IMAGE INTERPOLATION TECHNIQUES

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Abstract: In this paper interpolation techniques are presented. They are compared on an accuracy and computational efficiency base. Both the interpolation kernels and frequency responses are given.

Keywords: image interpolation, spline, Fourier transform

INTRODUCTION

Interpolation is the process of determining the values of a function at positions lying between its samples. It achieves this process by fitting a continuous function through the discrete input samples. This permits input values to be evaluated at arbitrary positions in the input, not just those defined at the sample points. While sampling generates an infinite bandwidth signal from one that is bandlimited, interpolation plays an opposite role: it reduces the bandwidth of a signal by applying a low-pass filter to the discrete signal. That is, interpolation reconstructs the signal lost in the sampling process by smoothing the data samples with an interpolation function.

The process of interpolation is one of the fundamental operations in image processing. The image quality highly depend on the used interpolation technique. The interpolation techniques are divided into two categories, deterministic and statistical interpolation techniques. The diference is that deterministic interpolation techniques assume a certain variability between the sample points, such as linearity in case of linear interpolation. Statistical interpolation methods approximate the signal by minimizing the estimation error. This approximation process may result in original sample values not being replicated. Since statisticsl methods are computationally inefficient, in this article only deterministic techniques will be discussed. A comparison betwwen methods will also be made.

In this article, we are mainly concerned about image interpolation. However, interpolation in two dimensions for the general case is sometimes difficult to describe. For gridded data, the *n*-dimensional interpolation function can be described as the product of *n* one-dimensional interpolation functions. Therefore it is permitted to look at one dimensional interpolation functions to discuss the behavior of the *n*-dimensional interpolation functions.

Deterministic interpolation techniques

Several interpolation techniques have been developed and can be found in the literature. The most commonly used methods are the nearest neighbor, linear and spline interpolation techniques. Less common are the polynomial and Lagrange interpolation methods.

INTERPOLATION KERNELS

The numerical accuracy and computational cost of interpolation algorithms are directly tied to the interpolation kernel. As a result, interpolationkernels are the target of design and analysis. Here, analysis is applied to the 1-D case. Interpolation in 2-D is a simple extension of the 1-D case. In addition, data samples are assumed to be eequally spaced along each dimension. This restriction causes no serious problem because images are usually defined on regular grids.

Nearest Neighbor

The simplest interpolation from a computational standpoint is the nearest neighbor, where each interpolated output pixel is assigned the value of the nearest sample point in the input image. This technique is also known as point shioft algorithm and pixel replication. The interpolation kernel for the nearest neighbor algorithm is defined as

$$h(x) = \begin{cases} 1 & 0 \le |x| < 0.5 \\ 0 & 0.5 \le |x| \end{cases} \tag{1}$$

The frequency response of the nearest neighbor kernel is

$$H(\omega) = \sin c\left(\frac{\omega}{2}\right) \tag{1a}$$

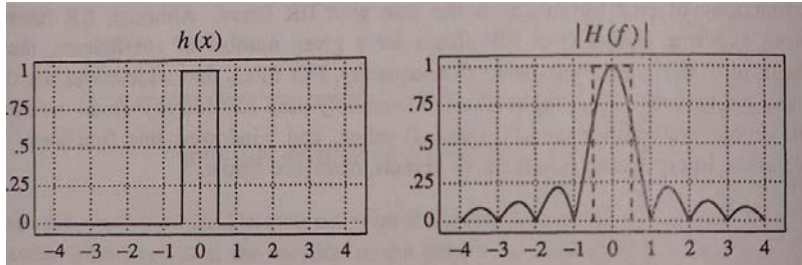The kernel and its Fourier transform are shown in Figure 1.

Figure 1

Convolution in the spatial domain with the rectangle function h is equivalent in the frequency domain to multiplication with a sinc function. Due to the prominent side lobes and infinite extent, a sinc function makes a poor low-pass filter. This technique achieves magnification by pixel replication, and minification by sparse point sampling. For large-scale changes, nearest neighbor interpolation produces images with blocky effects.

