Modeling Decision Making under Uncertainty and Vagueness

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Abstract: This paper discuss mainly issues related for modeling decision making under uncertain, vagueness, risky and imprecise information. There will be presented a description of five ordinal methods for modeling decision making under uncertainty in the context of linguistic data: Possibilistic Decisisonmaking, Revised Possibilistic Decisisonmaking, Commensurate L-Fuzzy Risk Minimization, Fuzzy relational Ordinal Risk Minimization, and Quadratic Ordinal Psychophysical Optimization. Finally, these five techniques are illustrated using a single example.

Keywords: Decision analysis, Fuzzy risk, Linguistic data

1 Decision Making and Decision Analysis

A decision analysis problem arises when we must choose between two or more alternative actions whose outcomes depend on which one of a collection of possible states of nature is the actual state. If it is possible to assign a well-defined subjective (or objective) probability distribution to the states of nature and to assign a numerical utility score measured on an interval scale to the outcome of each state-action pair, then we can compute the expected utility of each action and pick the action whose expected utility is greatest or disutility is least, Probabilities measured on a ratio scale are multiplied times utilities measured on an interval scale; adding the results for a given action gives expected utility measured on the same interval scale as the individual utilities. While this method makes no guarantees for any one decision, financial portfolio theory and the law of large numbers imply that a lifetime of making decisions by this rule has a very high probability of generating a higher lifetime utility than using any other rule when the scale assumptions are met. From a standpoint of computational theory of perception of "computing with words", this method begins by converting the natural perceptions of how usual or unusual the possible states are into probability estimates on a ratio scale, and converting the natural perceptions of how acceptable or unacceptable the possible outcomes of state-action pairs are into utility numbers on an interval scale using Von Neuman-Morgenstern utility theory or some similar method. A direct generalization of this is to use fuzzy numbers for probabilities and utilities, computing a fuzzy expected utility for each action by the extension principle of fuzzy mathematics. Since natural perceptions of usuality and acceptability are more likely to be in the form of words than numbers, the fuzzy method has the advantage of a more natural representation.

2 Methods for Modeling Decisionmaking

However, sometimes it is not reasonable to assume that the perceived usuality of the states of nature can be converted to a ratio scale of probability, even a fuzzy one, without excessive distortion. Similarly, sometimes it is not possible to assign utility scores on an interval scale, crisp or fuzzy, to adequately represent the perceived acceptability or unacceptability of outcomes. In such a case, it is necessary to rely on the ordinal properties of the perceived usuality and acceptability.

One method, widely used in practice even if frowned upon i theory, is to ignore the uncertainty about the states of nature, and base the decision entirely on the one state that is the most usual or likely one. Obviously, the appropriateness of this approach is directly proportional to the degree to which usuality is in fact concentrated in one state of nature.

Another ordinal method represents the attractiveness of each alternative action by the least desirable outcome for that action. The alternative selected is the one for which this worst-case value is most acceptable or least unacceptable.

The above two ordinal methods consider either the maximal usuality alone or the maximal disutility alone. The remainder of this paper is concerned with methods that try to capture the extremes of both utility and possibility within the confines of ordinal calculation.

2.1 Possibilistic Decisionmaking

The method of "possibilistic decisionmaking" which represented utility as membership in the fuzzy set of good outcomes and usuality as membership in the fuzzy set of possible states of nature. It was proposed by Yager. The potential utility of a state-action outcome is its membership in the fuzzy set of outcomes that are both possible and good, calculated as the minimum of the membership of the outcome in "good" and the state in "possible". The attractiveness of an action is then taken as the greatest potential utility of any of its possible outcomes. The choice of action is made by taking the alternative whose attractiveness is greatest.

2.2 Revised Possibilistic

Whalen pointed out that Yager's approach treated choice nodes and chance noder the same way, maximizing in both cases, and suggested an algorithm known as "revised possibilistic decisionmaking" that measured the disutility of each state-action outcome rather than its utility. The threat of a particular state-action outcome is its membership in the fuzzy set of outcomes that are both possible and bad, found by minimization; the risk of an action is the greatest threat of its possible outcomes. The choice of action is made by taking the alternative whose risk is least. Both of these varieties of possibilistic decisionmaking were introduced assuming that set memberships follow a complete weak order, represented without loss of generality by numbers in the unit interval. Thus, the nuances of natural perceptions of likelihood and acceptability were rapidly lost.

