Application of Stochastic Adding A/D Conversion in Adaptive Measurement and Fuzzyfication

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Abstract: Stochastic adding A/D conversion (SAADK) represents a method of measurement (or classification) which naturally can replace process of fuzzyfication of input quantities. The stochastic adding A/D conversion is an adaptive process. The accuracy of measurements using SAADK is proportional to the square root from measurement time interval, i.e. for the twice greater accuracy, the measurement time interval must be four times longer.

The adaptivity of the accuracy of measurement and the possibility of direct fuzzyfication decreases hardware complexity and increases the speed and reliability.

The main goal of this research work is to confirm that the fuzzy logic and SAADK are complementary, and to suggest the method of their combined application.

1 Introduction

In classical measurement in the whole measurement range absolute error is, more or less, is the same. This is characteristic of uniform quantizer, i.e. analog to digital converter. On the other hand, in case of control we know that all measurement subintervals are not of equal signification. This means that equal precision is not necessary for control applications. In less significant subintervals it is sufficient to confirm that the system is in it. No more information is needed. Fuzzy systems serve as good mathematical models for this simple situation.

An interesting question arises: does the fuzzyfication belong to domain of measurements and metrology? The answer can be found in the next several definitions.

Metrology: science of measurement. Metrology includes all aspects both theoretical and practical with reference to measurements, whatever their uncertainty, and in whatever fields of science or technology they occur.

Measurement: set of operations having the object of determining a value of a quantity.

Measurable quantity: attribute of a phenomenon, body or substance that may be distinguished qualitatively and determined quantitatively.

Analog to digital conversion: the conversion of an analog quantity in its digital counterpart.

The possible conclusion, from the above definitions, is that fuzzyfication can be accepted as a kind of measurement, since performing fuzzyfication we confirm membership functions, i.e. that fuzzyfied quantity has characteristics of defined fuzzy sets.

Human beings have one interesting characteristic. We are not capable to process too much information, so we process one information at a time, possible, most significant in that moment. The process of choosing this most significant information is a kind of control process. We have capability to do this.

Adaptive measurement system put this human characteristic in the area of automatic control systems.

Which is the way to solve this problem? Let's imagine that we have system with many inputs, one multipexer and one system for processing inputs. On inputs we have information with low resolution, defined with some synchronizing pulse, but using longer processing time we get information with higher resolution. If processing system works longer on specific input the information is more detailed and more reliable, in the opposite case information is rough and less reliable.

Processing cycle for all inputs lasts the same time, in every case, but work on specific inputs can be variable, depend on system state. So, there is a need for instrument capable to give rough and fast measurement information in the shorter time interval, or precise and reliable measurement information in longer time interval.

One such instrument is stochastic additive A/D converter.

2 Stochastic Additive Analog to Digital Converter

The recently developed measure method, stochastic adding A/D conversion has several advantages compared to standard digital instrumentation. Its main advantages are:

1. extremely simple hardware and, consequently, simple implementation of parallel measurements, and
2. possibility to trade speed for accuracy.

This instrument can be either fast and less accurate, or slow and very accurate. The choice can be made specifying the frequency of reading its output. This is a kind of adaptation which master processor performs. In the case of multichannel measurements, adaptation can be performed on each channel independently.

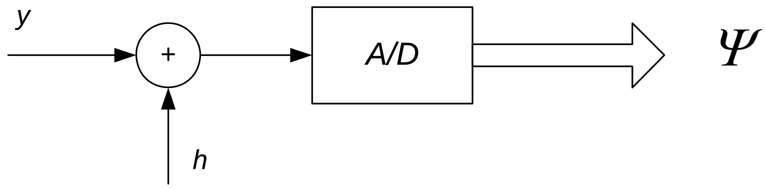

Figure 1
The most simple outline of the instrument

In the Fig. 1 a schematic of the instrument is shown. Dithering signal ***h*** is random, uniform and satisfy

$$0 \le |h| \le \frac{a}{2} \qquad \text{(Widrow's condition), and} \tag{1}$$

$$p(h) = \frac{1}{a}, \tag{2}$$

where ***a*** is a quantum of the uniform quantiser, and ***p***(***h***) is the corresponding probability density function of ***h***.

3 Theory of Operation - DC inputs

Let's observe the output of AD converter, say, Ψ. Let ***y*** = *const* = ***na*** + |Δ***a***| be the corresponding input voltage located between quantum level ***na*** and (***n*** + 1)***a***, at the distance |Δ***a***| ≤ ***a*** /2 from the closest quantum level ***na*** shown in Fig. 2.