Linear Interpolation

Linear interpolation is a first degree method that passes a straight line through every two consucutive points of the input signal.

In the spatial domain, linear interpolation is equivalent to convolvong the sampled input with the following kernel.

$$h(x) = \begin{cases} 1 - |x| & 0 \le |x| < 1 \\ 0 & 1 \le |x| \end{cases} \tag{2}$$

The frequency response of the linear interpolation kernel is

$$H(\omega) = \sin c^2 \left(\frac{\omega}{2} \right)$$

This kernel is also called triangle filter, roof function or Bartlett windoe. As shown in Figure 2 the frequency response of the linear interpolation kernel is superior to that of the nearest neighbor interpolation function. The side lobes are less prominent, so the performance is improved in the stopband. A passband is moderately attenuated, resulting in image smoothing.
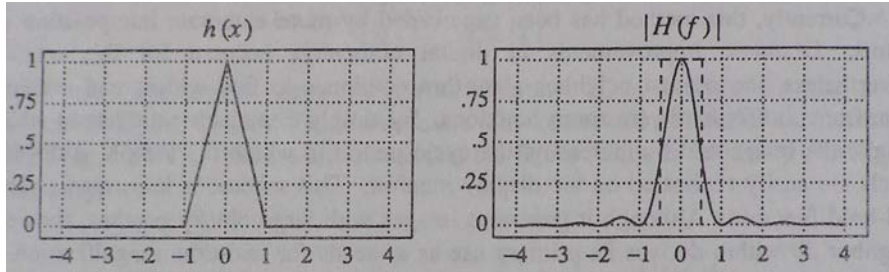
Figure 2

Linear interpolation produces reasonably good results at moderate cost. But for even better performance, more complicated algorithms are needed.

Cubic convolution

Cubic convolution is a third degree interpolation algorithm that fairly well approximates the theoretically optimum sinc interpolation function. The kernel is composed of piecewise cubic polynomials defined on subintervals (-2, -1), (-1, 0), (0, 1) and (1, 2). Outside this interval the interpolation kernel is zero. For deriving the cubic convolution kernel, we have to solve 8 linear equations with 7 unknown parameters, so the system has one "free" parameter that may be controlled by the user. The kernel is of form:

$$h(x) = \begin{cases} (a+2)|x|^3 - (a+3)|x|^2 + 1 & 0 \le |x| < 1 \\ a|x|^3 - 5a|x|^2 + 8a|x| - 4a & 1 \le |x| < 2 \\ 0 & 2 \le |x| \end{cases} \quad (3)$$

The frequency response is

$$R(\omega) = \frac{12}{\omega^2}\left(\sin c^2\left(\frac{\omega}{2}\right) - \sin c(\omega)\right) + a\frac{8}{\omega^2}\left(3\sin c^2(\omega) - 2\sin c(\omega) - \sin c(2\omega)\right)$$
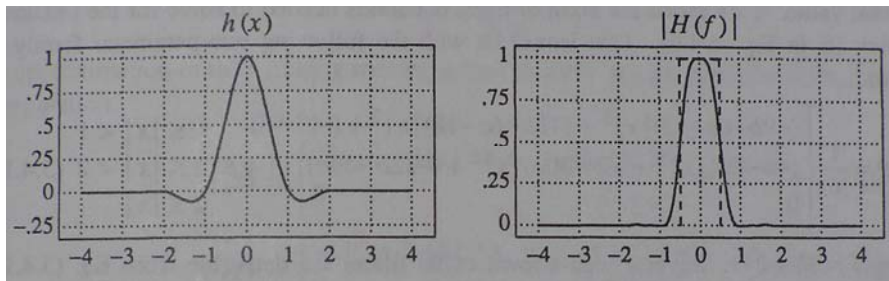

Figure 3

Choices for a are $a=-1$, $a=-0.75$ and $a=-0.5$. The performance of the interpolation kernel depends on a, and the frequency content of the image. For different images, different values of the parameter a gives the best performance.

