All Possible Path in a Credit Network

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Abstract: The Credit system compare with the traditional one is coplicated. Sometimes it is difficult to determine the possible path to reach the final result. A simple graph theoretical algorith is presented, which help to determine the different paths belongs to the actual situation.

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1 Introduction

Higher education in Hungary is undergoing changes these days. The earlier rigid educational system is being replaced by a more flexible system, following the patterns of countries with outstanding training achievements. The bulk of the subjects are worth credit scores. In this way, student performance can be measured as well as their advancement in their studies. The essence of the credit system means that students are required to accumulate a certain number of credits from the obligatory and optional subjects in a given period (e.g. each semester) so as to be able to proceed to the next stage, while the requirements of the curriculum must be taken into account.

There might be different constraints with regard to the credit system, for example:

– The accumulation of at least 30 credits is required in the first two semesters.

– 60 credits must be achieved during the first four semesters.

– The total number of credits to be obtained is 270 during the first nine semesters, etc.

Apart from those mentioned above other constraints may apply as well. For example, the total length of the training time may be limited; as well, there may be a maximum length of the period between the acquisition of all prerequisites and the final school leaving exam or the achievement of a certain average grade. We plan to introduce a method suitable for handling the transparency of this problem. Different methods have already been use for the analysis and mathematical modelling of the problems as well. These methods usually describe mathematical interdependence or sequential order. (See e.g. Gyaraki [1982], Kata [1993], Morgunov [1966]. However, these do not elaborate the possibilities of how to choose, nor do they give priority to the subjects, neither do they cover the methods of calculating the total credits, or the algorithm of flocking the subjects. Therefore, we will we attempt to solve these problems in the framework of this article. The method is based on an arrangement relation, namely on the ordinal relation of the different subjects. A circle free directed graph should be built up with the help of these relations. The points in the graph are events – they correspond to a certain subject. The directed edges represent the ordinal relation between the subjects. It is given a function at each point which represents the weight of the subjects in this point.

The tasks to be done are the following:

– the classification of subjects into sets belonging to the prescribed requirements (the priority function of a given graph),

– the determination of obtainable credit scores on a given route,

– the determination of all routes, and the subjects and the total weight of these routes.

In the following we define the arrangement relation, the rules of graph creation and the algorithms solving the above mentioned tasks.

2 The Formulation of the Task

Now we explain the graph theoretical description of the structure of subjects. The different subjects correspond to points on the plane. One subject may precede another in the following way:

If subject A precedes subject B, we will use the precedence relation for its nomination.

By means of the precedence relation, two given subjects are interlinked with a directed arc. From now on the subjects are called points and the directed arcs are called edges.

Let N = {A, B, …} be the set of subjects, and E{(A,B), (A,C), …} be the set of arcs the basis of the precedence relation. The [N,E] composition is usually called a directed graph or digraph in mathematics. The set N is finite, and consequently, the set E is finite too. A function is given at each point of the graph. Function S(x), $x \in N$ means the credit score belonging to a subject. The directed edges represent the precedence relations between the subjects and the number of credit scores belonging to the subjects are known.

Using these definitions the tasks given above are formulated as follows:

a) to rank subjects into as few groups as possible in a way that precedence (directed edge) leads only to the higher group (a priority function of a given graph)

b) to determine a route from the beginning of studies to graduation and the number of corresponding credit scores as well,

c) to determine all the possible routes from the starting point to the end must be with the corresponding credit scores.

Further we deal with the solution of tasks in a) – c).

3 The Classification of Subjects

In the previous section we have formulated the tasks in connection with the graph priority function. Assuming that subjects last one semester, the number of sets equals the maximum number of semesters needed for graduation. This assumption can be realized in the case of subjects consisting of more than one semester in that as many additional edges are given to the subject as the number of semesters. So, if subject A precedes subject B and A has two semesters, the accomplishment of the assignment is shown in the following figure.

Instead $A \rightarrow B$

$$A \rightarrow F \rightarrow B$$

Where F is a fictions subject, the number of its credit scores is 0, the number of credit scores of A and B is identical in both cases. The determination of priority function can be determined by the matrix method (see in Kaufmann [4]). As the size of the matrix is identical with the number of subjects, the size of the matrix will be large and its handling and storage may cause difficulties. At the same time this is a sparse matrix, so for the store of its nonzero elements some other method should be suggested. For example the tree structure can be used, requiring little storage room and being effective from the point of view of algorithm, (see e.g. Bakó [2]).

Instead of the matrix method the application of graph theoretical algorithm is suggested for use justified, where the task can be solved in n steps, where n is the number of all subjects. For the solution of the assigned tasks, a dual search algorithm is elaborated based on the determination of the shortest path. (See Bakó [1]).

