

# Deterioration-based Maintenance Management Algorithm

**Kornélia Ambrus-Somogyi**

Assoc. Prof.  
Institute of Media Technology, Budapest Tech  
Doberdó út 6, H-1034 Budapest, Hungary  
a\_somogyi.kornelia@rkk.bmf.hu

*Abstract: The Road Management Systems (and the PMS) usually do not take into consideration the future traffic change. The maintenance and rehabilitation actions and the development of the road network structure and the changing traffic structure modify the amount of the traffic on the road section. The deterioration process depends on mostly the volume of the traffic. That is why it is important to take into consideration the change of the traffic volume during the planning time horizon. In the lecture some techniques are shown which handle this problem: in multiperiod, long time model at each planning period the traffic volume change is take into consideration. In ranking models the problem could be handled and solved. In the case of one period Markov stabile model there is nothing to do. In the multiperiod model the problem could be solved also.*

## 1 PMS Models

### 1.1 The Main Questions of Decision Makers of the PMS

The main questions of decision makers of the Road Management System and the PMS:

- How much budget is required each year for maintenance and rehabilitation the elements for their whole lifetime to hold them in a certain condition level?
- Which AM elements condition distribution would be expected if the sum mentions above were to be available?
- What consequences would be expected if this sum were not available?
- What consequences would be expected if the maintenance expenditures were significantly increased?

- What are the optimum times for maintenance actions?
- What consequences would be expected from the delay of necessary actions?
- Who benefits and who loses, to what extent, etc?

## 1.2 The Models

A decision model for pavement management has been developed herein based on linear programming formulation. The engineering and deterioration model was created by Gaspar [5], the mathematical one by Bakó [2], Markov transition probability matrices are introduced to model the deterioration process of the road sections is determined by these matrices. To every type of road surfaces and class of traffic amount belongs a certain Markov matrix. The presented model and methodology is used to determine the optimal rehabilitation and maintenance policy in network level. Depending on the objective function two types of problems could be solved by the model: the necessary funds calculation and the optimal budget allocation for the entire network.

Several types of solution algorithms can be used depending on the given task, the available data, the budget constraints, etc. Two main types are the heuristic and the optimisation algorithms. The heuristic technique is usually used in project level, but it could be used in network level too. The optimisation models are solved by the traditional optimisation algorithms. Depending on the problem to be solved, we use integer, a linear or a dynamic programming algorithm.

There are several constraints to be fulfilled. We will denote the unknown variable by  $X_{ijk}$  which belongs to the pavement type  $i$ , to the traffic volume  $j$  and to the maintenance politics. The solution has to be Markov stable. The Markovian stability constraint is

$$\sum_{i=1}^s \sum_{j=1}^f \sum_{k=1}^t (Q_{ijk} - E) X_{ijk} = 0 \quad (1)$$

where  $E$  is a unit matrix.

Because the equality is usually not fulfilled or not desirable, we use  $\leq$  or  $\geq$  relation instead of equality in (1). There are several further constraints, which are connected with other suppositions. We suppose that the traffic volume will not change during the planning period:

$$\sum_{k=1}^t X_{ijk} = b_{ij}, i = 1, 2, \dots, s \quad (2)$$

$$j = 1, 2, \dots, f$$

where  $b_{ij}$  belongs to the pavement type  $i$  and the traffic volume  $j$ .

The total area of the road surface type  $i$  will remain the same at the end of the planning period

$$\sum_{j=1}^f \sum_{k=1}^t X_{ijk} = d_i, i = 1, 2, \dots, s \quad (3)$$

where  $d_i$  belongs to the pavement type  $i$  and  $\sum_{i=1}^s d_i = \underline{1}$ .

