e)$, where $D \in \mathcal{D}(G)$, then we have *balanced problem* $f_3(D) \xrightarrow[D\in\mathcal{D}(G)]{} \min$, which gives the result zero for given appreciation of edges in any spanning tree of the graph G. It seems that for classical problems of spanning trees unit values of the edges has not any importance.

If the edges of the graph G are separated into the disjunctive categories $S_1, \dots, S_p$ and we use modified objective function f in the form

$$f(D) = \max_{1\le i\le p} \sum_{e\in S_i\cap D} w(e) - \min_{1\le i\le p} \sum_{e\in S_i\cap D} w(e)$$

then unit values are more important. The solution of this type of problem is in paper [7].

Next example illustrates given problem (see Fig. 1).

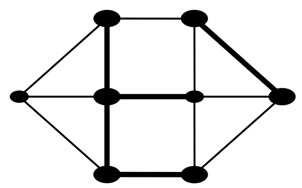

Figure 1

In Figure 1 is illustrated the graph G in which the edges are separated into two categories. The first category is illustrated by tin lines and the second of edges is illustrated by thick lines. The weigths of edges are not given because all edges have the same unit evaluation. The graph has eight vertices and every spanning tree of the graph G must has seven edges. So, the function f has the only minimal value one. The example of such minimal spanning tree of the graph G is in Fig. 2.

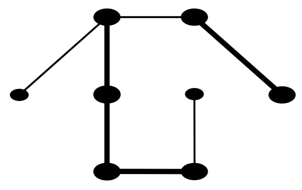

Figure 2

Maximal value of the function is in the case in which the edges of the spanning tree are from the same category. The example of such spanning tree is in Figure 3. This spanning tree has the edges from first category. The value of objective function for this spanning tree is $f(D) = 7$.

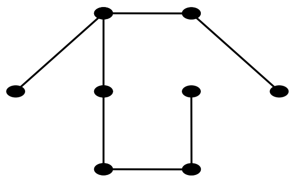

Figure 3

The solution of this problem for larger graphs with greater number of edges is more complicated. Detailed descriptions and algorithms were published in [7]. The problems with their complexity for the case when the weights of the edges are any nonnegative real numbers are described in [3].

Similar problems about categorisation of edges are analyzed in [1, 2, 4, 5, 6, 8, 9].

2 The Same Number of Colours

Let us have an electrical network, which is created by the junctions. We want to guarantee connection of all the junctions in this network. We can connect this network by two types of cables (for simplicity let us assume a red and a blue one). There is a scheme available, where the possible connections are drawn with the blue and the red cables between single junctions. Each pair of junctions can be connected with exactly one type of connecting cable. The technical equipment which provides the production of this electrical network has two assembly machines for each type of cable available. The connection of two junctions with this equipment takes some time but the middle value for the connection of any pair of junctions approaches to the real time needed for their connection. Therefore we can assume that the time needend for the connection of any pair of junctions is constant. These machines can work parallel and independent from one another at the installation of one network. At the start of installation of the next network one machine must wait for the other one, until the last connection will be performed and after that they can start with creation of the next network.The task is to propose the connections in the network the way that each junction will be connected to the network and the installation time of one such network will be as short as possible. This means that we want to find the connections in the network, if it is possible, with the same numberof the blue and the red connections.

We will solve the problem of such colour-balanced connection of single junctions in the network in this paper. We will describe the constructions of mathematical model that we are able to solve by the polynomial algorithm.

3 Matematical Model for Colour – balanced Network

Let us have ***P*** a combinatorial optimization problem by ***P*** = ($\mathcal{D}(G)$, f) on the graph $G=(V,E)$, where $\mathcal{D}(G)$ is a finite set of all feasible solutions of problem ***P*** and f is an objective function. We need to find the optimal feasible solution D from the system of solutions $\mathcal{D}(G)$ such, that:

$$f_2(D) \xrightarrow[D\in\mathcal{D}(G)]{} \min .$$

We are dealing with the optimization problem on graphs where feasible solutions are subsets of set of edges in given graph. This problem was formulated in [7]. Let us have the graph $G=(V, E)$ and the decomposition of set of edges E in p disjunctive categories $S_1, \ldots , S_p$. For each edge $e \in E$ is defined the weight where $w(e)$ is unit. Objective function f has the form:

$$f(D) = \max_{1\le i\le p} \sum_{e\in S_i\cap D} w(e) - \min_{1\le i\le p} \sum_{e\in S_i\cap D} w(e),$$

where solution $D \in \mathcal{D}(G)$ and we suppose that $\max\limits_{e\in\emptyset} w(e) = 0$.

