Choquet and Sugeno integrals-based functional representation

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Abstract: This paper discusses a representation by a difference of two Choquet integrals of comonotone additive and monotone functional L, defined on the class of functions with finite support. We propose a representation of a comonotone-$\check{\vee}$-additive and monotone functional L by two Sugeno integrals. It is also shown that the symmetric Sugeno integral is comonotone-$\check{\vee}$-additive functional on the class of functions with finite support.

Key words and phrases: fuzzy measures, Choquet integral, Sugeno integral, cumulative prospect theory.

1 Introduction

In the field of fuzzy measures (i.e. capacities, non additive measures), Choquet integral and Sugeno integral have been proposed to aggregate criteria in decision making problems. These integrals, constructed on the concept of fuzzy measure, are the most often used operators which are able to take into account the interaction among criteria.

The Choquet integral with respect to a fuzzy measure, proposed by Murofushi and Sugeno [11], is proved to be an adequate tool for the subjective evaluation and the decision analysis [19, 5]. The Sugeno integral, due to his ordinal nature, is suitable in qualitative decision making.

Tversky and Kahneman [20] brought into light the so-called Cumulative Prospect Theory (CPT), as a combination of cumulative utility (Choquet expected utility model) and generalization of expected utility (sign dependent expected utility model). We say that CPT holds if the preference $\preceq$ can be represented by a rank and sign dependent functional, i.e. it is expressed as

$$L(f) = C_{\mu^+}(f^+) - C_{\mu^-}(f^-),$$

where μ^+ and μ^- are two fuzzy measures, $f^+ = f \vee 0$ is the gain part of prospect f, and $-f^-$ is it's loss part, $f^- = (-f) \vee 0$. C_μ denotes a Choquet integral with respect to a fuzzy measure μ.

Schmeidler [21] proved that a commonotone additive and monotone functional L on $\mathcal{M}$, class of all nonnegative measurable functions can be represented by a Choquet integral with respect to a fuzzy measure. Narukawa et al. [12] proved that a comonotone-additive and monotone functional can be represented as a difference of two Choquet integrals and gave the conditions for which it can be represented by one Choquet integral.

Crucial properties of the Sugeno integral are monotonicity and comonotonic maxitivity, see [1, 2, 4, 15]. It is also homogeneous with respect to operation min $(\wedge)$.

The main aim of this paper is to consider the problem of whether or not a comonotone-$\check{\vee}$-additive functional L, defined on on the class of functions $f : X \to [-1, 1]$ with finite support, admits a Sugeno integrals-based representation. In the next section 2. the fuzzy measure, Choquet and Sugeno integrals and the concept of comonotonicity are recalled. In the section 3. we discussed representation of comonotone additive and monotone functional L by a difference of two Choquet integrals. In the section 4. we define fuzzy rank and sign dependent functional (functional of $\check{\vee}$-CPT type) and a representation of comonotone-$\check{\vee}$-additive and monotone functional L is obtained. Finally we shall prove comonotone-$\check{\vee}$-additivity of the symmetric Sugeno integral.

2 Choquet and Sugeno integral with respect to a fuzzy measure

Let X be an universal set. Let $\mathcal{A}$ be a σ-algebra of subsets of X.

A fuzzy measure μ is non-decreasing, non-negative real-valued set function $\mu : \mathcal{A} \longrightarrow [0, \infty]$, vanishing on the empty set. If the condition $\mu(X) = 1$ is satisfied, we say that μ is normalized fuzzy measure.

The conjugate fuzzy measure of μ is defined by $\bar{\mu}(A) = \mu(X) - \mu(\bar{A})$ where $\bar{A}$ denotes the complement set of A, $\bar{A} = X \setminus A$.

Let $\mathcal{M}(X)$ be the class of all measurable functions $f : X \to [0, \infty]$ and let $\mathcal{M}_1(X)$ denotes the class of all measurable functions $f : X \to [0, 1]$. We introduce the Choquet integral and Sugeno integral with respect to a fuzzy measure $\mu : \mathcal{A} \longrightarrow [0, \infty]$ $([0, 1])$, respectively, of a measurable function $f : X \to [0, \infty]$ $([0, 1])$, respectively.

