

# Features and Improvement of Non-Linear Robot-Controllers by Simulation

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*Abstract: Robot-controllers operate generally under remarkable change of values of inertia. In spite of this it is necessary to keep the critically damped, minimal time approach of target. The developed methods of adaptive control are in principle based on laws of linear controls, however the operation of the realized equipments go over to non-linear domain. Effects of this are appearing in the following features of path.*

*Keywords: Robot, position control, adaptive control, non-linear control, numeric simulation*

## 1 Introduction

This paper deal with the features of non-linear control and introduces with some model-process for improve the adaptive properties. Theses processes is based on the analysis of relationship in the functions through the use of control model simulation.

In position control of robots is desirable in the aperiodic approach of the target and to keep the robot from overshooting.

It is known that many robot controllers are non-linear. The gain of control loop for a critically damped case is calculable with classic methods for linear control loops, however we can only give an approach for non-linear control loops. Using advanced simulation methods and tools (eg. Matlab Simulink) we can model the real system nearly in the desired level of the operation, consequently we can derive such results from the simulation which are used in the design.

The aim of this thesis is to investigate and improve a model for adaptive non-linear position-control, in which an experimental function takes the influences of the changing inertia into consideration for the gain, while the robot is moving. These functions are based on running many simulations.

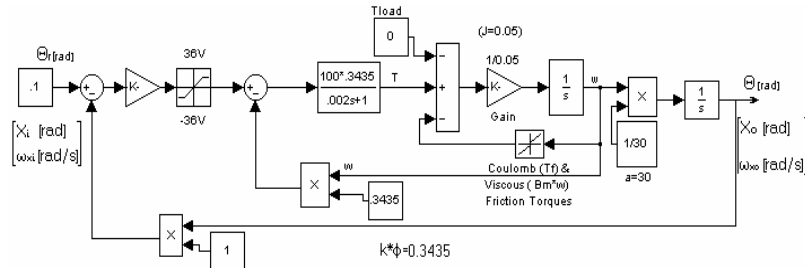


Figure 1

Model one of the position controlled robot link driven by permanent magnet DC motor

The Figure 1 shows the model one of the position controlled robot-link driven by a permanent magnet DC motor.

There is no current limitation in this model. The calculated heat-loss  $I^2Rt$  is negligible.

The gain here is P-type, the loop is 1-type because of the integration.  $X_i$  here is a reference step-signal. We can preset the value of Coulomb and Viscous friction in the model. With regarding to the effect of the friction reduction of an PWM controlled motor, we set the value of Coulomb friction low. Practically there is no backlash in modern robot-constructions.

The value of the load torque is zero in steady-state and there is no contact force during the movement of the robot TCP, since the movement of the robot is unrestricted. The gear-ratio is 1:30.

## 2 The Features of Non-Linearity

In this case the saturation-type non-linearity of the control-loop derives from the limit of the voltage (36V in our case). This 36V value permits a speed of 3.5 rad/s for the 1<sup>st</sup> link. This speed is sufficient to follow a relatively low frequency or amplitude input signals (Figure 2).

The one of the well-known features is that the value of the output signal depends on also the magnitude of the input signal.

The applicable method of the describing-function is based on the following conditions:

- parameter values of non-linear elements are stable in time,
- there is no constant or subharmonic components in the output signal,

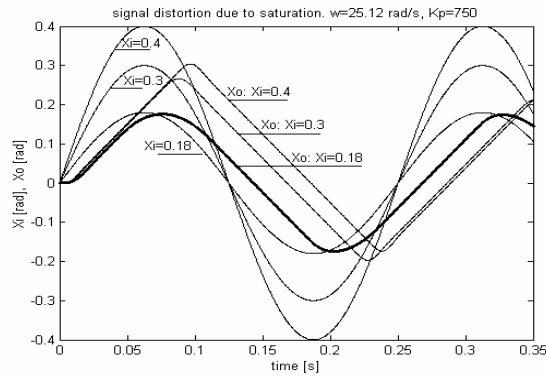


Figure 2

Distortion of output signal due to the saturation.  $\omega=25.12$  rad/s,  $X_i$  variable

- the output signal is periodic and its frequency is equal to the frequency of the input signal,
- there is only one non-linearity in the loop.