We have to apply one of the maintenance politic on every road section

$$\sum_{i=1}^s \sum_{j=1}^f \sum_{k=1}^t X_{ijk} = \underline{1} \quad (4)$$

We divide the segments into 3 groups: acceptable (good), unacceptable (bad) and the rest. Let us denote the tree set by  $J$  (good) by  $R$  (bad) and by  $E$  (rest of the segments) and by  $H$  the whole set of segments. The relations for these sets are given by

$$\begin{aligned} J \cap R &= \emptyset, & J \cap E &= \emptyset \\ R \cap E &= \emptyset, & J \cup R \cup E &= H \end{aligned}$$

The following conditions are related to these sets

$$\begin{aligned} \sum_{i,j,k \in J} X_{ijk} &\geq v_J \\ \sum_{i,j,k \in R} X_{ijk} &\leq v_R \end{aligned} \quad (5)$$

$$\underline{v}_E \leq \sum X_{ijk} \leq \overline{v}_E$$

where  $J, R, E$  are given above, and

$v_J$  the total length of the good road after the planning period

$v_R$  the total length of the bad road after the planning period

$v_E$  the lower bound of the other road

$\overline{v}_E$  the upper bound of the other road.

The meaning of the first condition is, that the amount of good segment has to be greater than or equal to a given value. The second relation does not allow more

bad roads than it is fixed in advanced. The third relation gives an upper and lower limit to the amount of the rest road.

Let us denote by  $c_{ijk}$  the unit cost of the maintenance politic  $k$  on the pavement type  $i$  and traffic volume  $j$ . Our objective is to choose such an  $X$  which:

- fulfils the conditions given above

with minimal rehabilitation cost.

The objective is

$$\sum_{i=1}^s \sum_{j=1}^f \sum_{k=1}^t X_{ijk} C_{ijk} \rightarrow \min! \quad (6)$$

Let us denote this value by  $C$ . The budget  $C^*$  which is available for the maintenance purpose usually less than  $C$ , so  $C^* < C$ . In this case we modify our model: the above mentioned rehabilitation cost function becomes constrained:

$$\sum X_{ijk} C_{ijk} \leq C^* \quad (7)$$

and we use an other objective. Let us denote the benefit by  $h_{ijk}$  where this is the benefit of the societies when we apply on the pavement type  $i$  and with the traffic volume  $j$  the maintenance politic  $k$ .

Our aim is to determine such a solution  $X$  which fulfils the constraints (1)-(5) and (7) and maximizes the total benefit of the society.

The objective in this case is

$$\sum X_{ijk} h_{ijk} \rightarrow \max! \quad (8)$$

Gaspar [6, 7] summarizes the detailed of the using algorithms.

The Road Management Systems (and the PMS) usually do not take into consideration the future traffic change. The deterioration process depends on mostly the volume of the traffic. That is why it is important to take into consideration the change of the traffic volume during the planning time horizon.

## 2 Traffic Planning and Forecasting

A lot of problems are known to be connected with traffic planning, forecasting, distribution and assignment:

- determination of the path with minimal travelling time between every pair of points (SP-problem);

- forecast of the future traffic starting from the source point (Forecasting);
- determination of the traffic on the road segments (Assignment problem).

## 2.1 SP Problem

At the SP problem let  $N$  be a finite set of points,  $E$  the set of edges, and  $d(x, y) > 0$  the travelling time on the edge  $(x, y)$ . The travelling time on path  $P = (x_1, x_2, \dots, x_r)$  is

$$L(P) = \sum_{i=1}^{r-1} d(x_i, x_{i+1}). \quad (9)$$

The multiterminal shortest path problem can be formulated as follows: determine the path  $P$  with minimal travelling time for every pair of points.

There are several algorithms for solving this problem. The best method was given by Warshall [9]. This algorithm consists of the following steps:

$$\begin{aligned} d_{ij}^{(0)} &= d(x_i, x_j), \\ d_{ij}^{(k)} &= \min(d_{ij}^{(r-1)}, d_{ir}^{(r-1)} + d_{rj}^{(r-1)}) \end{aligned} \quad (10)$$

where  $n$  is the number of points. This algorithm needs the least number of computational steps; only  $n^3$  additions and comparisons are needed. The matrix  $D^{(n)} = (d_{ij}^{(n)})$  gives the length of the minimal paths. In order to obtain the paths themselves we compute another so called labelling matrix  $S^{(k)}$ :

$$\begin{aligned} s_{ij}^{(0)} &= j \\ s_{ij}^{(k)} &= \begin{cases} s_{ij}^{(r-1)}, & \text{if } d_{ij}^{(r-1)} \leq d_{ir}^{(r-1)} + d_{rj}^{(r-1)}, \\ s_{ir}^{(r-1)} & \dots \text{otherwise} \end{cases} \end{aligned} \quad (11)$$