Definition 1 *([15, 1]) Let $(X, \mathcal{A})$ be a measurable space.*

i) The Choquet integral *w.r.t a fuzzy measure $\mu : \mathcal{A} \longrightarrow [0, \infty]$ is functional*

$C_\mu : \mathcal{M}(X) \longrightarrow [0,\infty]$ *defined by*

$$C_\mu(f) = \int_0^\infty \mu(\{x | f(x) \geq t\})\, dt$$

ii) The Sugeno integral *w.r.t. a fuzzy measure* $\mu : \mathcal{A} \longrightarrow [0,1]$ *is functional* $S_\mu : \mathcal{M}_1(X) \longrightarrow [0,1]$ *defined by*

$$S_\mu(f) = \bigvee_{t\in[0,1)} (t \wedge \mu(\{x \,|\, f(x) \geq t\}).$$

Let X be a finite set $X = \{x_1, \dots, x_n\}$. The Choquet integral of a function $f \in \mathcal{M}(X)$ w.r.t $\mu : \mathcal{A} \longrightarrow [0,\infty]$ can be expressed as

$$C_\mu(f) = \sum_{i=1}^{n} (f_{\alpha(i)} - f_{\alpha(i-1)})\mu(A_{\alpha(i)})$$

where f admits a comonotone-additive representation $f = \sum_{i=1}^n f_{\alpha(i)} \mathbf{1}_{A_{\alpha(i)}}$ and $\alpha = (\alpha(1), \alpha(2), \dots, \alpha(n))$ is a permutation of index space $\{1, 2, \dots, n\}$ such that $0 \leq f_{\alpha(1)} \leq \cdots \leq f_{\alpha(n)}$, $f_{\alpha(0)} = 0$, sets $A_{\alpha(i)}$ are $A_{\alpha(i)} = \{x_{\alpha(i)}, \dots, x_{\alpha(n)}\}$ and $\mathbf{1}_A$ is characteristic function of a crisp subset A of X.

The Sugeno integral of a function $f \in \mathcal{M}_1(X)$ w.r.t $\mu : \mathcal{A} \longrightarrow [0,1]$ can be written as

$$S_\mu(f) = \bigvee_{i=1}^{n} f_{\alpha(i)} \wedge \mu(A_{\alpha(i)}).$$

f admits a comonotone-maxitive representation $f = \bigvee_{i=1}^n f_{\alpha(i)} \wedge \mathbf{1}_{A_{\alpha(i)}}$, where $\alpha = (\alpha(1), \alpha(2), \dots, \alpha(n))$ is a permutation of index set $\{1, 2, \dots, n\}$ such that $0 \leq f_{\alpha(1)} \leq \cdots \leq f_{\alpha(n)} \leq 1$ and $A_{\alpha(i)} = \{x_{\alpha(i)}, \dots, x_{\alpha(n)}\}$.

There exist two extensions of Choquet integral to the whole real line $[-\infty, \infty]$, *the symmetric* Choquet integral, introduced by Šipoš and *the asymmetric* Choquet integral (more details can be found in [15, 1]). Let $\overline{\mathcal{M}}(X)$ denotes class of all measurable functions $f : X \to [-\infty, \infty]$.

Definition 2 *i)* The symmetric Choquet integral *w.r.t a fuzzy measure* $\mu : \mathcal{A} \longrightarrow [0,\infty]$ *is functional* $\hat{C}_\mu : \overline{\mathcal{M}}(X) \longrightarrow [-\infty, \infty]$ *defined by*

$$\hat{C}_\mu(f) = C_\mu(f^+) - C_\mu(f^-),$$

where $f^+ = f \vee 0$ *and* $f^- = (-f) \vee 0$.

ii) The asymmetric Choquet integral *w.r.t a fuzzy measure* $\mu : \mathcal{A} \longrightarrow [0,\infty]$, $\mu(X) < \infty$ *is functional* $C_\mu : \overline{\mathcal{M}}(X) \longrightarrow [-\infty, \infty]$ *defined by*

$$C_\mu(f) = C_\mu(f^+) - C_{\bar{\mu}}(f^-),$$

where $f^+ = f \vee 0$ *and* $f^- = (-f) \vee 0$.

When the expression $\infty - \infty$ *is occurred, the integrals are not defined.*

We give now a simple illustration.

Example 1 Let $X = \{1, 2, 3\}$ and

$f(1) = -1$	$\mu(\{1\}) = 0.2$	$\mu(\{1,2\}) = 0.4$
$f(2) = -0.4$	$\mu(\{2\}) = 0.3$	$\mu(\{1,3\}) = 0.45$
$f(3) = 0.5$	$\mu(\{3\}) = 0.2$	$\mu(\{2,3\}) = 0.6$

$C_\mu(f^+) = 0.5 \cdot 0.2 = 0.1$, $C_\mu(f^-) = 0.4 \cdot 0.4 + 0.6 \cdot 0.2 = 0.28$ and $C_{\bar{\mu}}(f^-) = 0.4 \cdot 0.8 + 0.6 \cdot 0.4 = 0.56$.
By Definition 2 i) and ii) we have $\hat{C}_\mu(f) = -0.18$ and $C_\mu(f) = -0.46$, respectivly.

