Fuzzy Set Approximation Using Polar Co-ordinates and Linguistic Term Shifting

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Abstract: Fuzzy systems built on sparse rule bases apply special inference techniques. A large family of them can be described by the concept of the general methodology of the fuzzy rule interpolation (GM) [1]. Accordingly to this the conclusion is produced in two steps. First a new rule is interpolated corresponding to the position of the reference point of the observation in each antecedent dimension. Secondly the conclusion is determined by firing this rule. This paper proposes a novel set approximation method (FEAT-p) applicable in the first step of the GM for the determination of the antecedent and consequent sets of the new rule. The suggested technique introduces the concept of the polar cut and calculates the points of the shape of the sets taking into consideration all sets belonging to the actual partition. The method can handle subnormal sets, too.

Keywords: fuzzy rule interpolation, polar cut, fuzzy set approximation, fuzzy set interpolation

1 Introduction

Systems working with fuzzy logic produce their output using a rule-based inference technique. One of the more important features of their knowledge base is the sparse or dense character of the rule base. The collection of the rules can be considered sparse when there exist possible observations (input values) that do not overlap the antecedent part of any rules.

The classical fuzzy reasoning techniques like Zadeh's, Mamdani's, Yager's or even Sugeno's cannot afford an acceptable output in such cases. Therefore such fuzzy inference methods are applied in systems built on sparse rule bases, which can estimate the result in lack of proper rules, too.

There are several applicable techniques in the literature, which more or less satisfy the general condition set introduced in [6] for the evaluation and comparison of the fuzzy rule interpolation methods based on the same fundamentals. These techniques can be divided into two groups depending on whether they are producing the estimated conclusion directly or they are interpolating an intermediate rule first.

Relevant members of the first group are among others the KH method [7] proposed by Kóczy and Hirota, which is the first developed one, the MACI [10], the FIVE [8] introduced by Kovács and Kóczy, the IMUL proposed by Wong, Gedeon and Tikk [12], the method based on the conservation of the fuzziness suggested by Gedeon and Kóczy and the interpolative reasoning based on graduality introduced by Bouchon-Meunier, Marsala and Rifqi [2]. The structure of the methods belonging to the second group can be described best by the generalized methodology (GM) defined by Baranyi et al. in [1]. Typical members of this group are e.g. the technique family proposed by Baranyi et al. in [1], the ST method [14] suggested by Yan, Mizumoto and Qiao and the IGRV [4] developed by Huang and Shen.

Most of the mentioned methods can be characterised by the feature that during the approximation they take into consideration only two rules which flank the observation. A new Fuzzy sEt Approximation Technique (FEAT) based on the shifting of the linguistic terms and its polar cut based implementation (FEAT-p) is suggested in this paper. This method was developed for the first step of the generalized methodology.

The rest of this paper is organized as follows. First the concept of the generalized methodology is recalled, followed by the presentation of the proposed FEAT method and some numerical examples outlining the sensitivity (tuning possibility) of the technique on the weighting factors of the rules.

2 Generalized Methodology of the Fuzzy Rule Interpolation

The method FEAT being introduced in this paper follows the concept of the generalized methodology. Its application area is the first step of this methodology. The generalized methodology of the fuzzy rule interpolation was introduced by Baranyi et al. in [1]. It defines a reference point (RP) for the characterization of the position of the fuzzy sets. There are several possible choices regarding to it, e.g. the centre of the core [1] [2], the centre of the support [2] or the centre of the gravity [4] can play this role. Choosing the reference point could be in strong relation with the selection of the defuzzification method. It can be viewed as a parameter (tuning point) of the technique. The distance of the fuzzy sets, which plays a crucial role in the determination of the approximated result is measured by the Euclidean distance of the reference points of the sets (1).

$$d(A_1, A_2) = |RP(A_1) - RP(A_2)| \tag{1}$$

where A_1 and A_2 are the fuzzy sets, RP is the reference point and d is the distance of the sets.

