Identification of Robot's Dynamic Model by Means of Neural Network

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Abstract: A design and simulation analysis of the neural network-based identification of the dynamic model of an educational robot is presented in the paper. The structure of identification module was designed using the same forms of non-linear functions as they are in the analytic dynamic model of robot. The analytic model is derived using Euler-Lagrange equations in two modes: for substitution of links masses of the robot arm with one mass point in the own centre of gravity of each links and for substitution with three mass points along the links. The weights of the neural network approximate the coefficients of the motion equations, which coefficients represent the kinematic and dynamic parameters of the robot's links.

Keywords: Mechatronic systems, Robotics, Dynamic Model, Neural Networks

1 Introduction

Robots belong to multi-variable, non-linear mechatronic systems with complicated interaction of the kinematic pairs. Same methods of adaptive motion control of robots use the dynamic model of robot for linearization of the control system.

The dynamics of robots is described with a set of differential equations. For deriving of the analytic motion equations of the rigid robot arm the so-called Euler-Lagrange equations is usually used, which leads to the vector equation of the robot arm motion and can be written in the next compact form:

$$\overline{M} = \overline{A}_M\left(\overline{q}, \overline{\xi}\right)\ddot{\overline{q}} + \overline{b}_M\left(\overline{q}, \dot{\overline{q}}, \overline{\xi}\right) \tag{1}$$

where: $\bar{q}, \dot{\bar{q}}, \ddot{\bar{q}}$ are vectors of the joints position, speed and acceleration, $\bar{\xi}$ is the vector of kinematic and dynamic parameters of the robot arm (lengths, moments of inertia and masses of the robot's links), $\bar{M}$ is the vector of generalised forces (torques) generated by or required for the motion of the joints, $\bar{A}_M$ is the inertia matrix and $\bar{b}_M$ is the matrix of Coriolis, centrifugal and gravitation forces or torques in the joints of the robot.

The real torque of robot's servo drives $\bar{M}_m$ has to get over in addition of $\bar{M}$ the friction torque $\bar{M}_{tr}$ and external load torque $\bar{M}_V$:

$$\bar{M}_m = \bar{M} + \bar{M}_{tr} + \bar{M}_V \tag{2}$$

Each components of equation (1) depend on the robot's kinematic and dynamic parameters $\bar{\xi}$. The masses of the links are distributed along the links non-homogeneously. Consequently, accurate derivation of the analytic dynamic model is very difficult. In robotics a substitution of the real mass of each link with an equivalent mass point to the centre of gravity of the link is usually used. It is obvious, if the mass of link is substituted with more masses along the link than the description of dynamics is more precise, however the computing is more complicated.

A new method to create the precise dynamic model of robots is using the neural networks for identification of the robot's dynamic model. The basic problem of neural networks is an appropriate network architecture design. Because the non-linearity of the dynamic model is represented by goniometric functions of the joints positions (see equation (3)), we can choice for the neural model the same form of activation functions as they are in the analytic model. One solution is presented in [5] where the non-linearity and parameters of the motion equations are approximated by the feed forward network consisting from one hidden layer of sinusoid neurons followed by an output layer of linear neurons.

Another solution is presented in this paper, where the non-linear elements of the analytic dynamic model are exactly used in analytic form and the not precisely known parameters of the dynamic model are identified by neural network.

2 Dynamic Analysis of Robot

In our case we compute the analytical dynamic model of an educational robot MA2000 for two solutions:

- Substitution of each link mass with one mass point in the centre of gravity of the link,
- Substitution of each link mass with three mass points along the link.

2.1 Dynamic Model of the Robot for Substitution of the Link Mass with One Mass Point in the Centre of Gravity

The kinematical scheme of our educational robot is in Figure 1. The first link is taken into consideration with its moment of inertia J_1, the second and third with they mass points m_2 and m_3 respectively. The location of mass points of links,

