Reconfiguration Flexibility Offered by Output State Feedback in Fault-Tollerant Control System

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Abstract: The questions addressed in this paper concern the applicability of the eigenstructure assignment to design reconfigurable control systems in structures unbiased on operating condition changes. An optimization approach to control reconfiguration for system with output state feedback and guaranteed system stability is presented to demonstrate the reconfiguration flexibility and some characteristics of the system modes in fault tollerant control.

Keywords: Model-based system fault diagnosis, system reconfiguration, eigenstructure assignment, computer aided control design

1 Introduction

Fault-tolerant control is concerned with the control of the faulty system. This can be done by changing the control law in the dependency on the structures of the plant which is operated. Presented approach to control reconfiguration is based on eigenstructure assignment for control systems with output state feedback. This technique is concerned with the placing of eigenvalues and their associated eigenvectors, via feedback control laws, to meet closed-loop specifications. Since the interest here is in control reconfiguration using a feedback law that preserves the eigenstructure characteristics describing the nominal closed-loop system, the design task was reformulated for the use of the singular value decomposition (SVD) principle.

The SVD is a tool of great importance in numerical analysis. It provides a stable, reliable transformation to canonical form which yields information about matrix rank, null space and range space not given by other decomposition. The SVD-based tasks can be then solved using a method which preserves of the left and right singular vectors. Furthermore, SVD takes place via orthogonal factors and so the conditioning of a matrix is completely unaffected.

The paper gives some background material on eigenvectors characterization where the idea behind defined system eigenstructure assignment is used, i.e. the output state feedback gain matrix are designed for prescribed nominal and impaired changed system closed-loop eigenvalues. The proposed scheme preserves the most dominant eigenvalues of the closed-loop system structure for nominal and faulty system.

2 Problem Formulation

Specifically, the linear continuous-time multiple input/multiple output (MIMO) system can be specified by the state-space description

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t) \tag{1}$$

$$\mathbf{y}(t) = \mathbf{C}\mathbf{x}(t) \tag{2}$$

where $\mathbf{x}(t) \in R^{n}$, $\mathbf{u}(t) \in R^{r}$, and $\mathbf{y}(t) \in R^{m}$ are the actual state variable, input and output vectors, respectively, $\mathbf{A} \in R^{nxn}$, $\mathbf{B} \in R^{nxr}$, and $\mathbf{C} \in R^{mxn}$ are the system matrices of appropriate dimensions. The system is assumed to be both controllable and observable and it is also assumed, that the input and output matrices are of full rank, that is $rank(\mathbf{B}) = r$, $rank(\mathbf{C}) = m$, and $r = m < n$, , $rank(\mathbf{A}) = n$. Then there exist the matrix $\mathbf{K}$ such that the static output feedback of the form

$$\mathbf{u}(t) = -\mathbf{K}\mathbf{y}(t) = -\mathbf{K}\mathbf{C}\mathbf{x}(t) \tag{3}$$

can be designed.

The freedom that characterizes the placing of the closed-loop system matrix eigenvalues and associated closed-loop eigenvectors, by eigenstructure assignment using output feedback, means that lease

(i) $max(r,m)$ closed-loop eigenvalues can be assigned,

(ii) $max(r,m)$ eigenvectors can be assigned with assigned closed-loop eigenvalues.

In view of (1), (2), (3), the closed-loop system is given by

$$\dot{\mathbf{x}}(t) = (\mathbf{A} - \mathbf{B}\mathbf{K}\mathbf{C})\mathbf{x}(t) \tag{4}$$

$$\mathbf{y}(t) = \mathbf{C}\mathbf{x}(t) \tag{5}$$

The solution of the aforementioned problem is a real $\mathbf{K} \in R^{rxm}$ matrix which can be designed using singular value decomposition (SVD) method for prescribed set of eigenvalues $\{r_j; j = 1,2,...,m\}$.