Before the description of the algorithm we would like to note that the graph contains only one point where the edges go outward (we will call it a source) and only one other point where the edges run in (we will call it a sink). The source means the beginnig of the studies, the sink their termination. This requirement can be easily accomplished. If in a college there are several initial subjects because of the different faculties or specializations, a fictions source is given to the graph and the real starting points are connected with the edges pointing to these fictions starting points.

It is easy to understand that a sole sink can be achieved with a similar method. The credit number of the source and sink points, that is to say the value of the function, is naturally zero.

Then the algorithm can be established. To solve the problem, the set of subjects are divided into two partial sets. One set – let's call it K – is the set of subjects that have been classified in a permanent way, the points of the other set is the set of all the other subjects – let's call it S -, that is to say S = N-K. In the course of the algorithm, to each $x \in K$ point is assigned by function h(x) showing the label of the set.

Source is marked by f, sink by w.

The H algorithm is then the following:

H0: Initial step: K=w, S=N-w, h(w) =1 and i = 2

H1: Let's take all $x \in N$, point for which

- $(x,y) \in E$, $x \in S$, $y \in K$ and
- all in the case $(x,z) \in E$, $z \in K$

It is easy to see that such a point exists. Otherwise the graph is not coherent and is devided into at least two sets, and the algorithm should be terminated.

H2: Let's assume that points x_1, x_2, …, x_p meet these requirements. Sets of S and K are modified and the label of the groups are determined, that new function value f is computed:

$K = K + x_1 + x_2 + \ldots + x_p,$ $S = N-K$

$h(x_1) = h(x_2) = \ldots = h(x_p) = i$ and $i = i+1$

H3: If S =0 we are ready, STOP, otherwise we can continue at H1.

The final result of algorithm H is shown in the example below:

Assume that T_1, T_2, …, T_9 are all the subjects and between them the following precedence relations exist:

$T_1 \prec T_2, T_1 \prec T_3, T_1 \prec T_5, T_1 \prec T_6$

$T_2 \prec T_4, T_2, \prec T_6, T_3 \prec T_5, T_4 \prec T_7$

$T_5 \prec T_7, T_5 \prec T_8, T_6 \prec T_4, T_6 \prec T_5$

$T_6 \prec T_8, T_7 \prec T_9, T_8 \prec T_9$

The directed graph belonging to the above precedence relation is shown in Fig. 1.

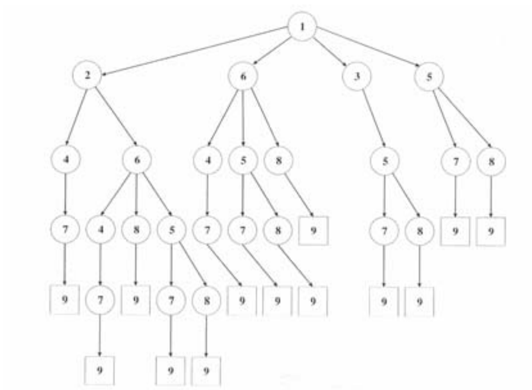

Figure 1

Algorithm H can be computed and the result of the algorithm is shown in the following table.

point	1	2	3	4	5	6	7	8	9
h	6	5	5	3	3	4	2	2	1

The result is that all subjects can be classified into 6 groups:

Group 1: T_9, Group 4: T_6,

Group 2: T_8, T_7, Group 5: T_2, T_3,

Group 3: T_5, T_4, Group 6: T_1

The algorithm H is quite siple. We have to store the subjects and the precedence relations. Beside this we need to compute and store 2 vectors which length is n where n is the number of the subjects. The maximum number of the steps H1-H3 is n.

4 Total Number of Possible Choice

The total number of the possible choice and the related credit points could be determined using the precedence relations and the credit points related to it. For this we have to determine all possible path from the from the source to the sink. After determination of these path it could be computed the possible feasible solutions where the total number of credit points is enough to fulfill the prescribed conditions.

The task is to determination such a trees, whose root is the source, and its final points constitute the sinks. To each x element of the tree the p(x) potential is assigned meaning the number of credit scores from the starting point to point x along the determined route. Three vectors have to be determined in H algorithm: vectors **A**, **B** and **C**.

The a_i element of vector **A** is a point of a determined P route (a subject), the corresponding b_i elements shows the point number which precedes point a_i along the route P and c_i is then sum of credit points along the road from the initial point (source) till the point a_i, that is $p(a_i)$.

For example in case of a_i=7, b_i=3, c_i=8 the point 7 is a fixed point of route P, the preceding point is to be found on the same route in a_3 and the total of credit scores is 8 up to point 7.

In the case of the graph shown in Figure 1, the tree belonging to all routes is shown in Figure 2.

The **A**, **B**, **C** vectors belonging to the route are shown in Figure 3.