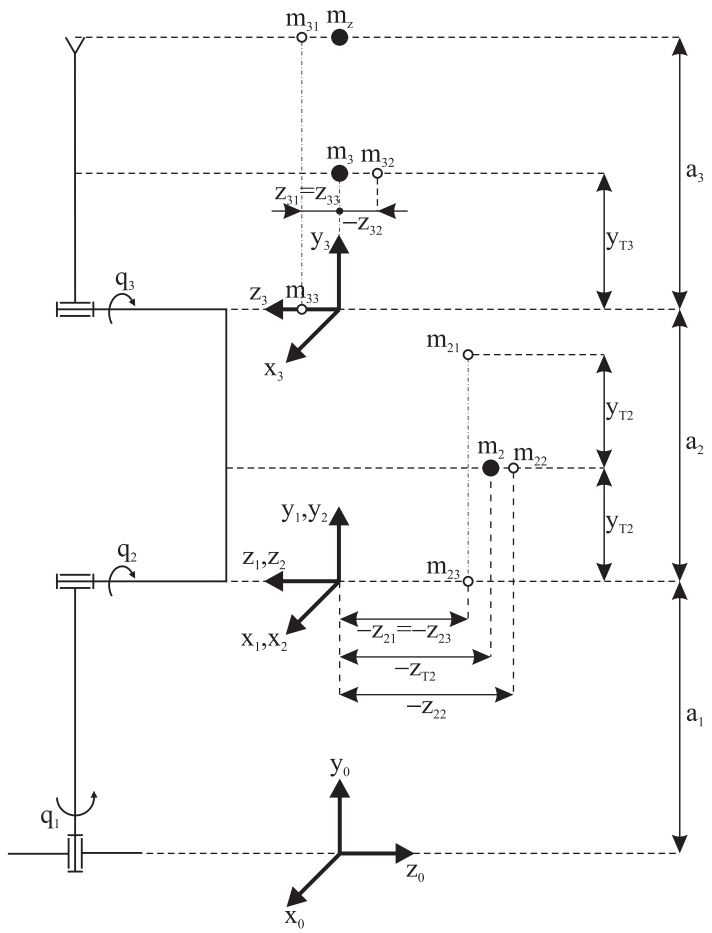

Figure 1
Kinematical scheme of educational robot

m_2, m_3 are valid for substitution with one mass point in the centre of gravity of the links.

m_{21}, m_{22}, m_{23}, m_{31}, m_{32}, m_{33} are valid for substitution with three mass points along the links.

the coordinate systems, the centre of gravity and length of the links are visible from the picture. The first three kinetic pairs of robot are taken into consideration, which represent the positioning mechanism of the robot. The other three kinetic pairs for orientation of the robot's end-effectors have a small masses and small influence to the robot dynamics and therefore are concentrated as m_z to the end point of the robot arm. The motion equations for the first three joints are (1), (2) and (3), where the non-linear element at the correspondent acceleration $\ddot{q}_i$ represents the variable moment of inertia with reference to the joint axis and the rest elements represent the variable load torque due to interaction of the kinematical pairs.

$$M_{11} = \begin{pmatrix} J_1 + m_2 z_{T2}^2 + \frac{1}{2} m_3 y_{T3}^2 + \frac{1}{2} m_z a_3^2 + \left(m_2 y_{T2}^2 + (m_3 + m_z) a_2^2\right)\sin^2(q_2) + \\ + (m_3 y_{T3} + m_z a_3) a_2 \cos(q_3) - (m_3 y_{T3} + m_z a_3) a_2 \cos(2q_2 + q_3) - \\ - \frac{1}{2}\left(m_3 y_{T3}^2 + m_z a_3^2\right)\cos(2q_2 + 2q_3) \end{pmatrix} \ddot{q}_1 +$$
$$+ m_2 z_{T2} y_{T2} \cos(q_2)\ddot{q}_2 - m_2 z_{T2} y_{T2} \sin(q_2)\dot{q}_2^2 +$$
$$+ \begin{pmatrix} \left(m_2 y_{T2}^2 + (m_3 + m_z) a_2^2\right)\sin(2q_2) + (m_3 y_{T3} + m_z a_3) 2a_2 \sin(2q_2 + q_3) + \\ + \left(m_3 y_{T3}^2 + m_z a_3^2\right)\sin(2q_2 + 2q_3) \end{pmatrix} \dot{q}_1 \dot{q}_2 +$$
$$+ \begin{pmatrix} -(m_3 y_{T3} + m_z a_3) a_2 \sin(q_3) + (m_3 y_{T3} + m_z a_3) a_2 \sin(2q_2 + q_3) + \\ + \left(m_3 y_{T3}^2 + m_z a_3^2\right)\sin(2q_2 + 2q_3) \end{pmatrix} \dot{q}_1 \dot{q}_3 \tag{3}$$