3 State Transformation

Given state model of the system (1), (2), it is possible to transform the state vector $\mathbf{x}$(t) to the input-closed state space by a matrix $\mathbf{T} \in R^{nxn}$ to yield the realization

$$\mathbf{x}(t) = \mathbf{T}\mathbf{x}_c(t) \tag{6}$$

Since (6) implies

$$\mathbf{T}\dot{\mathbf{x}}_c(t) = \mathbf{A}\mathbf{T}\mathbf{x}_c(t) + \mathbf{B}\mathbf{u}(t) \tag{7}$$

this follows as a consequence

$$\dot{\mathbf{x}}_c(t) = \mathbf{A}_c\mathbf{x}_c(t) + \mathbf{B}_c\mathbf{u}(t) \tag{8}$$

$$\mathbf{y}(t) = \mathbf{C}_c\mathbf{x}_c(t) \tag{9}$$

where

$$\mathbf{A}_c = \mathbf{T}^{-1}\mathbf{A}\mathbf{T}, \quad \mathbf{B}_c = \mathbf{T}^{-1}\mathbf{B} = \begin{bmatrix} \mathbf{I}_r \\ \mathbf{0}_{n-r,r} \end{bmatrix}, \quad \mathbf{C}_c = \mathbf{C}\mathbf{T}, \quad \mathbf{T} = \begin{bmatrix} \mathbf{B} & \begin{vmatrix} \mathbf{I}_r \\ \mathbf{0}_{n,n-r} \end{vmatrix} \end{bmatrix} \tag{10}$$

where $\mathbf{I}_r \in R^{rxr}$ is the identity matrix. With this transform of state variables the gain matrix $\mathbf{K}$ in control law (3) is not changed, i.e.

$$\mathbf{u}(t) = -\mathbf{K}\mathbf{C}\mathbf{T}\mathbf{T}^{-1}\mathbf{x}(t) = -\mathbf{K}\mathbf{C}_c\mathbf{x}_c(t) \tag{11}$$

and the closed-loop system equation (4) is transformed to

$$\dot{\mathbf{x}}_c(t) = (\mathbf{A}_c - \mathbf{B}_c\mathbf{K}\mathbf{C}_c)\mathbf{x}_c(t) \tag{12}$$

As one can see, the state-transformation does not affect the output feedback gain matrix and this is also true for the eigenvalues of the transformed system.

4 Feedback Gain Matrix

For any pair of closed-loop eigenvalues and their associated closed-loop eigenvectors $\{(r_j, \mathbf{h}_j),\ j = 1,2,...,m, m+1,...,n\}$, generally complex conjugate, holds

$$(\mathbf{A}_c - \mathbf{B}_c\mathbf{K}\mathbf{C}_c)\mathbf{h}_j = r_j\mathbf{h}_j, \quad j = 1,2,\ldots,m \tag{13}$$

where $\mathbf{h}_j$ is the j-th right eigenvector. This can be also written as

$$\begin{bmatrix} r_j\mathbf{I} & -\mathbf{A}_c & | & \begin{matrix}\mathbf{I}_r \\ \mathbf{0}_{n-r,r}\end{matrix} \end{bmatrix} \begin{bmatrix} \mathbf{h}_j \\ \mathbf{K}\mathbf{C}_c\mathbf{h}_j \end{bmatrix} = \mathbf{L}_{cj} \begin{bmatrix} \mathbf{h}_j \\ \mathbf{K}\mathbf{C}_c\mathbf{h}_j \end{bmatrix} = \mathbf{0}, \qquad j = 1,2,\ldots,m \tag{14}$$