The generalized methodology consists of two steps. First a new rule is determined whose antecedent part overlaps the observation at least partially and beside this in the case of a one dimensional antecedent universe the reference point of the antecedent set is the same as the reference point of the observation. In case of a multiple dimensional antecedent universe the last statement is valid for all the dimensions.

The approach of the generalized methodology reflects the assumption that there exists regularity in the rule base expressing a continuous mapping between the antecedent and consequent universes. In the first step the concrete form of this mapping is determined corresponding to the reference point(s) of the observation. The approximation process of the new rule can be divided into three stages. First the antecedent is determined using a set interpolation/approximation technique. In [1] the methods SCM, FVL, FPL, SRM I-II are suggested for this task. The method presented in [4], which uses scale and move transformations can also be applied. Secondly the position of the consequent is calculated using for example the fundamental equation of the fuzzy rule interpolation (FEFRI) [7][1] or applying a spline based interpolation. Thirdly the shape of the consequent is determined using the same technique and approach as in the first stage. The aim of this paper is to introduce the technique FEAT as a possible implementation of the first and third stage of the first step of the generalized methodology.

In the second step of the generalized methodology the newly determined rule is considered as a part of the extended rule base. The approximated conclusion, the result of the inference process is determined by firing this rule. Usually the antecedent part of the rule and the observation does not coincide perfectly, therefore some kinds of special single rule reasoning techniques are needed. For example the similarity transfer method introduced in [14], the revision principle based FPL and SRM techniques presented in [9] or the scale and move transformations based method [4] can be applied with success in this step.

3 Fuzzy Set Approximation by Shifting of Linguistic Terms (FEAT-p)

The proposed method is based on the assumption that a better approximation of the real relation between the antecedent and consequent universes can be attained by taking into consideration all the available rules in the rule base. This supposal appeared already right at the beginning of the history of the fuzzy rule interpolation e.g. in [7] or later e.g. in [13]. In spite of the possible advantages most of the methods use only two rules that surround the observation. The technique being presented serves the determination of the antecedent and consequent sets of the new rule in the first and third stage in the first step of the generalized methodology. The method is the same regardless of being applied for an antecedent or a consequent dimension.

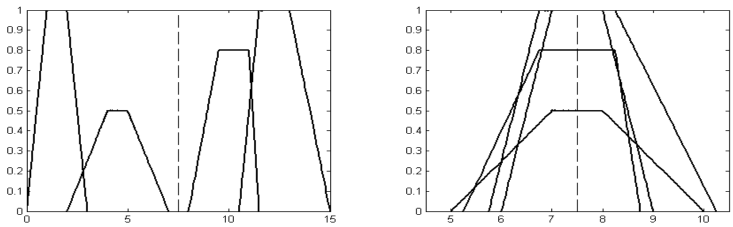


Figure 1

The original partition and the result of the shifting

The starting point is a fuzzy partition with the reference points of the sets determined in advance and the reference point of the observation in the actual dimension/partition. All the sets in the partition belong to the antecedent part of one or more rules.

First all the sets are shifted horizontally in order to reach the coincidence of their reference points with the reference point of the observation. This idea is similar to the concept in [2], but that method uses and translates only the two flanking sets into the location of the observation.

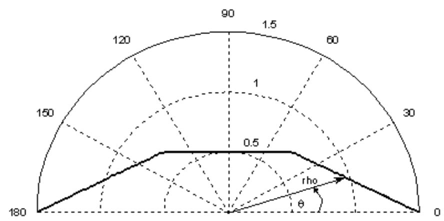


Figure 2

Next the shape of the new set is determined from the collection of the overlapped sets. There are several solutions for this task. In [5] the authors present a solution with low computational complexity based on α-cuts (FEAT-α) for the most popular case of the convex and normal fuzzy (CNF) sets.

Further on a polar cut based technique (FEAT-p) is presented. It can also be applied in cases when the normality condition is not satisfied for all the sets participating in the approximation process, i.e. the height of one or more sets is smaller than 1. Similar to the choice of the reference point the selection of the calculation mode of the shape is also a tuning point.