$$M_{21} = \left(m_2 y_{T2}^2 + m_3\left(a_2^2 + y_{T3}^2\right) + m_z\left(a_2^2 + a_3^2\right) + (m_3 y_{T3} + m_z a_3) 2a_2 \cos(q_3)\right)\ddot{q}_2 +$$
$$+ m_2 y_{T2} z_{T2} \cos(q_2)\ddot{q}_1 + \left(m_3 y_{T3}^2 + m_z a_3^2 + (m_3 y_{T3} + m_z a_3) a_2 \cos(q_3)\right)\ddot{q}_3 -$$
$$- \frac{1}{2}\begin{pmatrix} \left(m_2 y_{T2}^2 + (m_3 + m_z) a_2^2\right)\sin(2q_2) + (m_3 y_{T3} + m_z a_3) 2a_2 \sin(2q_2 + q_3) + \\ + \left(m_3 y_{T3}^2 + m_z a_3^2\right)\sin(2q_2 + 2q_3) \end{pmatrix} \dot{q}_1^2 -$$
$$- (m_3 y_{T3} + m_z a_3) a_2 \sin(q_3)\dot{q}_3^2 - (m_3 y_{T3} + m_z a_3) 2a_2 \sin(q_3)\dot{q}_2 \dot{q}_3 -$$
$$- \left(m_2 y_{T2} + (m_3 + m_z) a_2\right) g \sin(q_2) - (m_3 y_{T3} + m_z a_3) g \sin(q_2 + q_3) \tag{4}$$

$$M_{31} = \left(m_3 y_{T3}^2 + m_z a_3^2\right)\ddot{q}_3 + \left(m_3 y_{T3}^2 + m_z a_3^2 + (m_3 y_{T3} + m_z a_3) a_2 \cos(q_3)\right)\ddot{q}_2 -$$
$$- \frac{1}{2}\begin{pmatrix} -(m_3 y_{T3} + m_z a_3) a_2 \sin(q_3) + (m_3 y_{T3} + m_z a_3) a_2 \sin(2q_2 + q_3) + \\ + \left(m_3 y_{T3}^2 + m_z a_3^2\right)\sin(2q_2 + 2q_3) \end{pmatrix} \dot{q}_1^2 +$$
$$+ (m_3 y_{T3} + m_z a_3) a_2 \sin(q_3)\dot{q}_2^2 - (m_3 y_{T3} + m_z a_3) g \sin(q_2 + q_3) \tag{5}$$

2.2 Dynamic Model of the Robot for Substitution with Three Mass Points Along the Links

The kinematical scheme for substitution of the second and third links masses with three mass points along the links is in Figure 1, where the mass of the second link

m_2 is distributed to tree masses m_{21}, $m_{22,}$ m_{23} and the mass m_3 of the third link is distributed to tree masses m_{31}, $m_{32,}$ m_{33}. The distribution of the links masses is made by method of substitution of stiff entity by mass points published in [6], [10]. On the bases of rules for substitution we get the next relationships between masses

$$\begin{aligned} &m_2 = m_{21} + m_{22} + m_{23} && m_3 = m_{31} + m_{32} + m_{33} \\ &m_2 = 2m_{21} + m_{22} && m_3 y_{T3} = m_{31}(a_3 - y_{T3}) \\ &m_{22}(z_{22} - z_{T2}) = 2m_{21}(z_{T2} - z_{21}) && m_{32}z_{32} = (m_{31} + m_{33})z_{31} \end{aligned} \quad (6)$$