The disadvantages are detailed as follow:

- verification of accuracy is difficult in this way;
- the process can be used for only quantitative estimation;
- the behavior in time-domain is not can be estimated.

The feature of the saturation-type non-linearity, which is applied in our model is shown in Figure 3. In the describing-function we take only the 1<sup>st</sup> harmonic into account [5]:

$X_{o1}(t)=B_1 \sin \omega t+A_1 \cos \omega t$ . Be it:

$C_1=(B_1^2+A_1^2)^{1/2}$ ,  $\sin \varphi_1=A_1/C_1$ ,  $\cos \varphi_1=B_1/C_1$ . With this we describe:

$X_i(j\omega)=B$ ,  $X_{o1}(j\omega)=C_1 e^{j\varphi_1}$ .

According to the describing-function, the quotient of the signal amplitudes of output  $X_o$  and input  $X_i$  can be given by the following expression:

$N(B, j\omega)=C_1(B, j\omega) e^{j\varphi_1(B, j\omega)}/B$ , where  $X_i(j\omega)=B$ ,  $X_{o1}(j\omega)=C_1 e^{j\varphi_1}$ . The figure shows that in linear section the transfer constant is  $A_n$ . If the amplitude of input is sinusoidal, if  $X_i=B > b$

then output  $X_o$  is shaped cut away.

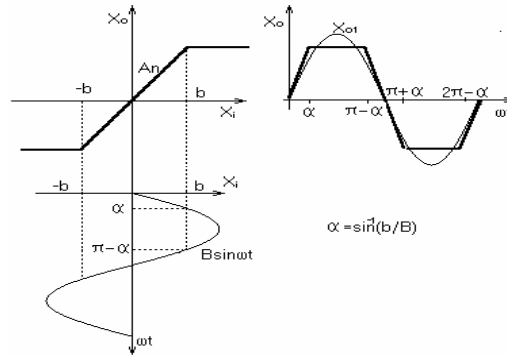


Figure 3

The describing-function of saturation-type non-linearity

$$X_o(t) = A_n X_i(t) = A_n B \sin \omega t, \quad 0 \leq \omega t \leq \alpha,$$

$$X_o(t) = A_n b, \quad \alpha \leq \omega t \leq \pi - \alpha,$$

$$X_o(t) = -A_n X_i(t) = -A_n B \sin \omega t, \quad \pi - \alpha \leq \omega t \leq \pi,$$

where  $b$  is an input-value that produces the saturation, and

$$\alpha = \sin^{-1}(b/B), \quad A_1 = 0,$$

$$B_1 = (2/\pi) \int_0^\alpha A_n B \sin^2 \omega t d\omega t + 2/\pi \int_{\pi-\alpha}^\pi A_n b \sin \omega d\omega t +$$

$$+ 2/\pi \int_{\pi-\alpha}^\pi A_n B \sin^2 \omega t d\omega t. \text{ After integration}$$

$$B_1 = (2/\pi) A_n B [\alpha + \sin 2\alpha/2].$$

$$\text{Here } C_1 = B_1 \text{ and } \phi_1 = 0, \quad N(B, j\omega) = (2/\pi) A_n [\alpha + \sin 2\alpha/2] = N(B), \quad B \geq b.$$

If  $B > b$ , the value of the function is less than  $A_n$ , but if  $B \leq b$ , the behavior of the function is the same as the behavior of the linear element with value  $A_n$ .

Here the maximum speed of angle is  $\omega = 3.5$  rad/s, and is unable to follow a bigger speed input signal. The maximum gradient of output  $X_o$  means the limit of speed, derived from the voltage limit of the motor.

Another important feature of control is the effect of saturation-type non-linearity for on accuracy.