Figure 2
The situation for y = const

For the situation depicted in Fig. 2 the quantized level of ***y*** + ***h*** 's $\Psi \in \{$***na,*** (***n*** + ***1***)***a***$\}$. The expectation $\overline{\Psi}$ is given by

$$\overline{\Psi} = \Psi_1 \cdot p_1 + \Psi_2 \cdot p_2 = (n+1) \cdot a \cdot \frac{|\Delta a|}{a} + na \frac{(a - |\Delta a|)}{a} =$$

$$= n \cdot |\Delta a| + |\Delta a| + na - n \cdot |\Delta a| = na + |\Delta a| = y$$

$$\overline{\Psi} = y \qquad (3)$$

The corresponding variance is:

$$\overline{e^2} = \sigma_\Psi^2 = (\Psi_1 - \overline{\Psi})^2 \cdot p_1 + (\Psi_2 - \overline{\Psi})^2 \cdot p_2 = (a - |\Delta a|)^2 \cdot \frac{|\Delta a|}{a} + |\Delta a|^2 \cdot \frac{(a - |\Delta a|)}{a} =$$

$$\sigma_\Psi^2 = (a - |\Delta a|) \cdot \left((a - |\Delta a|) \cdot \frac{|\Delta a|}{a} + \frac{|\Delta a|^2}{a} \right) = (a - |\Delta a|) \cdot (|\Delta a|)$$

$$\sigma_\Psi^2 = (a - |\Delta a|) \cdot (|\Delta a|) \qquad (4)$$

An interesting question is: what is the error if we have finite number N of dithered samples? The answer give theory of samples and central limit theorem. Suppose that we have next set of samples: $\Psi_1, \Psi_2, ..., \Psi_n$. Then measurement quantity is

$$\overline{\Psi} = \frac{1}{N} \sum_{i=1}^{N} \Psi_i \approx const. \qquad (5)$$

Central limit theorem gives next result

$$\sigma_{\overline{\Psi}}^2 = \frac{\sigma_{\Psi}^2}{N}. \qquad (6)$$

The equations (4), (5) and (6) completely define the situation when we have a finite number *N* of dithered samples is given.

The shape of σ_{Ψ}^2 as function of Δa, for the case of $y = const,$ is given in Fig. 3.

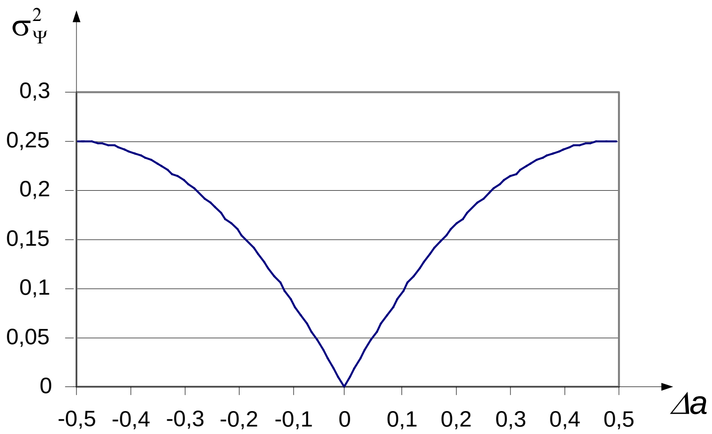

Figure 3
The diagram of σ_{Ψ}^2 (Δa) for y = const and a = 1

From the Fig. 3 is obvious that the maximum value of error is in the case of $y = c = \frac{a}{2} + na$, i.e. when measurement voltage has value of threshold voltage, but the minimum value of error is when $y = c = na$, i.e. when measurement voltage has value of quantum level.

In the literature it is described how to manage fuzzy logic variables, how they can be modified but mainly the shape is supposed. In this paper we will give one way how to obtain these functions based on measurements and the numeric processing of the measurements.

Stochastic additive A/D converter basically works with equidistant comparators, but this is not obligatory. If we make A/D converter with nonequidistant threshold levels, then we have stochastic additive analog to fuzzy converter (SAAFC).

4 Membership Functions on the Output of the Stochastic Additive Analog to Fuzzy Converter

Measurement range is divided in to *n* intervals. There is one deciding threshold in every interval. (*n* + 1) sets are defined by *n* deciding thresholds. The first set is „ Approximately the minimal value of the measurement range“, (*n* + 1)st set is „ Approximately the maximum value of the Measurement range“. Between these two sets there are (*n* - 1) sets „Approximately A_j“ where x_{A_j} represents the middle of the two deciding thresholds: PO_j and PO_{j-1} where j = 2,...,*n*.. The deciding thresholds are marked from PO_1 to PO_n. There exists the following limitation: $PO_1 > PO_{j-1}$ where j = 2,...,*n*.

In general case, the membership of the defined sets can be described in the following way:

$$A_j(x) = \{x_i | x_i \geq PO_{j-1} \wedge x_i < PO_j\} \qquad \text{where } j = 2,...,n \tag{7}$$

For the first and the last set:

$$A_1(x) = \{x_i | x_i < PO_1\} \tag{8}$$

$$A_{n+1}(x) = \{x_i | x_i \geq PO_n\} \tag{9}$$

The fuzzy intervals or fuzzy numbers are defined on these sets.