The SVD procedure applied to the matrix $\mathbf{L}_{cj}$ results

$$\begin{bmatrix} \mathbf{q}_{j1}^T \\ \vdots \\ \mathbf{q}_{jn}^T \end{bmatrix} \mathbf{L}_{cj} \begin{bmatrix} \mathbf{p}_{j1} & \cdots & \mathbf{p}_{jn} & \mathbf{p}_{j,n+1} & \cdots & \mathbf{p}_{j,n+r} \end{bmatrix} = \\ = \begin{bmatrix} \sigma_{j1} & & & & & \\ & \ddots & & \mathbf{0}_{j,n+1} & \cdots & \mathbf{0}_{j,n+r} \\ & & \sigma_{jn} & & & \end{bmatrix} \tag{15}$$

where $\{\mathbf{q}^T_{jl} \in C^n,\ l = 1,2,...,n\}$, $\{\mathbf{p}_{jl} \in C^{n+r},\ l = 1,2,...,n+r\}$, are the sets of left and right singular vectors, respectively, $\{\sigma_{jl} \in R^n,\ l = 1,2,...,n\}$ is the set of singular values of $\boldsymbol{L}_j$ matrix, and $\{\mathbf{0}_{jl} \in R^n,\ l = n+1,\ ...\ \ n+r\}$ is the set of zero-values column vectors. Using this, the number of input variables, r, (r = rank($\mathbf{B}$)) determines the dimension of the subspace in which achievable eigenvectors must reside and orientation of the subspace is determined by the matrices ($\mathbf{A}$, $\mathbf{B}$) and the desired eigenvalue r_j.

It is evident from (15), that all column vectors $\{\mathbf{p}_{jk},\ k = r+1, \ldots, n+r\}$, obtained by SVD procedure, satisfy the condition $\mathbf{L}_{cj}\mathbf{p}_{jk} = \mathbf{0}$ and can be partitioned as in (14). Then using the SVD-based solutions for all desired eigenvalues it is possible to compute matrix $\mathbf{K}$. If only one vector $\mathbf{p}_{jk}$, $k = n+1,\ldots, n+r$ from the set associated with r_j is selected for design (no multiple eigenvalue r_j be chosen), this vector can be partitioned as

$$\mathbf{p}_{jk} = \begin{bmatrix} \mathbf{v}_j \\ \mathbf{w}_j \end{bmatrix} = \begin{bmatrix} \mathbf{h}_j \\ \mathbf{K}\mathbf{h}_j \end{bmatrix} = \begin{bmatrix} \mathbf{v}_j \\ \mathbf{K}\mathbf{v}_j \end{bmatrix} \tag{16}$$

$$\mathbf{p}_{jk} = \begin{bmatrix} \mathbf{v}_j + i\mathbf{m}_j \\ \mathbf{w}_j + i\mathbf{n}_j \end{bmatrix} = \begin{bmatrix} \mathbf{h}_j \\ \mathbf{Kh}_j \end{bmatrix} = \begin{cases} \begin{bmatrix} \mathbf{v}_j \\ \mathbf{w}_j \end{bmatrix} = \begin{bmatrix} \mathbf{v}_j \\ \mathbf{Kv}_j \end{bmatrix} \\ \begin{bmatrix} \mathbf{m}_j \\ \mathbf{n}_j \end{bmatrix} = \begin{bmatrix} \mathbf{v}_{j+1} \\ \mathbf{Kv}_{j+1} \end{bmatrix} \end{cases} \tag{17}$$

for real or for complex desired closed-loop eigenvalue r_j , respectively. Then using the SVD-based solutions for all desired eigenvalues one can construct the matrix equation

$$\mathbf{W} = \begin{bmatrix} \mathbf{w}_1 & \mathbf{w}_2 & \cdots & \mathbf{w}_m \end{bmatrix} = \mathbf{KC}_c \begin{bmatrix} \mathbf{v}_1 & \mathbf{v}_2 & \cdots & \mathbf{v}_m \end{bmatrix} = \mathbf{KC}_c\mathbf{V} \tag{18}$$

Consequently, the feedback gain matrix **K** is given by

$$\mathbf{K} = \mathbf{W}(\mathbf{C}_c\mathbf{V})^{-1} \tag{19}$$

where $\mathbf{W} \in R^{\mathrm{rxm}}$, $\mathbf{V} \in R^{\mathrm{nxm}}$, respectively.