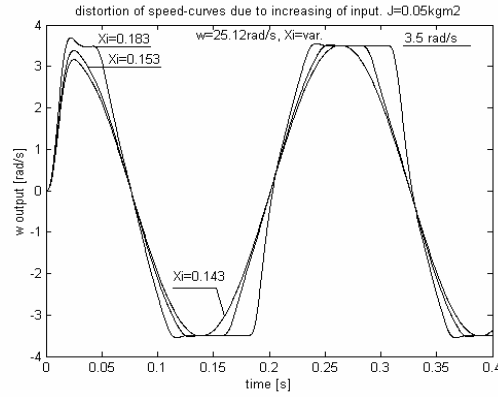


Figure 4

Distortion of speed-curves due to non-linearity

If the input of the control demands a greater-than possible speed at the output due to the saturation, then the control output speed can cause very significant distortion (see Figure 2). Parameters of employed sinusoid signals for investigation and the computed maximum values of speeds from this are shown in title of figures.

## 2.1 The Failures of the Describing-function Methods

This is an approximate method because it disregards the higher harmonics appeared on of the output of the non-linear element, so assume that signal output is entirely sinusoid. This method gives a good solution if the neglected harmonics affect no the operation of the control significantly. For example at the describing-function of the saturation:

$$N(\mathbf{B}j\omega) = (2/\pi)A_n[\alpha + \sin 2\alpha/2].$$

Compose the rate of the 3<sup>rd</sup> harmonic and the sinusoid input signal, that is to say compose the describing-function relative to 3<sup>rd</sup> harmonic:

$$N_3(\mathbf{B}) = 4A_n (b/\mathbf{B}) [1 - (b/\mathbf{B})^2]^{3/2} / 3 \pi.$$

The curves of the  $N(\mathbf{B})/A_n$  and  $N_3(\mathbf{B})/A_n$  relative describing-functions computed by Matlab are shown in Figure 5.

One can see that the 3<sup>rd</sup> harmonic can be neglected at low input signals (where  $\mathbf{B} \leq b$ ), but at higher input signals the value of the 3<sup>rd</sup> harmonic agrees the based-harmonic in order of magnitude. The Figure 5 shows the speed curves with constant circle-frequency  $\omega_{in} = 25\text{ rad/s}$  and variable amplitude input signals, by near the non-linearity.

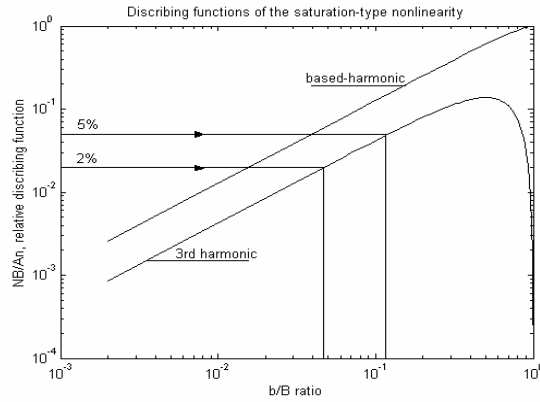


Figure 5  
The ratio of the based and the 3<sup>rd</sup> harmonics

The analyze done in an analogue case in Figure 6. It is shown well the attendance of 3<sup>rd</sup> and 5<sup>th</sup> harmonics, according to the aforementioned features of the describing-function and its applications.

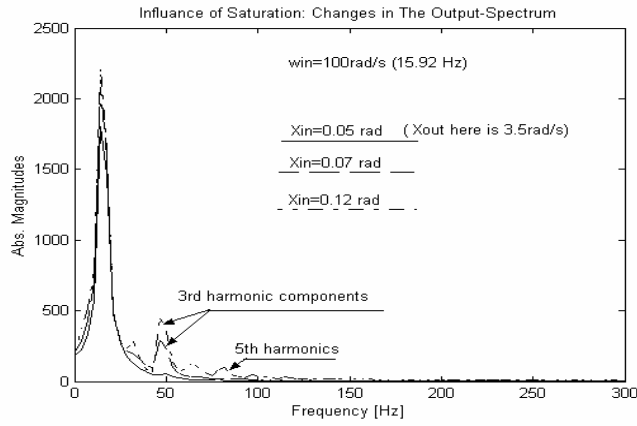


Figure 6  
FFT-analyzes of the distortion speed-curves

The FFT-analyze is done by Matlab-FFT process by imported dates to Matlab from the model running under Matlab-Simulink.