The suggested shape calculation technique is based on the assumption that an extension and a resolution principle of the fuzzy sets can be defined for polar cuts. The method uses a polar co-ordinate system whose origin coincides with the abscissa of the reference point of the observation. A polar cut is defined by a value pair {*rho*, θ} (Fig. 2) that determines a point on the shape of the linguistic term.

For each polar cut of the approximated set the value *rho* is calculated as weighted average of the *rho* values of the shifted sets for the same θ angle using the formula (2).

$$rho\left\{A_{j\theta}^{a}\right\} = \frac{\sum_{k=1}^{n_j} w_{jk} \cdot rho\left\{A_{jk\theta}\right\}}{\sum_{k=1}^{n_j} w_{jk}} \tag{2}$$

where *rho* denotes the length of a polar cut, j is the actual antecedent dimension, θ is the angle of the actual cut, n_j is the number of the sets in the partition, $A_{jk\theta}$ is the polar cut of the k^{th} set, w_{jk} is the weighting factor of the k^{th} set and $A_{j\theta}^{a}$ is the approximated polar cut.

It seems to be natural that the sets whose original position were in the neighbourhood of the reference point of the observation to exercise higher influence as those ones situated in farther regions of the universe of discourse. Therefore the weighting factor should be dependent on distance. The simplest weighting factor is the reciprocal value of the distance, which can be expressed by the formula (3) with p=1, but there are several recommendations in the literature for more or less analogue cases. For example in [7] the square of the reciprocal value of the distance is suggested (p=2). The authors of [11] propose the use of the reciprocal value of the distance on the m^{th} power (p=m), where m is number of the antecedent dimensions.

$$w_{jk} = \frac{1}{d\left(A_j^*, A_{jk}\right)^p} \tag{3}$$

where A_j^* is the fuzzy set corresponding to the j[th] dimension of the observation. In [13] three variants of the weighting factor called extensibility functions are introduced. These can be described by (4) with p=1 respective p=2 and (5).

$$w_{jk} = \frac{1}{\lambda \cdot d\left(A_j^*, A_{jk}\right)^p} \tag{4}$$

$$w_{jk} = e^{-\lambda \cdot d\left(A_j^*, A_{jk}\right)} \tag{5}$$

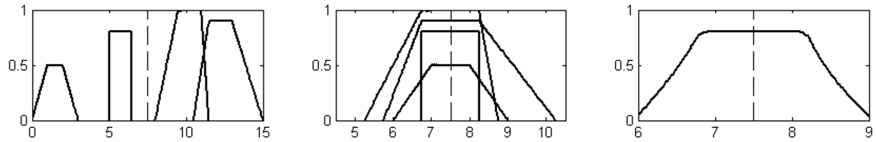
where λ is a positive constant determining the effective extensibility distance. The choice of the weighting factor can add a free parameter to the formula (2) to adjust the sensitivity.

In case of the weighting factors (3) and (4) the distance between the observation and the k[th] set of the partition is in the denominator. Therefore these weighting factors require the supplement of the formula (2), namely if the distance is zero the polar cuts of the approximated set should coincide with the respective polar cuts of the set A_{jk}. Thus the endpoints of the polar cuts are calculated using the formula (6). Hereby the method FEAT-p becomes a fuzzy set interpolation technique. It ensures the fulfilment of the condition 4 from [6], namely the compatibility with the rule base, for the rule interpolation method based on the above mentioned method. It means that if the observation coincides with the antecedent part of a rule, the estimated conclusion should coincide with the consequent part of that rule, too.

$$rho\left\{A_{j\theta}^a\right\} = \begin{cases} \dfrac{\sum_{k=1}^{n_j} w_{jk} \cdot rho\left\{A_{jk\theta}\right\}}{\sum_{k=1}^{n_j} w_{jk}} & d\left(A_j^*, A_{jk}\right) > 0 \\ rho\left\{A_{jk\theta}\right\} & d\left(A_j^*, A_{jk}\right) = 0 \end{cases} \tag{6}$$

4 Numerical Examples

This section intends to present some relevant features of the suggested method outlining the effect of the weighting factor.