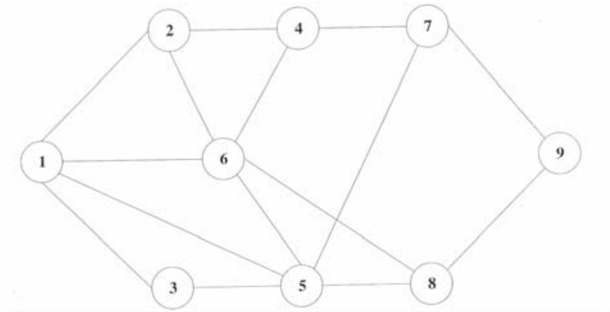

Figure 2

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
A	1	2	6	3	5	4	6	4	5	8	5	7	8	7	4	5	8	7	7	8
B	1	1	1	1	1	2	2	3	3	3	4	5	5	6	7	7	7	8	9	9
C	0	2	3	4	1	7	5	8	4	7	5	7	5	13	10	6	9	14	10	8

	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38
A	9	7	8	9	9	9	7	7	8	9	9	9	9	9	9	9	9	9
B	10	11	11	12	13	14	15	16	16	17	18	19	20	22	23	27	28	29
C	7	11	9	7	5	13	16	12	10	9	14	10	8	11	9	16	12	10

Figure 3

To determine all these, a specific potential method and labeling technique is used. At the beginning, the end point of all the possible edges going out of the sink is determined. These are entered in the successive elements of vector **A**. The vector **B** is the so called label vector, it signifies the starting point belonging to the termination point. This will be used for the determination of the route later on. The source, the starting point is 1 and the endpoints of the edges going out of it are points 2,6,3,5, that is $a_1=1$, $a_2=2$, $a_3=6$, $a_4=3$, $a_5=5$.

The label in source is the source itself, that is $b_1=1$. As points 2,6,3,5 have been reached from point 1, so $b_2=b_3=b_4=b_5=1$. The related element of vector C: $c_1=0$, $c_2=2$, $c_3=3$, $c_4=4$ and $c_5=1$ because these are the potentials.

The next elements of A from a_2 contain the directed termination points of the directed edges from point 2, the corresponding b_1 elements mean the starting points of the directed edges, that is to say 2, while vector **C** contains the sum of the obtainable credit scores on the route.

In the algorithm, a further vector, vector **D** is used to flag the actual position. Its value is zero at the beginning. If a point enters A, the corresponding element of D is changed into 1, indicating that the construction of the routes have to be accomplished from this point. If route construction has ended from this point, the corresponding value is changed into zero again.

Furthermore, we show the algorithm P which is used to solve the the above task.

Steps of the algorithm P are the following:

P0 Initial step $a_1=1$, $b_1=1$, $c_1=0$, $d_1=1$, $j=2$

P1 If all the elements of D are 0, we are ready, otherwise we have to find the first nonzero element of D, (its value is equal to1), let its index be i, the corresponding node a_i.

P2 Let's determine the set H of endpoints of edges going out of point a_i

$H = \{x \,|(a_i,x) \in E\}= (x_1,x_2, \ldots, x_K)$

Let k=1

P3 $a_j=x_k$, $b_j=a_i$, $c_j=c_i+s(x_k)$

If x_k is an endpoint then $d_j=0$ otherwise $d_j=1$

P4 $j=j+1$, $k=k+1$

If k>K let's continue at P5, otherwise at P3.

P5 $d_i=0$, let's continue at P1.

In algorithm P we take all the points from which route construction is still possible (stepP1). All the outgoing edges are determined for such an x_1 point (step P2). The termination points of the outgoing edges are placed in the successive blank places in A, the starting point of these edges are entered in the appropriate place of vector **B**, that is to say a_i, and the total of credit scores accumulated on the route is entered into the appropriate elements of C. Finally, we indicate in the appropriate elements of vector **D** whether the termination point is reached or not (in this case the appropriate D element value is 0) or the construction of routes can be further built from the termination point (the appropriate value of D is 1 (see step P3).

Steps P3-P4 is carried out as long as we have accomplished the above operation with all edge termination points. Then the value of element **D** belonging to the starting point is changed into 0, indicating that route construction has been accomplished from this point of view and then we start the procedure from the beginning at step P1. (see step P5)

The resultant vectors provide answers for the questions raised at the beginning of this work.

a The elements of vector **A** containing the termination point are the termination points of all possible routes. The **C** values belonging to these elements contain the total of credit scores belonging to the routes. This for example in the vector of the presented graph $a_{3,1}$=9, that is to say, a termination point and the number of credit scores belonging to the route is 14.

b The routes themselves can also be determined from the algorithm using a modified version of the usual labeling technique. We introduce it for the above route: $a_{3,1}$=9, so it is the termination point of the route. The preceding point can be found in a_{18} (its value is 7), as a_{18}=18. The point proceeding point 7 is 4, as b_{18}=8 and a_8=4. The next preceding point is 6, as b_8=3 and a_3=6. At last we have reached the starting point, as b_3=1.

The points of the route obtaining in this way: 1,6,4,7,9 and the member of credit scores on this route is 14.

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