The following features are of minor importance in usual tasks of robotics when the robot approaches their target.

However in case of the trajectory following it's very important to follow punctually the input of the control, mostly in the movement rectilinear or circular, otherwise this link of the robot will no in the position required in time prescribed.

Hence maybe no realized the coordinated movement among in axis of robot, consequently the movement will no on the shape prescribed.

Its known that there is a strict speed-control under the linear movement of the robot. If the prescribed speed and the voltage of the motor isn't high, the operation will be in the quasi linear range. The behavior nonlinear there is in the cases of the higher voltages.

Accordance with the FFT-analysis the value of the 3<sup>rd</sup> harmonic may be attain 14% in Figure 6 when the motor voltage is the highest 36V and so the speed of angle is 3.5 rad/s. The distortions of the speed that can be shown in Figure 4 with sinusoid input develop on the 3-times frequency with near 14% amplitude.

The curves on the Figure 4 are  $\omega_{out}$  output signals from the loop of control with sinusoid input before last integration.

The operation of the control incapable to reduce the distortions in the speed-curves, because this accompaniments are the features of the non-linear controls.

In the course of the linear movement the axis of the robot must to move according to a speed-time function previously calculated. If any link of the robot differ from the actual values of this speed-time function with some percents, this fact obstruct to reach the value of angle in time. Therefore the arrival of the link to the point calculated and in a predetermined time it is not realized, in other words the robot-TCP will not be in the spatial position calculated. The path-following errors arose at high speed linear movements from this cause too. At low speeds the effects of non-linearity appear imperceptibly and the accuracy of the path-following is higher. Improvement the features of the control non-linear are possible with the modification of element non-linear. In this occurrence it is need to change the parameters of motor that can raise the mass and the costs.

Because of the saturation the possible highest value of speed is  $\omega_{out}=3.5$  rad/s on 1<sup>st</sup> axis as mentioned in introduction. The Figure 4 shows the curves of speed. On its peaks there are distortions if the speed is in the domain of non-linearity.

### **3 Improving the Adaptivity of Control**

Control can shift from overdamped to underdamped mode while following a single trajectory. Underdamped control is undesirable in an industrial robot, as overshoot can cause damage to the objects the robot is manipulating, hence, controllers are tuned to give near critically damped response at normal operating

speed. At high speed the inertial loads change rapidly, and at low speed some robots can move with noticeable vibration [2]. All these dynamic effects generate disturbances which cause errors in following of trajectory, hence robot designers try to keep the loop-gains as high as possible.

Adaptive control systems adjust the gain of the control loops in order to maintain critical damping over a range of manipulator configuration. If we take axis 1<sup>st</sup> of an R6-type robot into consideration when the 2<sup>nd</sup> and 3<sup>rd</sup> axis are moving, the value of inertia may be changed by the variation of configuration in the 1:100 ratio.

### 3.1 Finding the Values of Function $Kp(\Theta_r)$ and $Kp(J)$

For improve the adaptivity of non-linear control we need to know the relationship between the suitable gain and the actual values of inertia.

Running the model, the gain ( $Kp$ ) was variable, while input ( $X_i$ ) and inertia ( $J$ ) were constant. Here  $J$  means  $J_{reduced}$ . We have done numerous trial-runs on our model to find the value of gain which produces a minimal time aperiodic course. In these cases the value of  $X_i$  was changed between 0.0001 and 6 radians, and  $J$  was changed between 0.0005 and 0.05 kgm<sup>2</sup>. The resultant curves  $Kp(J)$  at  $X_i=const$  are shown in Figure 7.

Note that the influence of change of inertia decreases at higher values of  $X_i$ . We can draw the conclusion that in strongly saturated non-linear position control the function between the critical gain values and inertia is approximately hyperbolic.