SAAFC makes the quantification on this sum:

$$x_i = x(t) + h(t) \tag{10}$$

where:

x_i *i*th value of input sum into the flash A/D converter

i the serial number of quantification in the measurement cycle.

$x(t)$ value of the variable which is fuzzyficated in the moment of sampling.

$h(t)$ value of random variable of uniform distribution in the moment of sampling.

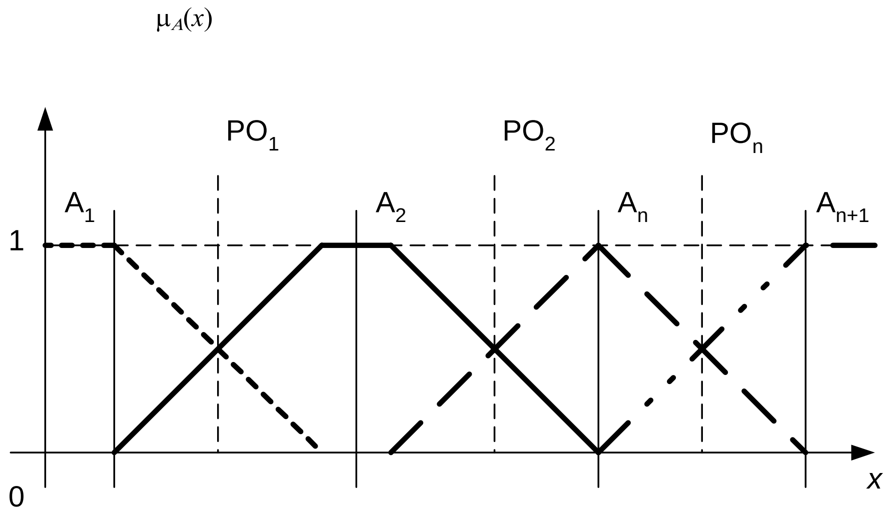

Figure 4
Showing the form of membership function

Membership function of the tested value of these fuzzy sets is determined using the following formula:

Membership function to these fuzzy sets **(A_j)** is defined by the relative frequency of appearing of the measuring result in **(A_j)** during the measuring cycle.

$$\mu_{A_j}(x) = \frac{a_j}{N} \tag{11}$$

where:

j the serial number of the fuzzy set, $j = 1, 2, ..., (n + 1)$

a_j number of appearing value on the output of the fuzzy set A_j

N the total quantification number during the measuring cycle.

The following limitation is in force for h:

$$h \le \min(PO_j - PO_{j-1}) \text{ where } j = 2, ..., n. \tag{12}$$

where h must be smaller or equal with the minimum difference between the two neighbouring deciding thresholds.

The elements of continuous set are ordered to fuzzy sets in the membership function. This operation can be called fuzzyfication.

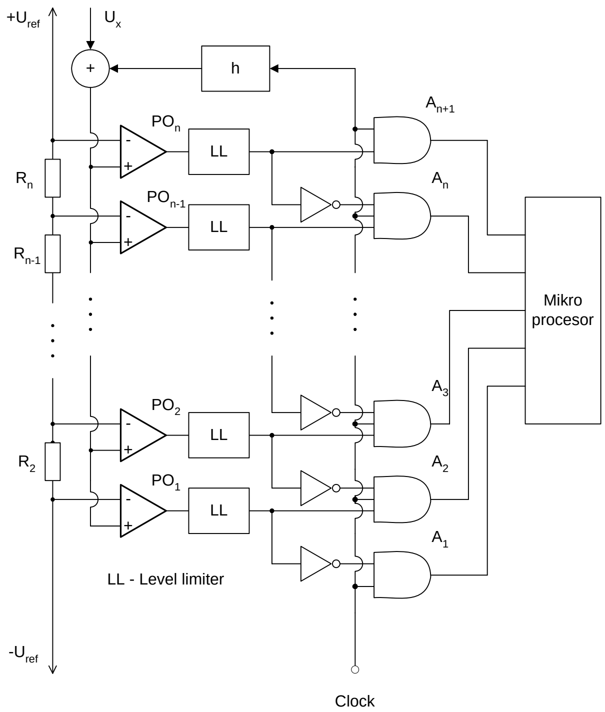

Figure 5

The stochastic additive analog to fuzzy converter

Conclusions

If we deal with system with great number of inputs and with limited capability, not only for measurements, but for processing as well, we can use this system more efficiently if we apply above mentioned idea: if processing system works longer on specific input the information is more detailed and more reliable, in the opposite case information is rough and less reliable. Similarly to human reasoning, system can concentrate on most significant input and most significant information is processed. For other inputs, system only confirms that they are under control.

The appearance of the signal in the first instant of quantization indicates rough estimation of specific input and it can define processing time interval. If estimation tells that input is under control, we can process immediately the next input. But if estimation tells that input is not under control, system pay more attention (longer processing time) to go back under control.

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