5 Control Reconfiguration

The system faults modify the system properties, which can be now described by state-space equations

$$\dot{\mathbf{x}}(t) = \mathbf{A}_f\mathbf{x}(t) + \mathbf{B}_f\mathbf{u}(t) \tag{20}$$

$$\mathbf{y}(t) = \mathbf{C}_f\mathbf{x}(t) \tag{21}$$

where $\mathbf{A}_\mathrm{f} \in R^{\mathrm{nxn}}$, $\mathbf{B}_\mathrm{f} \in R^{\mathrm{nxr}}$, and $\mathbf{C}_\mathrm{f} \in R^{\mathrm{mxn}}$ are the system matrices of the same dimensions with the matrices of the nominal state-space model.

The reconfiguration task is to include a new stabilizing feedback control law

$$\mathbf{u}(t) = -\mathbf{K}_f\mathbf{y}(t) = -\mathbf{K}_f\mathbf{C}_f\mathbf{x}(t) \tag{22}$$

such that the new closed-loop system matrix $\mathbf{A}_f - \boldsymbol{B}_f\boldsymbol{K}_f\boldsymbol{C}_f$ can capture as much of the eigenstructure of the nominal closed-loop system matrix as possible.

Analogously to (10) another transform matrix $\mathbf{T}_f$ can be defined e.g. as

$$\mathbf{B}_{fc} = \mathbf{T}_f^{-1}\mathbf{B}_f = \begin{bmatrix} \mathbf{I}_r \\ \mathbf{0}_{n-r,r} \end{bmatrix}, \quad \mathbf{T}_f = \begin{bmatrix} \mathbf{B}_f & \begin{matrix} \mathbf{I}_r \\ \mathbf{0}_{n,n-r} \end{matrix} \end{bmatrix} \tag{23}$$

In the new state-coordinates, the impaired system is described by

$$\mathbf{A}_{fc} = \mathbf{T}_f^{-1}\mathbf{A}_f\mathbf{T}_f, \quad \mathbf{B}_{fc} = \mathbf{T}_f^{-1}\mathbf{B}_f, \quad \mathbf{C}_{fc} = \mathbf{C}_f\mathbf{T}_f \tag{24}$$

In order to maintain the performance of the nominal closed-loop system, the control law (22) must be determined in such a way that the set of the impaired system matrix eigenvalues $eig(\mathbf{A}_{fc} - \mathbf{B}_{fc}\mathbf{K}_f\mathbf{C}_{fc})$ includes the m most dominant eigenvalues $\{r_j,\ j = 1,2,...,m\}$ of the nominal system and the corresponding eigenvectors of the impaired system have to be as closed as possible to the corresponding eigenvectors of nominal system.

For desired eigenvalues r_j, $j = 1,2,...,$m, now holds

$$(\mathbf{A}_{fc} - \mathbf{B}_{fc}\mathbf{K}_f\mathbf{C}_{fc})\mathbf{h}_{fj} = r_j\mathbf{h}_{fj}, \quad j = 1,2,\ldots,m \tag{25}$$

$$\begin{bmatrix} r_j\mathbf{I} - \mathbf{A}_{fc} & | & \begin{matrix}\mathbf{I}_r \\ \mathbf{0}_{n-r,r}\end{matrix}\end{bmatrix}\begin{bmatrix}\mathbf{h}_{fj} \\ \mathbf{K}_f\mathbf{C}_{fc}\mathbf{h}_{fj}\end{bmatrix} = \mathbf{L}_{fcj}\begin{bmatrix}\mathbf{h}_{fj} \\ \mathbf{K}_f\mathbf{C}_{fc}\mathbf{h}_{fj}\end{bmatrix} = \mathbf{0}, \quad j = 1,2,\ldots,m \tag{26}$$

respectively. Using the same design step as (15) – (19) are, starting from (26) one can compute gain matrix $\mathbf{K}_f$ for control law (22).

Designed eigenstructure characterizes the behavior of the closed-loop system, since the eigenvalues determine the stability and the eigenvectors the controlability and observability of each system mode.