2.3 Commensurate L-Fuzzy Risk Minimization

The next stage in generalization is the use of L-fuzzy (variously rendered "lattice fuzzy" of "linguistic fuzzy") sets for bad outcomes and possible states, with set memberships defined on an incompletely ordered abstract lattice. If all memberships are measured on a common lattice, it is still possible in principle to take the minimum of the membership of a state-action outcome in the L-fuzzy set of bad outcomes and the membership of the corresponding state in the L-fuzzy set of possible states. But since the ordering is incomplete, it is not always possible to find an explicit minimum or maximum. Furthermore, it may not be meaningful to compare the membership of a state in the set of usual states with the membership of an outcome in the set of bad outcomes, since the two are qualitatively so different. The Commensurate L-Fuzzy Risk Minimization technique deals with this by placing usuality set memberships on one incompletely ordered lattice and disutility set memberships on a separate incompletely ordered lattice. The version presented in uses an ordered pair to specify the membership of a state-action outcome in the set of outcomes that are both possible and bad. The risk of an alternative action is found by symbolically maximizing these pairs similarly to the way the previous method maximizes unresolved minima; the selected action is the one for which this symbolic structure is least risky. Recognizing that this process is not highly decisive, the method successively falls back to the L-fuzzy approach with a common incomplete lattice, and the revised possibilistic approach with a single complete weak order of membership grades.

2.4 Fuzzy Relational Ordinal Risk Minimization

Whalen & Schott have suggested an approach to commensurate ordinal decision making based on recent insights in knowledge granulation, computing with words, fuzzy relations as fuzzy x-y graphs, and second order fuzzy sets. In this approach, the disutility of a state-action outcome is represented by a linguistic variable. The base values of this linguistic variable may be well-defined utility numbers, or they may be a purely abstract ordered set. The usuality of a state is similarly represented by a linguistic variable defined on a base variable of degrees of possibility.

The **second order fuzzy threat of a state-action pair** $T(\tilde{A},s)$ is s second order fuzzy set of disutilities. The membership of a disutility base value *x* in this set is itself a (first-ordet) fuzzy set of possibilities $\mu_{\tilde{T}(\tilde{A},s)}(x)$ (x). The membership of a possibility base value p in the fuzzy possibility of disutility x (that is, its membership in the fuzzy set of possibilities that the disutility if (A,s) is x) is denoted $\mu_{\mu_{\tilde{T}(\tilde{A},s)}(x)}(p)$. The second-order mambershp function is defined by the Cartesian minimum of the fuzzy disutility of the state-action pair and the fuzzy possibility of the state: the value of $\mu_{\mu_{\tilde{T}(\tilde{A},s)}(x)}(p)$ is the minimum of the membership of *x* in the fuzzy disutility of the state-action pair (A,s),

$_{\tilde{d}(A,s)}(x)$ and the membership of *p* in the fuzzy possibility of state p, $_{\tilde{\pi}(s)}(p)$

$$\mu_{\mu_{\tilde{T}(\tilde{A},s)}}(p) = \min \begin{Bmatrix} \mu_{\tilde{d}(A,s)}(x), \\ \mu_{\tilde{\pi}(s)}(p) \end{Bmatrix} \tag{1}$$

The **second order fuzzy risk profile of an action**, $\tilde{r}A$, is a second order fuzzy set of disutilities formed by the union of the fuzzy relations corresponding to the threat of all the state-action pairs for that action. The possibility that the disutility of a given action is *x* is a fuzzy set of possibilities formed as the union of the fuzzy possibility that each possible outcome has disutility *x*. This is equivalent to a fuzzy *x-y* graph for each action; the knowledge granules making up this graph have fuzzy coordinates defined by the disutility (*x*) and usuality (*y*) of the possible outcomes.