The different between this solution and the previous one mass point substitution is expressed by torques ΔM_1, ΔM_2 and ΔM_3. Than the final motion equations for three mass points substitution are:

$$M_{13} = M_{11} + \Delta M_1 \quad (7)$$

$$M_{23} = M_{21} + \Delta M_2 \quad (8)$$

$$M_{33} = M_{31} + \Delta M_3 \quad (9)$$

where

$$\begin{aligned} \Delta M_1 = &\begin{pmatrix} 2m_{21}\left(z_{21}^2 + z_{T2}z_{22} - z_{21}z_{22} - z_{T2}z_{21}\right) + \frac{1}{2}m_{31}a_3(a_3 - y_{T3}) + \\ + (m_{31} + m_{33})z_{31}^2 + m_{32}z_{32}^2 + 2m_{21}y_{T2}^2 \sin^2(q_2) - \\ - \frac{1}{2}m_{31}a_3(a_3 - y_{T3})\cos(2q_2 + 2q_3) \end{pmatrix}\ddot{q}_1 + \\ &+ \left(2m_{21}y_{T2}^2 \sin(2q_2) + m_{31}a_3(a_3 - y_{T3})\sin(2q_2 + 2q_3)\right)\dot{q}_1\dot{q}_2 + \\ &+ m_{31}a_3(a_3 - y_{T3})\sin(2q_2 + 2q_3)\dot{q}_1\dot{q}_3 \end{aligned} \quad (10)$$

$$\begin{aligned} \Delta M_2 = &\left(2m_{21}y_{T2}^2 + m_{31}a_3(a_3 - y_{T3})\right)\ddot{q}_2 + m_{31}a_3(a_3 - y_{T3})\ddot{q}_3 - \\ &- \left(m_{21}y_{T2}^2 \sin(2q_2) + \frac{1}{2}m_{31}a_3(a_3 - y_{T3})\sin(2q_2 + 2q_3)\dot{q}_1^2\right) \end{aligned} \quad (11)$$

$$\begin{aligned} \Delta M_3 = &m_{31}a_3(a_3 - y_{T3})\ddot{q}_3 + m_{31}a_3(a_3 - y_{T3})\ddot{q}_2 - \\ &- \frac{1}{2}m_{31}a_3(a_3 - y_{T3})\sin(2q_2 + 2q_3)\dot{q}_1^2 \end{aligned} \quad (12)$$

From comparision of motion equations (3), (4) and (5) with equations (7), (8) and (9) follows that the motion equations of both dynamic models contain components with the same form of goniometric functions (the same non-linearity). The different between the solution for one mass and mullty mass subtitution of the link´s inertia is only in the different constant coeficients at the components, which coeficients exprese the kinematic and dynamic parameters of the links (see coeficients at goniometric functions *sin* q_i, *cos* q_i, *sin(2* q_i*)*, *sin*^{2} q_i, *sin* $(2q_i + 2q_j)$ and so on).

3 Identification of the Dynamic Model

The principal structure of the dynamic model identification is in Fig. 2. The robot is represented by its analytic dynamic model.

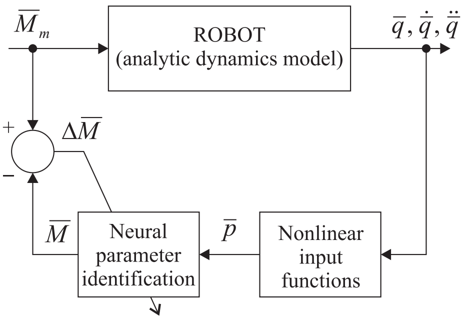

Figure 2
Structure of identification

To minimise the number of neurons and layers we design the neural network resemble to analytic model. Let we compare the motion equations of the first join for one mass (3) and tree mass model (7), (10). As we can see both equation can be written in the form

$$\begin{aligned} M_{13} = {} & w_{1,1}\ddot{q}_1 + w_{1,2}\sin^2(q_2)\ddot{q}_1 + w_{1,3}\cos(q_3)\ddot{q}_1 + w_{1,4}\cos(2q_2 + q_3)\ddot{q}_1 + \\ & + w_{1,5}\cos(2q_2 + 2q_3)\ddot{q}_1 + w_{1,6}\cos(q_2)\ddot{q}_2 + w_{1,7}\sin(q_2)\dot{q}_2^2 + \\ & + w_{1,8}\sin(2q_2)\dot{q}_1\dot{q}_2 + w_{1,9}\sin(2q_2 + q_3)\dot{q}_1\dot{q}_2 + w_{1,10}\sin(2q_2 + 2q_3)\dot{q}_1\dot{q}_2 + \\ & + w_{1,11}\sin(q3)\dot{q}_1\dot{q}_3 + w_{1,12}\sin(2q_2 + q_3)\dot{q}_1\dot{q}_3 + w_{1,13}\sin(2q_2 + 2q_3)\dot{q}_1\dot{q}_3 + \\ & + w_{1,14}\dot{q}_1 + b_1 \end{aligned} \tag{13}$$