The extension of the Sugeno integral to the bipolar scale $[-1, 1]$ in the spirit of the symmetric extension of Choquet integral, has been proposed by M. Grabisch in [8]. We recall now the symmetric Sugeno integral of measurable function $f : X \longrightarrow [-1, 1]$, where X is finite set.

Definition 3 *[8] Let μ be a fuzzy measure, $\mu : \mathcal{A} \longrightarrow [0, 1]$.*
The symmetric Sugeno integral *is functional $\hat{S}_\mu : \overline{\mathcal{M}}_1(X) \longrightarrow [-1, 1]$ defined by*

$$\hat{S}_\mu(f) = \Big(S_\mu(f^+)\Big) \check{\vee} \Big(-S_\mu(f^-)\Big)$$

where $f^+ = f \vee 0$ and $f^- = (-f) \vee 0 = -(f \wedge 0)$.

An operation $\check{\vee}$ is the *symmetric maximum*, originally introduced by Grabisch, and it is given by: $a \check{\vee} b = \mathrm{sign}(a + b)(|a| \vee |b|)$. $\check{\wedge}$ denotes an operation *symmetric minimum*: $a \check{\wedge} b = \mathrm{sign}(a \cdot b)(|a| \wedge |b|)$.

The symmetric Sugeno integral of the function $f \in \overline{\mathcal{M}}_1(X)$ can be considered as it is proposed in [8] by:

$$\hat{S}^1_\mu(f) = \Big\langle \Big(\check{\bigvee}_{i=1}^{s} f_{\alpha(i)} \check{\wedge} \mu(\{x_{\alpha(1)}, \ldots, x_{\alpha(i)}\}) \Big) \\ \check{\bigvee} \Big(\check{\bigvee}_{i=s+1}^{n} f_{\alpha(i)} \check{\wedge} \mu(\{x_{\alpha(i)}, \ldots, x_{\alpha(n)}\}) \Big) \Big\rangle \tag{1}$$

where α is a permutation of index set such that $-1 \le f_{\alpha(1)} \le \cdots \le f_{\alpha(s)} < 0$ and $0 \le f_{\alpha(s+1)} \le \cdots \le f_{\alpha(n)} \le 1$.

Two approaches, the equation (1) and Definition 3, are different in general. The reason for that is non-associativity of the operation $\check{\vee}$ on $[-1, 1]$ and M. Grabisch proposed in [8] several rules of computation to avoid this problem (see [7, 8]).

The most representative axiomatic characterization of the Choquet integral and Sugeno integral-based functionals, C_μ and S_μ are based on the concept of comonotonic additivity. At the end of this section, let we recall, briefly, comonotonicity (common monotonicity) of two measurable functions.

Two measurable functions f and g on X are called comonotone [4] if for all x , $x_1 \in X$ we have $f(x) < f(x_1) \;\Rightarrow\; g(x) \leq g(x_1)$, i.e. if they are measurable with respect to the same chain $\mathcal{C}$ in $\mathcal{A}$ ($\mathcal{A}$ is a σ-algebra of subsets of X).

3 Comonotone additive functional

In this section we consider comonotone additive functional L and briefly present some results obtained in [12].

Let X be a countable set $X = \{x_1, x_2, \ldots, \}$ and let μ be a fuzzy measure $\mu : \mathcal{P}(X) \longrightarrow [0, \infty]$. Let $\mathcal{S}(X)$ denotes class of functions $f : X \longrightarrow [-\infty, \infty]$ with finite support, where $supp(f) = \{x \in X | f(x) \neq 0\}$.

We say that a functional L on class $\mathcal{S}(X)$ is

i) comonotone additive iff $L(f+g) = L(f) + L(g)$ for all comonotone functions f, $g \in \mathcal{S}(X)$,

ii) monotone iff $f \leq g \implies L(f) \leq L(g)$ for all functions f, $g \in \mathcal{S}(X)$,

iii) comonotone monotone iff $f \leq g \implies L(f) \leq L(g)$ for all comonotone functions f, $g \in \mathcal{S}(X)$.

Let we consider comonotone additive and comonotone monotone functional L on $\mathcal{S}(X)$.