$$\mu_{\mu_{\tilde{r}A}(x)}(p) = \max_{s} \left\{ \mu_{\mu_{\tilde{r}A}(x)}(p) \right\} \tag{2}$$

The **first order fuzzy risk profile of an action**, $\tilde{r}A$, is created from the action' s second order risk profile using a Sugeno integral. The first order possibility that the disutility of an action is x is the maximum over possibility grades (0 to 1) of the minimum of each possibility grade with its membership in the second order possibility that the disutility of the action is *x*.

$$\mu_{\tilde{r}A}(x) = \max_{p} \left\{ \min(p, \mu_{\mu_{\tilde{r}A}(x)}(p) \right\} \tag{3}$$

The **linguistic risk assessment** is the final stage in the process, found by converting the first order fuzzy risk profile of each alternative action back into words. This allows the user to exercise judgment as to what action should be chosen and what rhetorical argument to use to justify that choice. Wenstop's classic approach to linguistic approximation does not work well for this purpose because the possibility distributions tend to be multimodal, so a method based on linear integer programming is used instead. Future research will evaluate the effectiveness of genetic algorithms and other methods for linguistic approximation in the context of risk minimization using computing with words.

2.5 Quadratic Ordinal Psychophysical Optimization

Whalen and Wang apply quadratic programming to an ordinal interpretation of linguistic probability and utility terms. This interpretation incorporates a well-established finding of psychophysics: the degree to which stimuli must differ physically to be discerned perceptually is proportional to the magnitude for the stimuli according to logarithmic law of human (and animal) perception. If the utility of each state-action pair is known only roughly, for instance as "good", "fair", or "poor", we will represent this ordinally; each fair utility is greater than any poor utility and less than any good utility. Over a century of research in the field of psychophysics indicates a very strong tendency for human perception to operate on a logarithmic scale, in which the "just noticeable difference" between stimuli is a constant proportion of the magnitude of the stimulus rather than a fixed incremental amount. Based on this, we require that each "fair" utility is greater than or equal to a fixed constant called the distinguishability ratio times any "poor" utility, and each "good" utility is greater than or equal to the same fixed constant times any "fair" utility. The distinguishability ratio is a generalization of the psychophysical concept of a decibel. To represent the fact that two quantities with the same rough description need not be identical as long as their difference is not psychologically significant, the quantitative representation of each "fair" utility must be less than or equal to the distinguishability constant times that of every other "fair" utility, and similarly among "good" and "poor". This automatically entails that each utility in a class is also greater than or equal to any other utility divided by the distinguishability ratio. If the probability of each state is specified numerically, then the overall utility of an action is the sum of the unknown numeric representations of the utility of each of the action's state-action pairs, weighted by the numerical probability weights. This is a linear function with linear inequality constraints; thus, it is possible to find the maximum and minimum overall utility for each action by linear programming. More importantly, we can find the minimum and maximum difference between the overall utility of two actions. Alternative A dominates Alternative B if max{utility(A)}<0; it is not sufficient that max{utility(A)}> max{utility(B)} and min{utility(A)}>min{utility(B)}, but Alternative A can dominate alternative B even if max{utility(B)}>min{utility(A)}.

A dominated action can be removed from consideration, leaving a short list of nondominated actins. These can be re-analyzed in several ways. One can use a finer grid of linguistic terms like "very low" or "upper medium" and repeat the analysis. One can also introduce additional inequalities into the linear programming formulation. For example, there may be two actins that both generate "low" utility in a particular state, but further introspection and/or economic analysis may indicate one is discernibly lower than the other. Finally, one can move to methods that are more decisive than the ordinal ones, but which require stronger assumptions. If the utility of each state-action pair is specified numerically but the probability of the different attributes is only specified roughly, the linear programming problem is very similar to the one described above. If utility is known only as good, fair, poor and probability is known only as low, medium, high, finding the maximum and minimum of the difference between two alternative actions, and thus identifying dominated alternatives, becomes a problem in quadratic programming. The difference in overall utility of between action i and action j is a weighted sum,

$$D_{ij} = \sum_k \left(U_{ik} * P_k - U_{jk} * P_k \right) \quad (4)$$

in which each term is the product of two variables, the utility of action i in state k and the unknown probability, where these variables U•• and P• are subject to a set of linear inequality constraints. If the maximum value of the quadratic function Dij, subject to the linear constraints, is negtive then action dominates action i.