Where $w_{1,1},\dots,w_{1,14}$ are the weights, which express the kinematic and dynamic parameters of the robot and their value is different for one mass point or tree mass point configuration. The component $w_{1,14}\dot{q}_1 + b_1$ presents the friction torque, where b_1 is the initial value of friction and w_{14} express the speed dependence of friction.

The identification procedure consist of the:

- Input transformation, which transform the inputs of the identification module thought non-linearity of the dynamic model according to equations (14).
- The neural network with linear transfer function, which creates the output torque as the sum of the waited output of input transformation according equation (15). The weights are learned using method of back propagation.

$$
\begin{array}{lll}
p_{1,1} = \ddot{q}_1 & p_{1,6} = \cos(q_2)\ddot{q}_2 & p_{1,11} = \sin(q3)\dot{q}_1\dot{q}_3 \\
p_{1,2} = \sin^2(q_2)\ddot{q}_1 & p_{1,7} = \sin(q_2)\dot{q}_2^2 & p_{1,12} = \sin(2q_2 + q_3)\dot{q}_1\dot{q}_3 \\
p_{1,3} = \cos(q_3)\ddot{q}_1 & p_{1,8} = \sin(2q_2)\dot{q}_1\dot{q}_2 & p_{1,13} = \sin(2q_2 + 2q_3)\dot{q}_1\dot{q}_3 \\
p_{1,4} = \cos(2q_2 + q_3)\ddot{q}_1 & p_{1,9} = \sin(2q_2 + q_3)\dot{q}_1\dot{q}_2 & p_{1,14} = \dot{q}_1 \\
p_{1,5} = \cos(2q_2 + 2q_3)\ddot{q}_1 & p_{1,10} = \sin(2q_2 + 2q_3)\dot{q}_1\dot{q}_2 &
\end{array} \tag{14}
$$

$$
M_1 = purelin \sum_{i=1}^{14} w_{1,i} p_{1,i} + b_1 \tag{15}
$$

In the similar way we design the network for approximation of torque M_2. The basic equation for design of the neural network topology is

$$
\begin{aligned}
M_{23} = {} & w_{2,1}\ddot{q}_2 + w_{2,2}\cos(q_3)\ddot{q}_2 + w_{2,3}\cos(q_2)\ddot{q}_1 + w_{2,4}\ddot{q}_3 + \\
& + w_{2,5}\cos(q_3)\ddot{q}_3 + w_{2,6}\sin(2q_2)\dot{q}_1^2 + w_{2,7}\sin(2q_2 + q_3)\dot{q}_1^2 + \\
& + w_{2,8}\sin(2q_2 + 2q_3)\dot{q}_1^2 + w_{2,9}\sin(q_3)\dot{q}_3^2 + w_{2,10}\sin(q_3)\dot{q}_2\dot{q}_3 + \\
& + w_{2,11}\sin(q_2) + w_{2,12}\sin(q_2 + q_3) + w_{2,13}\dot{q}_2 + b_2
\end{aligned} \tag{16}
$$

The input functions are:

$$
\begin{array}{lll}
p_{2,1} = \ddot{q}_2 & p_{2,6} = \sin(2q_2)\dot{q}_1^2 & p_{2,11} = \sin(q_2) \\
p_{2,2} = \cos(q_3)\ddot{q}_2 & p_{2,7} = \sin(2q_2 + q_3)\dot{q}_1^2 & p_{2,12} = \sin(q_2 + q_3) \\
p_{2,3} = \cos(q_2)\ddot{q}_1 & p_{2,8} = \sin(2q_2 + 2q_3)\dot{q}_1^2 & p_{2,13} = \dot{q}_2 \\
p_{2,4} = \ddot{q}_3 & p_{2,9} = \sin(q_3)\dot{q}_3^2 & \\
p_{2,5} = \cos(q_3)\ddot{q}_3 & p_{2,10} = \sin(q_3)\dot{q}_2\dot{q}_3 &
\end{array} \tag{17}
$$