We create matematical model which will best describes problem from the second section. Firstly, we will describe an electrical network that should be produced. This network is represented by the graph G. Vertex set of this graph will present the junctions of the network and the edges of the graph G will present the connections between the junctions. There are two different types of cables for connecting the junctions available. The edges which correspond the blue cables connections will make the category Nr. 1 and the edges corresponding the red cables connections will create the category Nr. 2. We will get the set of edges E which contains edges of two categories, S_1 and S_2. We assign the weight $w(e) = 1$ to each edge. This way is the construnction of the graph finished. Problem ***P*** is given by graph G, by decomposition of edges into categories S_1 and S_2, and by objective function f. System of feasible solutions $\mathcal{D}(G)$ contains all spanning trees D of the graph G. If there was no spanning tree in the graph G, then there would not exist the connection between all junctions in the given network. It means that graph G contains subgraph which represents a part of network without connection to electrical current. Next, we suppose that graph G is connected. So, the graph has spanning tree and system of feasible solutions $\mathcal{D}(G)$ is nonempty. The task is to find such spanning tree D from the set $\mathcal{D}(G)$ for which value of objective function is minimal with respect to weights and categories S_1 and S_2. It means the number of used red and blue connecting cables should be the same if possible or it should differ minimally. The spanning tree with minimal difference between the number of blue and red connecting cables used by production of given network is an optimal solution. It means the both assembly machines will work parallel as long time as possible. It will minimize the waiting time of one machine for another one by transition to the installation of the next electrical network.

To find such spanning tree we will use "Algorithm L_3 – Spanning Tree(+)" published in [7]. In this paper is also decsribed computing complexity of this algorithm. With respect to the results shown in chapter 4 in paper [7] we can generalize the description of our problem from the second chapter. The number of used colours does not have to be limited to 2 colours. We can increase the number of colours to the maximal number of machines which are able to work parallel and independent by production of one network. The finding of solution remains polynomial also by such modified problem definition.

Conclusion

In this paper we have shown the specific option how to apply the mathematical optimization model using graph theory. It is possible to use similar approach not only for our problem, but also for various technical or industrial aplications. There are known many other objective functions exept for the mentioned objective function also. Mathematical modelling of similar problems optimization finds a large versatility in praxis.

References

[1] Averbakh, I., Berman, O.: *Categorized bottleneck-minisum path problems on networks*, Operation Research Letters **16** (1994) 291-297

[2] Berežný, Š., Cechlárová K. and Lacko, V.: *Optimization problems on graphs with categorization of edges*, in: Proc. SOR 2001, eds. V. Rupnik, L. Zadnik-stirn, S. Drobne (Predvor, Slovenia, 2001) 171-176

[3] Berežný, Š., Cechlárová, K. and Lacko, V.: *A polynomially solvable case of optimization problems on graphs with categorization of edges*, in: Proc. of MME′99 (Jinřichův Hradec, 1999) 25-31

[4] Berežný, Š., Lacko, V.: *Balanced problems on graphs with categorization of edges*, Discussiones Mathematicae Graph Theory **23** (2003) 5-21

[5] Berežný, Š., Lacko, V.: *Special problems on graphs with categorization*, in: Proc. of MME′2000 (Praha, 2000) 7-13

[6] Berežný, Š., Lacko, V.: *Easy (polynomial) problems on graphs with categorization*, in: Proc. of New trends of aviation development (Air Force Academy of gen. M. R. Štefánik, Košice, 2000) 36-46

[7] Berežný, Š., Lacko, V.: *The Color-balanced Spanning Tree Problems*, Kybernetika vol. **41** (2005) 539-546

[8] Punnen, A. P.: *Traveling salesman problem under categorization*, Operation Research Letters **12** (1992) 89-95

[9] Richey, M. B., Punnen, A. P.: *Minimum weight perfect bipartite matchings and spanning trees under categorizations*, Discrete Appl. Math. **39** (1992) 147-153