### 3.2 Approximations with Hyperbolic Function

In the case  $X_i=const$  we can fit a hyperbolic function on the  $Kp(J)$  curve. The function given by expression  $Kp=[350/J^{0.21} + 130]$  on  $J$  can be fitted. This function was derived from the investigations in Figure 7. The shape of this curve reflects better the monoton nature of the function  $Kp(J)$ .

The adaptive position control model used a hyperbolic approximation can be shown in Figure 9.

The control loop given by an actual value of inertia calculates actual value of gain  $Kp$ . Deficiency of that model is it does not take into consideration the function  $Kp(X_i)$ , so it is necessary to reduce  $Kp$ .



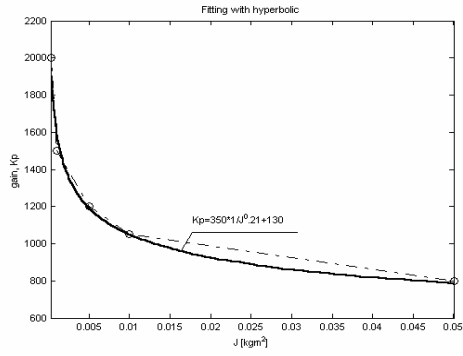


Figure 7  
 $Kp(J)$  functions of control loop resulted minimal time aperiodic approaches

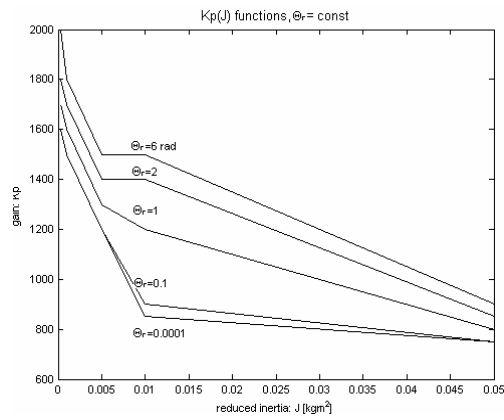


Figure 8  
 Approximation of the  $Kp(J)$  function with hyperbolic approaches

### Conclusions

Simulation methods can be used to properly model and investigate the operation of saturation-type non-linear position control.

Considering that the operation of robot-link go in the linear domain by termination of the control process when decreases of the speed next to the target, the non-linearity disturbs no the operation in significant degree. In this cases lags behind the 3<sup>rd</sup> harmonic and its influences.

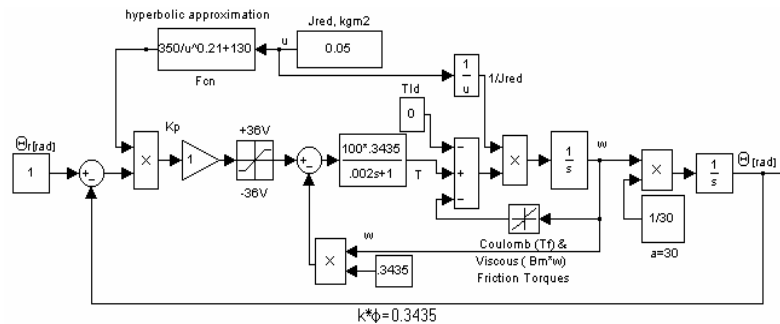


Figure 9

Inertia-adaptive control model operating with a  $Kp(J)$  function with hyperbolic approaches

However if the robot have a task for trajectory following in the course, and the axis of the robot are speed-controlled separately, the following accuracy of the robot depend on the failures and the troubles of the speed control of axis.

In this cases an axis operating in non-linear domain may come along with a failure in the measure of the 3<sup>rd</sup> harmonics, accordingly may increases the inaccuracy of the trajectory following.

It was found that the target's minimum time of aperiodic approach can be solved even with a wide variety of parameters. These functions can be determined, and their applied can realize better behavior in the course of operation of the control loop. With a suitable fitting of these functions one can design a non-linear adaptive position controller for hardly changing conditions.

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