The neural network with linear transfer function is

$$
M_2 = purelin \sum_{i=1}^{13} w_{1,i} p_{2,i} + b_2 \tag{18}
$$

For the torque M_{33} is valid the next motion equation:

$$
\begin{aligned}
M_{33} = {} & w_{3,1}\ddot{q}_3 + w_{3,2}\ddot{q}_2 + w_{3,3}\cos(q_3)\ddot{q}_2 + \\
& + w_{3,4}\sin(q_3)\dot{q}_1^2 + w_{3,5}\sin(2q_2 + q_3)\dot{q}_1^2 + w_{3,6}\sin(2q_2 + 2q_3)\dot{q}_1^2 + \\
& + w_{3,7}\sin(q_3)\dot{q}_2^2 + w_{3,8}\sin(q_2 + q_3) + w_{3,9}\dot{q}_3 + b_3
\end{aligned} \tag{19}
$$

The input functions are:

$$
\begin{array}{lll}
p_{3,1} = \ddot{q}_3 & p_{3,4} = \sin(q_3)\dot{q}_1^2 & p_{3,7} = \sin(q_3)\dot{q}_2^2 \\
p_{3,2} = \ddot{q}_2 & p_{3,5} = \sin(2q_2 + q_3)\dot{q}_1^2 & p_{3,8} = \sin(q_2 + q_3) \\
p_{3,3} = \cos(q_3)\ddot{q}_2 & p_{3,6} = \sin(2q_2 + 2q_3)\dot{q}_1^2 & p_{3,9} = \dot{q}_3
\end{array} \tag{20}
$$

The neural network with linear transfer function is

$$M_3 = purelin \sum_{i=1}^{9} w_{1,i} p_{3,i} + b_3 \tag{21}$$

3.1 Learning Neural Network

The network was learned as the inverse neural dynamic model to the robot analytical dynamic model Fig. 2. To the analytic dynamic model the inputs are the torques of the joint servo drives and the outputs are the position, speed and acceleration of the robot joints. The outputs of the analytic model are the input for the neural identification module including the input transformation. The error for learning is the different between the real torque of the robot and the approximated torque by neural network.

For learning of the network the torques were generated according to diagrams in Fig. 3. The corresponding motion variables $\overline{q}, \dot{\overline{q}}, \ddot{\overline{q}}$ as the output of the analytic dynamic model are on the Fig. 4. The motion was generated so that all joints are running up with constant acceleration, after that are running with constant speed and braking with constant deceleration.

For training of the network the Levenberg-Marquardt algorithm was used. The learning was finished after the fourth epoch with error less then $1e^{-25}$.