6 Optimization

Transformation matrices $\mathbf{T}$ or $\mathbf{T}_f$ are not unique and there exist other structures of this matrices given by permutations in rows and coloms of the basic structure, i.e.

$$\mathbf{T} = \begin{bmatrix}\mathbf{B} & \begin{vmatrix}\mathbf{I}_r \\ \mathbf{0}_{n,n-r}\end{vmatrix}\end{bmatrix} \approx \mathbf{T}_h = \begin{bmatrix}\mathbf{B} & \left|\begin{pmatrix}\mathbf{I}_r \\ \mathbf{0}_{n,n-r}\end{pmatrix}_h\right|\end{bmatrix} \tag{27}$$

The optimization criterion in the frame of the transformed impaired system is

$$J = \min_h \left\| ([\mathbf{h}_1\,\mathbf{h}_2 \ldots \mathbf{h}_m] - [\mathbf{h}_{if}\,\mathbf{h}_{2f} \ldots \mathbf{h}_{mf}])_h \right\|_F^2 \tag{28}$$

where $\mathbf{h}_i$, i = 1,2,…,m are the right eigenvectors associated with desired eigenvalues.

The procedure outlined above easily is extended to the case of state-feedback, but its reconfiguration flexibility is limited, since all the eigenvalues of the nominal closed-loop system have to be preserved in both control structures.

7 Illustrative Example

The continuous time system models are given by (1), (2), and (20), (21), where

$$\mathbf{A}=\begin{bmatrix} -0.0582 & 0.0651 & 0.0000 & -0.1710 \\ -0.3030 & -0.6850 & 1.1090 & 0.0000 \\ -0.0715 & -0.6580 & -1.9470 & 0.0000 \\ 0.0000 & 0.0000 & 1.0000 & 0.0000 \end{bmatrix}, \quad \mathbf{B}=\begin{bmatrix} 0.0000 & 1 \\ -0.0541 & 0 \\ -1.1100 & 0 \\ 0.0000 & 0 \end{bmatrix}$$

$$\mathbf{A}_f=\begin{bmatrix} -0.0582 & 0.1000 & 0.0000 & -0.1710 \\ -0.1030 & -0.6850 & 1.1090 & 0.0000 \\ -0.0715 & -0.6580 & -0.9800 & 0.0000 \\ 0.0000 & 0.0000 & 1.5000 & 0.0000 \end{bmatrix}, \quad \mathbf{B}_f=\begin{bmatrix} 0.00 & 0.9 \\ -0.09 & 0.0 \\ -1.11 & 0.0 \\ 0.00 & 0.0 \end{bmatrix}$$

$$\mathbf{C}=\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \quad \mathbf{C}_f=\begin{bmatrix} 0.9 & 0 & 0 & 0.0 \\ 0.0 & 0 & 0 & 0.7 \\ 0.0 & 0 & 1 & 0.0 \end{bmatrix}$$

Solving for all matrices with possible permutation of matrix elements in (27) the next nonsingular transform matrices were obtained

$$\mathbf{T}_1=\begin{bmatrix} 0.0000 & 1 & 0 & 0 \\ -0.0541 & 0 & 0 & 0 \\ -1.1100 & 0 & 1 & 0 \\ 0.0000 & 0 & 0 & 1 \end{bmatrix}, \mathbf{T}_2=\begin{bmatrix} 0.0000 & 1 & 0 & 0 \\ -0.0541 & 0 & 0 & 0 \\ -1.1100 & 0 & 0 & 1 \\ 0.0000 & 0 & 1 & 0 \end{bmatrix}, \mathbf{T}_3=\begin{bmatrix} 0.0000 & 1 & 0 & 0 \\ -0.0541 & 0 & 1 & 0 \\ -1.1100 & 0 & 0 & 0 \\ 0.0000 & 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{T}_4=\begin{bmatrix} 0.0000 & 1 & 0 & 0 \\ -0.0541 & 0 & 0 & 1 \\ -1.1100 & 0 & 0 & 0 \\ 0.0000 & 0 & 1 & 0 \end{bmatrix}, \mathbf{T}_{1f}=\begin{bmatrix} 0.00 & 0.9 & 0 & 0 \\ -0.09 & 0.0 & 0 & 0 \\ -1.11 & 0.0 & 1 & 0 \\ 0.00 & 0.0 & 0 & 1 \end{bmatrix}, \mathbf{T}_{2f}=\begin{bmatrix} 0.00 & 0.9 & 0 & 0 \\ -0.09 & 0.0 & 0 & 0 \\ -1.11 & 0.0 & 0 & 1 \\ 0.00 & 0.0 & 1 & 0 \end{bmatrix}$$