Two set functions, μ_L^+ and μ_L^+, are induced by a functional L, defined by:

$$\mu_L^+(C) = L(\mathbf{1}_C) \text{ and } \mu_L^-(C) = -L(-\mathbf{1}_C), \quad \text{for all finite sets } C \subset X.$$

Let $A \subset X$ be infinite set. μ_L^+ and μ_L^- are defined by

$$\mu_L^+(A) = \sup\{\, \mu_L^+(C) \,|\, C \subset A\,\} \text{ and } \mu_L^-(C) = \sup\{\, \mu_L^-(C) \,|\, C \subset A\,\}$$

Then for $A \subset B$ we have $\mathbf{1}_A \leq \mathbf{1}_B$ and because of monotonicity of the functional L we obtain $\mu_L^+(A) \leq \mu_L^+(B)$ and $\mu_L^-(A) \leq \mu_L^-(B)$, i.e., μ_L^+ and μ_L^- are fuzzy measures.

We give now the theorem of representation of comonotone additive and comonotone monotone (c.a.c.m. for short) functional L. Proof can be found in [12].

Proposition 1 *[12] L is c.a.c.m. functional on $\mathcal{S}(X)$ if and only if there exist fuzzy measures μ_L^+ and μ_L^- such that*

$$L(f) = C_{\mu_L^+}(f^+) - C_{\mu_L^-}(f^-),$$

for all $f \in \mathcal{S}(X)$.

In the case of finite set X, we have the next corollary:

Corollary 1 *[12] L is c.a.c.m. functional on $\mathcal{S}(X)$ if and only if there exists a fuzzy measure μ_L such that*

$$L(f) = C_{\mu_L}(f),$$

for all $f \in \mathcal{S}(X)$.

The results, presented above, have application in preference representation. An axiomatization for the preference $\preceq$ to by CPT is given in [12], but we will not discuss this topic.

4 Comonotone-$\check{\vee}$-additive functional and its $\check{\vee}$-CPT representation

In this section we shall adopt a rule of computation $\lfloor\ \rfloor$ proposed in [8]:

$$\left\lfloor \check{\bigvee}_{i=1}^{n} a_i \right\rfloor := \left(\check{\bigvee}_{a_i \geq 0} a_i\right) \check{\vee} \left(\check{\bigvee}_{a_i < 0} a_i\right)$$

to avoid problem of non-associativity of $\check{\vee}$ and in accordance with it, definition 1 (ii) for symmetric Sugeno integral.

Let we assume that X is countable set. Let $\mathcal{S}_1(X)$ denotes class of functions $f : X \longrightarrow [-1, 1]$ with finite support and let μ be a normalized fuzzy measure. We define a fuzzy rank and sign dependent functional and comonotone-$\check{\vee}$-additive functional.

Definition 4 *A functional, $L : \mathcal{S}_1(X) \longrightarrow [-1, 1]$ is a* fuzzy rank and sign dependent functional (f.r.s.d., or functional of $\check{\vee}$-CPT type) *on $\mathcal{S}_1(X)$ if there exist two fuzzy measures μ^+ and μ^- such that for all $f \in \mathcal{S}_1(X)$*

$$L(f) = \Big(S_{\mu^+}(f^+)\Big) \check{\vee} \Big(- S_{\mu^-}(f^-)\Big),$$

where $f^+ = f \vee 0$ and $f^- = (-f) \vee 0$.

If $\mu^+ = \mu^-$ than the functional of $\check{\vee}$-CPT type is the symmetric Sugeno integral, i.e. it is symmetric, hence, $L(-f) = -L(f)$.

Let L be a functional on $\mathcal{S}_1(X)$, $L : \mathcal{S}_1(X) \longrightarrow [-1, 1]$. We say that a functional L on class $\mathcal{S}_1(X)$ is

i) comonotone-$\check{\vee}$-additive iff $L(f \check{\vee} g) = L(f) \check{\vee} L(g)$ for all comonotone functions f, $g \in \mathcal{S}_1(X)$,

ii) L is positively $\check{\wedge}$-homogeneous iff $L(a \check{\wedge} f) = a \check{\wedge} L(f)$ for every $f \in \mathcal{S}_1(X)$ and $a \geq 0$.

Remark 1 Note that the Sugeno integral S with respect to fuzzy measure μ is comonotone-$\check{\vee}$-additive functional which maps $\mathcal{S}_1(X)^+$ into $[0,1]$. This implies that for all comonotone functions $f,\, g \in \mathcal{S}_1(X)$ we have

$$S_\mu((f^+ \check{\vee} g^+)) = \Big(S_\mu(f^+)\Big) \check{\vee} \Big(S_\mu(g^+)\Big),$$

and analogously, equation holds for f^- and g^-.