B-splines

      A *B-spline* of degree n is derived through n convolutions of the box filter, B_0. Thus, $B_1=B_0*B_0$ denotes a B-spline of degree 1, yielding the familiar triangle filter. (Letter B may stand for basic or basis). The B-spline of degree 1 is equivalent to linear interpolation. The second degree B-spline B_2 is produced by convolving B_0*B_1.

      The cubic B-spline B_3 is generated from convolving B_0*B_2. That is $B_3=B_0*B_0*B_0*B_0$.

Figure 4 summarizes the shapes of these low-order B-splines.

The cubic B-spline interpolation kernel is defined as:

$$h(x) = \frac{1}{6}\begin{cases} 3|x|^3 - 6|x|^2 + 4 & 0 \le |x| < 1 \\ -|x|^3 + 6|x|^2 - 12|x| + 8 & 1 \le |x| < 2 \\ 0 & 2 \le |x| \end{cases} \quad (4)$$
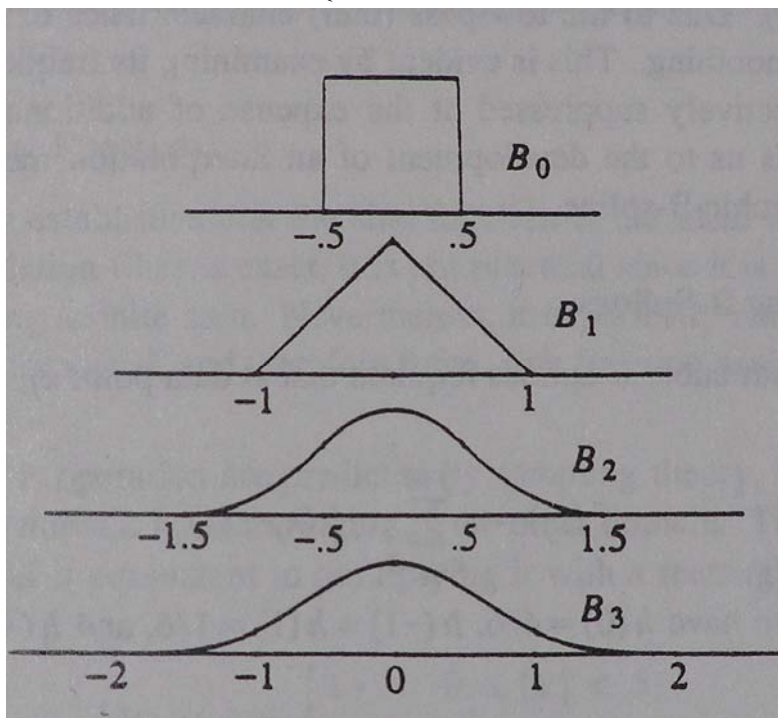

Figure 4

Unlike cubic convolution, the cubic B-spline kernel is not interpolatory since it does not satisfy the necesssary constraint that h(0)=1 and h(1)=h(2)=0. Instead, it is an approximating function that passes near the points but not necessarily through them. This is due to the fact that the kernel is strictly positive.

The positivity of the B-spline kernel is attractive for our image processing application. When using kernels with negative lobes, it is possible to generate negative values while interpolating positive data. Since negative intensity values are meaningless for display, it is desirable to use strictly positive interpolation kernelsto guarantee the positivity of the interpolated image.

References

[1]   George Wolberg: Digital Image Warping, IEEE Computer Society Press, Los Alamitos, California, 1990

[2]   Michael Unser: Splines – A Perfect Fit for Signal and Image Processing, IEEE Signal Processing Magazine, November 1999, pp. 22-38

[3]   Robert G. Keys: Cubic Convolution Interpolation for Digital Image Processing, IEEE Transactions on Acoustics, Speech, and Signal Processing, Vol. ASSP-29, No. 6, December 1981, pp.1153-1160

[4]   Hsieh S. Hou, Harry C. Andrews: Cubic Splines for Image Interpolation and Digital Filtering, , IEEE Transactions on Acoustics, Speech, and Signal Processing, Vol. ASSP-26, No. 6, December 1978, pp.508-517