3 Example for Illustration

To illustrate the four ordinal approaches discussed above, here is a simple abstract decision problem in which utility is granulated to good, fair, and poor, and usuality is granulated to usual, plausible, and rare. If there are just three states of nature, assessed as a usual state, a plausible state, and a rare state, there are six possible risk profiles without ties in utility. In the illustrative example, we will compare some alternative actions.

Table 1

Linguistic Utility & Probability

Actions:	Usual State	Plausible State	Rare state
A1	good	fair	poor
A2	good	poor	fair
A3	fair	good	poor
A4	fair	poor	good
A5	poor	good	fair
A6	poor	fair	good

3.1 Commensurate L-Fuzzy Risk Minimization

For this method, we assume that low, medium, high disutility and usual, plausible, rare usuality are on two separate scales, Table 2.

In this case, actions A1, A2 and A3 are difficult to rank among themselves without additional information, but all three are discernibly better than A4, while A5 and A6 are the two least attractive alternatives.

Table 2

Commensurate L-Fuzzy Risk Minimization

	Usual State	Plausible State	Rare state	Overall Risk
A1	(low, usual)	(medim, plausible)	(high, rare)	(low, usual) or (medium, plausible) or (high, rare)
A2	(low, usual)	(high, plausible)	(medium,rare)	(low, usual) or (high,plausible)
A3	(medium, usual)	(low, plausible)	(high, rare)	(medium, usual) or (high,rare)
A4	(medium, usual)	(high, plausible)	(low, rare)	(medium, usual) or (high, plausible)
A5	(high, usual)	(low, plausible)	(medium, rare)	(high, usual)
A6	(high, usual)	(medium, plausible)	(low, rare)	(high, usual)

3.2 Fuzzy Relational Ordinal Risk Minimization

Suppose that low, medium and high disunity are represented by the following vectors giving the compatibility of utility base variable scores with membership grades shown in Table 3. Also suppose that rare, plausible, and usual have the same vector representations, respectively, though of course applying to possibility base vlues rather than disutility. Then the three second-order fuzzy sets representing the three threats associated with action A2 are as shown in Tables 4, 5 and 6.

Table 3

Membership Functions for Fuzzy Disutilities

1	0.75	0.5	0.25	0	0	0	0	0	low
0	0.25	0.5	0.75	1	0.75	0.5	0.25	0	medium
0	0	0	0	0	0.25	0.5	0.75	1	high
0	**0.125**	**0.250**	**0.375**	**0.500**	**0.625**	**0.750**	**0.875**	**1**	**disutilities**

Table 4

Second-Order Fuzzy Threat of (low, usual)

Poss	Disutility:								
	0.000	0.125	0.250	0.375	0.500	0.625	0.750	0.875	1.000
0.000	0	0	0	0	0	0	0	0	0
0.125	0	0	0	0	0	0	0	0	0
0.250	0	0	0	0	0	0	0	0	0
0.375	0	0	0	0	0	0	0	0	0
0.500	0	0	0	0	0	0	0	0	0
0.625	0.25	0.25	0.25	0.25	0	0	0	0	0
0.750	0.5	0.5	0.5	0.25	0	0	0	0	0
0.875	0.75	0.75	0.5	0.25	0	0	0	0	0
1.000	1	0.75	0.5	0.25	0	0	0	0	0

to the fuzzy set of possibilities compatible with disutility .625 in the risk profile of Action A2. This is because:

- possibility .250 belongs 0% to the fuzzy set of possibilities compatible with disutility .625 in the fuzzy threat of (low, usual) associated with the usual state and action A2 (Table 4),
- possibility -250 belongs 25% to the fuzzy set of possibilities compatible with disutility .625 in the fuzzy threat of (high, plausible) associated with the plausible state and action A2 (Table 5),
- possibility .250 belongs 50% to the fuzzy set of possibilities compatible with disutility .625 in the fuzzy threat of (medium, rare) associated with the rare state and action A2 (Table 6),

The first-order risk profile of action A2, found by Sugeno integral, is shown in Table 8.