$$\mathbf{T}_{3f}=\begin{bmatrix} 0.00 & 0.9 & 0 & 0 \\ -0.09 & 0.0 & 1 & 0 \\ -1.11 & 0.0 & 0 & 0 \\ 0.00 & 0.0 & 0 & 1 \end{bmatrix}, \mathbf{T}_{4f}=\begin{bmatrix} 0.00 & 0.9 & 0 & 0 \\ -0.09 & 0.0 & 0 & 1 \\ -1.11 & 0.0 & 0 & 0 \\ 0.00 & 0.0 & 1 & 0 \end{bmatrix}$$

and used for given transformation of the input matrices. Performing the calculations wit the transform matrix $\mathbf{T}_2 = \mathbf{T}$, to illustrate the design step procedure, the numerical values for the $\mathbf{A}_c$, $\mathbf{B}_c$, and $\mathbf{C}_c$, from (10), are given by

$$\mathbf{A}_c = \begin{bmatrix} 22.0690 & 5.6007 & -20.4991 & 0.0000 \\ -0.0035 & -0.0582 & 0.0000 & -0.1710 \\ 26.6948 & 6.1453 & -24.7010 & 0.0000 \\ -1.1100 & 0.0000 & 1.0000 & 0.0000 \end{bmatrix}$$

$$\mathbf{B}_c = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad \mathbf{C}_c = \begin{bmatrix} 0.00 & 1 & 0 & 0 \\ 0.00 & 0 & 0 & 1 \\ -1.11 & 0 & 1 & 0 \end{bmatrix}$$

This nominal controller is to be designed using the desired eigenvalues

$\mathbf{r} = [-0.5973,\ -1.5 + 2.0i, -1.5 - 2.0i]$

Having obtained numerical values for the $\mathbf{A}_c$, $\mathbf{B}_c$, and $\mathbf{C}_c$, the matrices $\mathbf{L}_{c1}$, $\mathbf{L}_{c2}$, be construct from (14) using eigenvalue r_1 = - 0.5973, r_2 = -1.5 + 2.0i, as

$$\mathbf{L}_{c1} = \begin{bmatrix} -22.6663 & -5.6007 & 20.4991 & 0.0000 & 1 & 0 \\ 0.0035 & -0.5391 & 0.0000 & 0.1710 & 0 & 1 \\ -26.6948 & -6.1453 & 24.1037 & 0.0000 & 0 & 0 \\ 1.11000 & 0.0000 & -1.0000 & -0.5973 & 0 & 0 \end{bmatrix}$$

$$\mathbf{L}_{c2} = \begin{bmatrix} -23.5690 + 2.0i & -5.6007 & 20.4991 & 0.0000 & 1 & 0 \\ 0.0035 & -1.4418 + 2.0i & 0.0000 & 0.1710 & 0 & 1 \\ -26.6948 & -6.1453 & 23.2010 + 2.0i & 0.0000 & 0 & 0 \\ 1.11000 & 0.0000 & -1.0000 & -1.5000 + 2.0i & 0 & 0 \end{bmatrix}$$