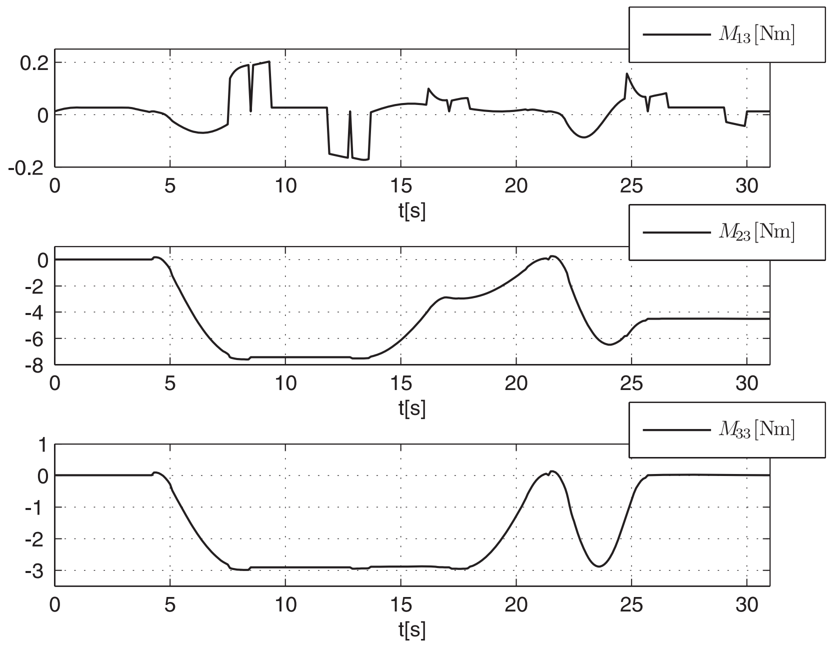

Figure 3
Reference torques of joints

Even though the traning date was generated using the analytic dynamic model, the learning proccese will be the same for the traning data obtained by measurement on the real robot.

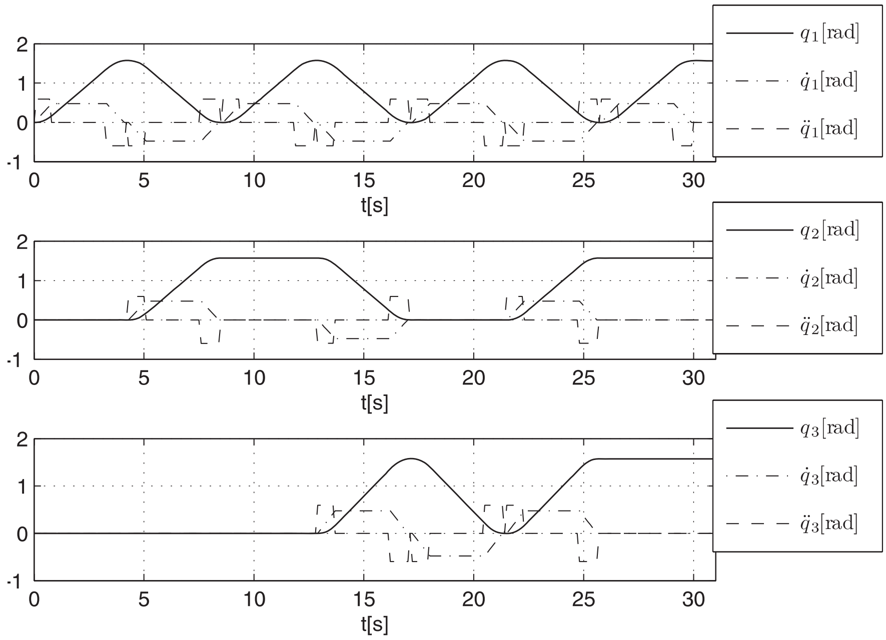

Figure 4
Motion variables $\overline{q}, \dot{\overline{q}}, \ddot{\overline{q}}$

Conclusions

Methods of analysis of robot´s kinematics and dynamics are generally known and published in many teaching books of robotics. The direct kinematic analysis is usually made by Denavit-Hartenberger method, which uses the matrix transformation between joints coordinates and three-dimensional coorinates and by maens of MATLAB Toolbox is quite easy and quick to compute. This solution of robot´s kinematic analysis is implemented to the dynamic analysis, where the Euler-Lagrange equations are often used. The dynamic analysis using MATLAB Toolbox is quite easy and quick too.

A problem of analytic dynamic models is they complexity and inaccuracy. We get the less complex model when the links inertia are assumed as the roll moment of inertia with respect of joint axis and with the mass point in the center of gravity of the links. However this dynamic model in many causes is not precise inaf. More precise models is available at multi mass point substitution of links inertia, but the model is more complex than before. Other problem is to compute the masses distribution on links, because in many cases of robots same machanical and electrical components are instaled inside of the links (sensors, gears, servodrives, condutors and so on). The best precisity is available if we identify the robot´s dynamic parameters by measurement directly on the robot.

One of the possible solution using neural network is presented in the paper. The designed neural model observe the structure of the analytic model and thereby the physical meaning of the derived torque components are remained. It means that the sum of componets at the joint acceleration in the torque equation gives the variable moment of inertia and the sum of less components give the variable load torque consisting from centrifugal, coriolis, gravitation and friction torgues (for examle compare the eq. (13) with its general form (1)). The advantege of that structure is that same of adaptive controls [1], [5] use these physical moments of inertia and load torque for adaptation of the feedback control system.

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