Table 8

First-Order Fuzzy Risk Profile of Action A2

	Disutility:								
	0.000	0.125	0.250	0.375	0.500	0.625	0.750	0.875	1.000
Poss.	1.000	0.750	0.500	0.250	0.250	0.250	0.500	0.625	0.625

Table 5

Second-Order Fuzzy threat of (high, plausible)

Poss	Disutility:								
	0.000	0.125	0.250	0.375	0.500	0.625	0.750	0.875	1.000
0.000	0	0	0	0	0	0	0	0	0
0.125	0	0	0	0	0	0.25	0.25	0.25	0.25
0.250	0	0	0	0	0	0.25	0.5	0.5	0.5
0.375	0	0	0	0	0	0.25	0.5	0.75	0.75
0.500	0	0	0	0	0	0.25	0.5	0.75	1
0.625	0	0	0	0	0	0.25	0.5	0.75	0.75
0.750	0	0	0	0	0	0.25	0.5	0.5	0.5
0.875	0	0	0	0	0	0.25	0.25	0.25	0.25
1.000	0	0	0	0	0	0	0	0	0

Table 6

Second-Order Fuzzy Threat of (medium, rare)

Poss	Disutility:								
	0.000	0.125	0.250	0.375	0.500	0.625	0.750	0.875	1.000
0.000	0	0.25	0.5	0.75	1	0.75	0.5	0.25	0
0.125	0	0.25	0.5	0.75	0.75	0.75	0.5	0.25	0
0.250	0	0.25	0.5	0.5	0.5	0.5	0.5	0.25	0
0.375	0	0.25	0.25	0.25	0.25	0.25	0.25	0.25	0
0.500	0	0	0	0	0	0	0	0	0
0.625	0	0	0	0	0	0	0	0	0
0.750	0	0	0	0	0	0	0	0	0
0.875	0	0	0	0	0	0	0	0	0
1.000	0	0	0	0	0	0	0	0	0

The second-order fuzzy risk profile of action A2 is represented by the union of these three second-order fuzzy sets:

Table 7

Second-Order Fuzzy risk Profile of Action A2

Poss	Disutility:								
	0.000	0.125	0.250	0.375	0.500	0.625	0.750	0.875	1.000
0.000	0	0.25	0.5	0.75	1	0.75	0.5	0.25	0
0.125	0	0.25	0.5	0.75	0.75	0.75	0.5	0.25	0.25
0.250	0	0.25	0.5	0.5	0.5	0.5	0.5	0.5	0.5
0.375	0	0.25	0.25	0.25	0.25	0.25	0.25	0.75	0.75
0.500	0	0	0	0	0	0.25	0.5	0.75	1
0.625	0.25	0.25	0.25	0.25	0	0.25	0.5	0.75	0.75
0.750	0.5	0.5	0.5	0.25	0	0.25	0.5	0.75	0.75
0.875	0.75	0.75	0.5	0.25	0	0.25	0.25	0.25	0.25
1.000	1	0.75	0.5	0.25	0	0	0	0	0

Conclusions

As an illustrative example, the entry 0.75 in the row for .250 possibility and column for .625 disutility in Table 9 means that the possibility .250 belongs 50% linguistic aporoximator is under development, but ths suffices for an initial demonstration. The linguistic approximations to the risk profiles of the six alternative actions are as follows:

A1: low or possibly (lower medium or upper medium)đ

A2: low or high

A3: medium or possibly low

A4: medium or possibly low

A5: high or low

A6: high or possibly (lower medium or upper medium)

Action A1 seems to be indicated by this output; however, the user has more flexibility to construct and evaluate alternative theoretical arguments for and against various action, combining the output of the method with orther information. Thus, computing with words is well suited here to a decision support application rather than an automated decision mechanism.

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