Applying MATLAB file *svd* to the matrices $\mathbf{L}_{c1}$, $\mathbf{L}_{c2}$, yields (the last two right singular column vectors)

$$[\mathbf{p}_{15} \quad \mathbf{p}_{16}] = \begin{bmatrix} 0.59810 & -0.3153 \\ 0.2617 & 0.7137 \\ 0.7290 & -0.1673 \\ -0.1092 & -0.3059 \\ 0.0762 & 0.2787 \\ 0.1576 & 0.4382 \end{bmatrix}$$

$$
[\mathbf{p}_{25} \quad \mathbf{p}_{26}] = \begin{bmatrix} 0.6365+0.0732i & 0.0317-0.1377i \\ 0.0241-0.0541i & 0.2270+0.2848i \\ 0.7393+0.0062i & 0.0888-0.0907i \\ -0.0319+0.0075i & 0.0070-0.0321i \\ 0.1288+0.0228i & -0.0769+0.1446i \\ -0.0703-0.1278i & 0.8955-0.0374i \end{bmatrix}
$$

and matrices **V** and **W** can be form using $\mathbf{p}_{15}$ and $\mathbf{p}_{26}$ to get

$$
\mathbf{V} = \begin{bmatrix} 0.5980 & 0.0317 & -0.1377 \\ 0.2617 & 0.2270 & 0.2848 \\ 0.7290 & 0.0888 & -0.0907 \\ -0.1092 & 0.0070 & -0.0321 \end{bmatrix}, \quad \mathbf{W} = \begin{bmatrix} 0.0762 & -0.0769 & 0.1446 \\ 0.1576 & 0.8955 & -0.0374 \end{bmatrix}
$$

Using (19) to compute the control gain matrix **K** gives result

$$
\mathbf{K} = \mathbf{W}(\mathbf{C}_c\mathbf{V})^{-1} = \begin{bmatrix} 6.6606 & -2.2639 & -29.3457 \\ -28.7214 & 11.4608 & 136.8414 \end{bmatrix}
$$

The eigenvalues we are looking for are

$$
\mathbf{r} = eig(\mathbf{A}-\mathbf{BKC}) = [-0.5973,\ -1.5+2.0i, -1.5-2.0i, -3.0853]
$$

i.e. the desired eigenvalues are dominant.

Supposing that the impaired system was changed due the system fault and the state-space matrices of this model are $\mathbf{A}_f$, $\mathbf{B}_f$,and $\mathbf{C}_f$, respectively, similarly can be used the transform matrix $\mathbf{T}_{3f}$ and the numerical values for the $\mathbf{A}_{cf}$, $\mathbf{B}_{cf}$, and $\mathbf{C}_{cf}$, from (24) are

$$
\mathbf{A}_{fc} = \begin{bmatrix} -1.0334 & 0.0580 & 0.5928 & 0.0000 \\ -0.0100 & -0.0582 & 0.1111 & -0.1900 \\ -1.2623 & 0.0875 & -0.6316 & 0.0000 \\ -1.6650 & 0.0000 & 0.0000 & 0.0000 \end{bmatrix}, \quad \mathbf{B}_c = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}
$$

$$
\mathbf{C}_c = \begin{bmatrix} 0.00 & 0.81 & 0 & 0.0 \\ 0.00 & 0.00 & 0 & 0.7 \\ -1.11 & 0.00 & 0 & 0.0 \end{bmatrix}
$$

Using the same desired eigenvalues, resulting control gain matrix $\mathbf{K}_f$, optimal to **K** in the sence of minimal performace index (28), is

$$
\mathbf{K}_f = \begin{bmatrix} 0.2464 & -16.1160 & -5.0640 \\ 3.2194 & -49.3948 & -78.0429 \end{bmatrix}
$$

and the eigenvalues we are looking for are

$$\mathbf{r} = eig(\mathbf{A}_f - \mathbf{B}_f\mathbf{K}\mathbf{C}_f) = [-0.5973,\ -1.5+2.0i, -1.5-2.0i, -6.3550]$$

One can verify that some solutions for some nonsingular transform matrices are unstable.

Conclusions

The paper presents some properties of the singular value decomposition and its application in eigenstructure assignment for output state feedback controller design tasks. This approach was taken in designing the reconfigurable control system with output feedback, where some special considerations were given to eigenvector/eigenvalue optimization for a system structures unbiased on system faults. The presented method offer a powerful way to select robust reconfigurable control based on known state-space models of a dynamic system.

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