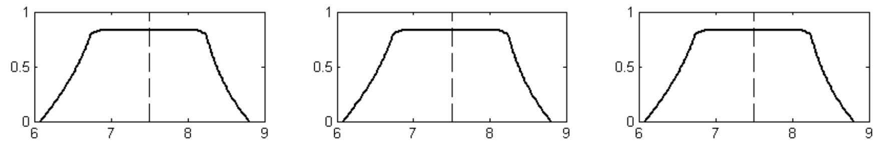
a) The original partition b) Shifted sets c) w=1

Figure 3

Figure 3a shows a partition containing four trapezoidal shaped fuzzy sets with the characteristic points given by (5) and (6). The shifted sets corresponding to the reference point of the observation *RP(A*)*=7.5 are presented in figure 3b. Figure 3c contains the approximated set using the simplest weighting factor (w_i=1).

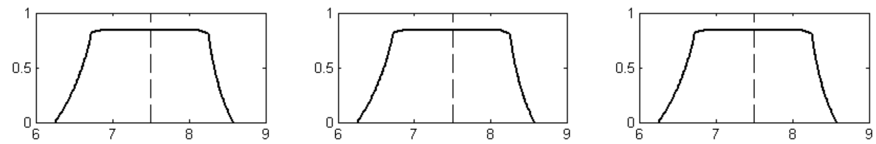
$$A_1 = \begin{bmatrix} 0.0 & 0.0 \\ 1.0 & 0.5 \\ 2.0 & 0.5 \\ 3.0 & 0.0 \end{bmatrix}, \qquad A_2 = \begin{bmatrix} 5.0 & 0.0 \\ 5.0 & 0.8 \\ 6.5 & 0.8 \\ 6.5 & 0.0 \end{bmatrix} \tag{5}$$

$$A_3 = \begin{bmatrix} 8.0 & 0.0 \\ 9.5 & 1.0 \\ 11.0 & 1.0 \\ 11.5 & 0.0 \end{bmatrix}, \qquad A_4 = \begin{bmatrix} 10.5 & 0.0 \\ 11.5 & 0.9 \\ 13.0 & 0.9 \\ 15.5 & 0.0 \end{bmatrix} \tag{6}$$



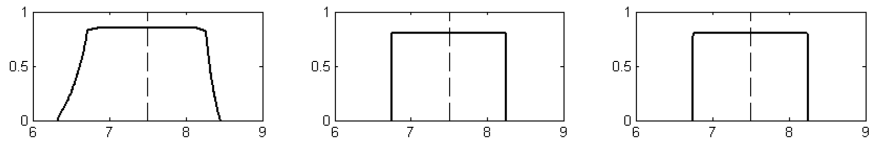
a) λ=1 b) λ=10 c) λ=100

Figure 4



a) λ=1 b) λ=10 c) λ=100

Figure 5



a) λ=1 b) λ=10 c) λ=100

Figure 6

The results presented in Fig. 4 were obtained using the weighting factor (3) with p=1. The results presented in Fig. 5 were obtained using the same weighting factor with p=2. Fig. 6 contains the approximated sets by weighting factor (5). In each case (Fig. 4-6) three λ values were tried. It can be clearly observed that increasing λ the second set (A_2), which is the nearest one to the observation becomes more and more dominant. The weighting factor (5) is the most sensitive to the value of λ.

Conclusions

The rule interpolation based fuzzy inference techniques ensure the acceptable output even in such cases when there are no rules whose antecedent part would overlap the observation at least partially. A group of these techniques follow the general methodology of the fuzzy rule interpolation [1].

In this paper a new method is proposed for the first step of the GM aiming the approximation of the antecedent and consequent parts of the new rule. The technique FEAT-p introduces the concept of the polar cut and uses all the sets belonging to the partition for the calculation of the points of the new set. Its main advantage is that it can handle subnormal sets, too. Moreover its important characteristics are its low computational complexity, comprehensibility and its assumed positive influence on the approximation capability of the GM.

Some numerical examples were presented in order to outline the sensitivity of the technique to the selection of the type and parameters of the weighting factors.

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