In the case of finite set X and $\mathcal{S}_1(X)$ class of functions $f : X \longrightarrow [-1,1]$ we have next result, obtained by authors [17].

Proposition 2 *Let X be a finite set. If $L : \mathcal{S}_1(X) \longrightarrow [-1,1]$ is an idempotent comonotone-$\check{\vee}$-additive and monotone functional on $\mathcal{S}_1(X)$, then L is a f.r.s.d functional, i.e., there exist two fuzzy measures μ_L^+ and μ_L^- such that*

$$L(f) = \Big(S_{\mu_L^+}(f^+)\Big) \check{\vee} \Big(-S_{\mu_L^-}(f^-)\Big).$$

The condition of monotonocity of L in Proposition 2 can be replaced with a less restrictive condition, i.e. with comonotone monotonicity.

If X is finite set, a functional of $\check{\vee}$-CPT type on $\mathcal{S}_1(X)$ is not always comonotone-$\check{\vee}$-additive.

Example 2 *[18]* Let $X = \{1,2\}$ and

$f(1) = 0.5$	$g(1) = 0.5$	$\mu(A) = 0$ if $A = \emptyset$
$f(2) = 0$	$g(2) = -0.5$	$\mu(A) = 1$ if $A \neq \emptyset$

We consider functional of $\check{\vee}$-type L defined by

$$L(f) = \Big(S_\mu(f^+)\Big) \check{\vee} \Big(-S_\mu(f^-)\Big).$$

$f,\, g \in \mathcal{S}_1(X)$ are comonotone functions, $L(f) = 0.5$ and $L(g) = 0$, but $L(f \check{\vee} g) = 0$.

Proposition 3 *Let X be a countable set. If $L : \mathcal{S}_1(X) \longrightarrow [-1,1]$ is f.r.s.d functional than it is comonotone-$\check{\vee}$-additive functional.*

`Proof.` Let f i g be two comonotone functions with finite support $f,\, g : X \longrightarrow [-1,1]$. We have that $f(x) > 0$ implies $g(x) \geq 0$, hence $(f \check{\vee} g)^+ = f^+ \check{\vee} g^+$ and $(f \check{\vee} g)^- = f^- \check{\vee} g^-$.
By Remark 1 and the commutativity of operation $\check{\vee}$ we obtain

$L(f \check{\vee} g) =$

$$
\begin{aligned}
&= \Big(S_{\mu_L^+}((f\check{\vee}g)^+)\Big)\check{\vee}\Big(-S_{\mu_L^-}((f\check{\vee}g)^-)\Big)\\
&= \Big(S_{\mu_L^+}((f^+\check{\vee}g^+))\Big)\check{\vee}\Big(-S_{\mu_L^-}((f^-\check{\vee}g^-))\Big)\\
&= \Big(S_{\mu_L^+}(f^+)\Big)\check{\vee}\Big(S_{\mu_L^+}(g^+)\Big)\check{\vee}\Big(-\Big(\Big(S_{\mu_L^-}(f^-)\Big)\check{\vee}\Big(S_{\mu_L^-}(g^-)\Big)\Big)\Big)\\
&= \Big(S_{\mu_L^+}(f^+)\Big)\check{\vee}\Big(S_{\mu_L^+}(g^+)\Big)\check{\vee}\Big(-S_{\mu_L^-}(f^-)\Big)\check{\vee}\Big(-S_{\mu_L^-}(g^-)\Big)\\
&= \Big(S_{\mu_L^+}(f^+)\Big)\check{\vee}\Big(-S_{\mu_L^-}(f^-)\Big)\check{\vee}\Big(S_{\mu_L^+}(g^+)\Big)\check{\vee}\Big(-S_{\mu_L^-}(g^-)\Big)\\
&= L(f)\check{\vee}L(g).
\end{aligned}
$$

Corollary 2 *Let X be a countable set. The symmetric Sugeno integral is comonotone-$\check{\vee}$-additive functional on $\mathcal{S}_1(X)$.*

`Proof.` We obtain the claim if we take in Proposition 3 for $L(f) := \hat{S}_\mu(f)$.

5 Concluding remarks

The problem of representation comonotone $\check{\vee}$-additive and monotone functional L has been considered. We have focused on countable state space X, what is not so unusual, and the prospects with finite support. Comonotone $\check{\vee}$ additivity of symmetric Sugeno integral has been obtained.

The motivation is based mainly on axiomatic characterization of of the preference relation $\preceq$ to be CPT, stated in [12] and our approach may be viewed as adequate base for an axiomatization for the preference representation in qualitative decision making.

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