## 2.2 Traffic Forecasting and Assignment

Fratar problem: Given a matrix  $A = (a_{ij})$ . The element  $a_{ij}$  means the amount of traffic from point  $i$  to point  $j$ . Let us denote by  $k_i$  respectively by  $l_i$  the amount of traffic leaving respectively entering to point  $i$ . The Fratar problem (F-problem) is to determine the future traffic matrix from the matrix  $A$  using the given  $k_i$  and

$l_i$  values. In the case of Gravity problem Instead of matrix  $A$  matrix  $D$  is given where the element  $d_{ij}$  means the minimal travelling time from point  $i$  to point  $j$ . These two problems seem to be different but both problems can be solved by the forecasting of the input-output (I/O) table.

Let  $K_1, K_2, \dots, K_m$  be sources and let  $L_1, L_2, \dots, L_m$  be sinks. The element  $a_{ij} \geq 0$  of matrix  $A$  means the traffic from  $K_i$  to  $L_j$ . Let us denote by  $k_i$  respectively by  $l_j$  the total traffic leaving  $K_i$  respectively  $L_j$ :

$$k_i = \sum_{j=1}^n a_{ij}, \quad (i = 1, 2, \dots, n),$$

$$l_j = \sum_{i=1}^n a_{ij}, \quad (i = 1, 2, \dots, n).$$
(12)

The matrix  $A$  is called I/O table, and elements  $k_i, l_j$  are called the marginal values of I/O table  $A$ .

Let  $k_1, k_2, \dots, k_n$  and  $\lambda_1, \lambda_2, \dots, \lambda_n$  be new marginal values ( $k_i \geq 0, \lambda_i \geq 0, i = 1, 2, \dots, n$ ). The forecasting of the I/O table is to determine the new I/O table  $X$  from  $A$  and given marginal values  $k_i, \lambda_j$  so that the tables  $A$  and  $X$  are 'similar'. The two tables are similar, when  $x_{ij}$  can be determined in the following form

$$x_{ij} = \xi_i a_{ij} \eta_j$$
(13)

where  $\xi_i, \eta_j$  are unknown. Klafszki [8] Bakó [1] has shown by the help of geometric programming that this problem is equivalent to the minimalization of information divergence. The table  $X$  has to satisfy the equations (12).

Let  $k_1, k_2, \dots, k_n$  and  $\lambda_1, \lambda_2, \dots, \lambda_n$  be new marginal values ( $k_i \geq 0, \lambda_i \geq 0, i = 1, 2, \dots, n$ ). The forecasting of the I/O table is to determine the new I/O table  $X$  from  $A$  and given marginal values  $k_i, \lambda_j$  so that the tables  $A$  and  $X$  are 'similar'. The two tables are similar, when  $x_{ij}$  can be determined in the following form

$$x_{ij} = \xi_i a_{ij} \eta_j$$
(14)

where  $\xi_i, \eta_j$  are unknown. Klafszki [8] Bakó [1] has shown by the help of geometric programming that this problem is equivalent to the minimalization of information divergence. The table X has to satisfy the equations (13).

$$\sum_{j=1}^n x_{ij} = k_i, \quad (i = 1, 2, \dots, n) \tag{15}$$

$$\sum_{i=1}^n x_{ij} = \lambda_j, \quad (j = 1, 2, \dots, n)$$

Using equations (14) and (15) we obtain

$$\xi_i \sum \eta_j a_{ij} = k_i, \quad \eta_j \sum \xi_i a_{ij} = \lambda_j \tag{16}$$

D'Esopo [4] suggested the following method for solving the equations (16):

Step 0  $\eta_j^{(0)} = \lambda_j, \quad (j = 1, 2, \dots, n)$

$$\xi_i^{(r)} = \frac{k_i}{\sum_j \eta_j^{(r)} a_{ij}}, \quad (i = 1, 2, \dots, n),$$

Step r  $\eta_j^{(r)} = \frac{\lambda_j}{\sum_i \xi_i^{(r)} a_{ij}}, \quad (j = 1, 2, \dots, n)$  (17)

Bergmann [3] proved the convergence of the algorithms. The algorithm is terminated, when the error

$$\max_{1 \leq i \leq \pi} \left| \frac{\sum \xi_i a_{ij}^{n_j - k_i}}{k_i} \right| \tag{18}$$

is sufficiently small.

Traffic assignment is the process of allocating all the trips in one or more O/D matrices to their route in the network, resulting the flows on its links. The traffic assignment methods employ three basic steps: SP problem, assignment a part of the traffic on the segments and check for convergence.

### 3 Combined Algorithms

The maintenance and rehabilitation actions and the development of the road network structure and the changing traffic structure modify the amount of the traffic on the road section. The deterioration process depends on mostly the volume of the traffic.

That is why it is important to take into consideration the change of the traffic volume during the planning time horizon. In the first part the PMS models are summarised. In the second part we illustrate the traffic forecast, distribution any assignment algorithm (TFDA).

We proposed a new model with: new segments each time period

– traffic forecasting and assignment each planning period. Now the basic idea of combined algorithm is demonstrated. The algorithms are a long range multitime period ones, we compute the maintenance and rehabilitation actions for each year. After determination the actions in a year, a TFDA (*traffic forecast, distribution any assignment algorithm*) step is taken, where the new traffic on the segments are determined.

After that new traffic categories is formed and the next PMS step (for the next time period) use this new traffic categories. The algorithms demonstrate is both cases the steps of the algorithm.

The first algorithm is shown in the case of heuristic (ranking) method:

- Step 1            This is a PMS ranking for the first time period.
- Step 2            A TFDA step is given.
- Step 3            This computation continues until the last period is finished.  
                    The number of the time period is n.

The combined algorithm is shown in the case of Markov deterioration model is the following:

- Step 1            The first step is to compute the Markov stable solution. Here traffic change is not taken into consideration.
- Step 2            The yearly deterioration and maintenance process is determined.
- Step 3            After in each year a TFDA algorithm is taken and the result of that is used to form the new traffic categories for the next year.
- Step 4            The algorithm is finished when we compute the deterioration and maintenance actions for the last time period.

The procedures given above serve better results because the traffic changes are taken into consideration in each year.

## References

- [1] Bakó, A.: Mathematical and Computational Evaluation and Development of Traffic Planning and Forecasting, Candidate dissertation, HAS, 1980, p. 220
- [2] Bakó, A.: Decision Supporting Model for Highway Maintenance, Acta Politechnica Hungarica, 2004, pp. 96-108
- [3] Bergman, L.: Dokazateľstvo sztodimosztyi G. B., Selekkozskov dlja zadaci sz transzportnúi organicsamijami, Zsurnal Vücsiszl. Mat. I. Mat. Fiziki 1, 1967, pp. 147-156
- [4] D'Esopo, D. A.: Lekkowitz, An Algorithm for Computing Intersonal Transfers Using the Gravity Model, OPNS, Res. 11, 1963, pp. 901-907
- [5] Gaspar L.: Development of the First Hungarian PMS, Transportation Research, 1992, pp. 132-141 (in Hungarian)
- [6] Gaspar, L.: Ein netzbezogenes Managementsystem für die Shassenerhaltung in Ungarn, Strasse und Autobahn 1992/8, pp. 123-127
- [7] Gaspar, L.: Road Management, Hungarian Academy of Sciences, 2003, p. 361 (in Hungarian)
- [8] Klafszky, E.: On the Prediction of the Input-Output Table, Report of Computig and Aut. Inst. 10, 1973, pp. 1-13, (in Hungarian)
- [9] Warshall, S.: A Theorem of Boolean Matrices, J. of ACM 9, 1962, pp. 11-12