Motion Control of Wheeled Mobile Robots

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Abstract: The paper deals with the modeling and control strategies of the motion of wheeled mobile robots. The model of the vehicle has two driving wheels and the angular velocities of the two wheels are independently controlled. First, the vehicle kinematics model and the control strategies using a feedforward compensator are analyzed. Second, fuzzy reactive control of a mobile robot motion in an unknown environment with obstacles are proposed. Finally, the mobile robot simulation is illustrated.

Keywords: autonomous motion, wheeled mobile robot, feedforward compensator, fuzzy reactive control

1 Introduction

A wheeled mobile robot is a wheeled vehicle which is capable of autonomous motion. Autonomous mobile robots are a very interesting subject both in scientific research and practical applications. First the control strategies using the controller for each motor including the simple PI feedback controller and the feedforward compensator which has the inverse vehicle dynamics are proposed. Second, the fuzzy reactive control of a wheeled mobile robot motion in an unknown environment with obstacles is analyzed. In this paper the model of the vehicle has two driving wheels (which are attached to both sides of the vehicle) and the angular velocities of the two wheels are independently controlled. The center of the driving wheels is regarded as the gravity center. This model is the simplest and the most suitable for a small-sized and light, battery-driven autonomous vehicle.

The paper is organized as follows: Section 1: Introduction. The modeling of the wheeled mobile robots given in Section 2. In Section 3: The Control Strategies of Wheeled Mobile Robots Using Feedforward Compensator is illustrated. In Section 4: Fuzzy reactive control of a mobile robot motion in an unknown environment with obstacles are proposed.

2 Modeling of the Wheeled Mobile Robots

2.1 Kinematics Constraints

We consider a mechanical system with n generalized coordinate's q subject to m kinematics constraints:

$$A(q)\dot{q} = 0 \tag{1}$$

where: $A \in R^{mxn}$ is a full rank matrix [1]. A large class of mechanical systems, such a wheeled vehicle and mobile robots involve kinematics constraints. In the literature these kinematics constraints can generally be classified as nonholonomic or holonomic. A mobile robot involving two actuator wheels is considered as a system subject to nonholonomic constraints.

2.2 Kinematics Model

Let's consider the kinematics model for an autonomous vehicle. The position of the mobile robot in the plane is shown in Fig. 1.

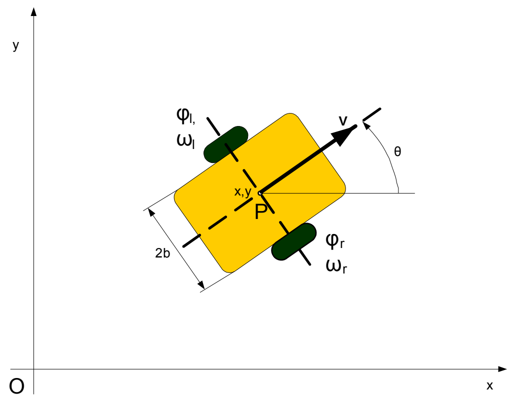

Figure 1
Position of mobile robot in plane

The inertial-based frame (Oxy) is fixed in the plane of motion and the moving frame is attached to the mobile robot. In this paper we will assume that the mobile robots are rigid cart equipped, with non-deformable conventional wheels, and they are moving on a non-deformable horizontal plane. During the motion: the contact between the wheel and the horizontal plane is reduced to a single point, the wheels are fixed, the plane of each wheel remains vertical, the wheel rotates about its horizontal axle and the orientation of the horizontal axle with respect to the cart can be fixed.

The contact between the wheel of the mobile robots and the non-deformable horizontal plane supposes both the conditions of pure rolling and non-slipping during the motion. This means that the velocity of the contact point between each wheel and the horizontal plane is equal to zero. For low rolling velocities this is a reasonable wheel moving model. The center of the fixed wheel is a fixed point of the cart and b is the distance of the center of the wheel from P.

The rotation angle of the wheel about its horizontal axle is denoted by $\varphi(t)$ and the radius of the wheel by R. Hence, the position of the wheel is characterized by two constants: b and R and its motion by a time-varying angle: $\varphi_r(t)$ – the rotation angle of the right wheel and $\varphi_l(t)$ – the rotation angle of the left wheel.

The configuration of the mobile robot can be described by five generalized coordinates such as:

$$q = [x,\ \ y,\ \ \theta,\ \ \varphi_r,\ \ \varphi_l]^T \tag{2}$$

where:

x and y are the two coordinates of the origin P of the moving frame (the geometric center of the mobile robot),

θ is the orientation angle of the mobile robot (of the moving frame),

$\varphi_r(t)$ – the rotation angle of the right driving wheel,

$\varphi_l(t)$ – the rotation angle of the left driving wheel.

The vehicle velocity v can be found in equation (3):

$$v = R(\omega_r + \omega_l)/2 \tag{3}$$

where:

$\omega_r = \dfrac{d\varphi_r}{dt}$ – angular velocity of the right wheel,

$\omega_l = \dfrac{d\varphi_l}{dt}$ – angular velocity of the left wheel,

The position and the orientation of the mobile vehicle are determined by a set of differential equations (4-6) in the following form:

$$\dot{x} = (R \cos\theta\ (\omega_r + \omega_l))/2 \tag{4}$$

$$\dot{y} = (R \sin\theta\ (\omega_r + \omega_l))/2 \tag{5}$$

$$\dot{\theta} = R\ (\omega_r - \omega_l)/2b \tag{6}$$

Here,

$$\dot{x} = v \cos\theta \tag{7}$$

$$\dot{y} = v \sin\theta \tag{8}$$

Then the matrix form is:

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \cos\theta & 0 \\ \sin\theta & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v \\ \dot{\theta} \end{bmatrix} \tag{9}$$

Finally, the kinematics model of the vehicle velocity v and the angular velocity $\dot{\theta}$ can be represented by the matrix as follows:

$$\begin{bmatrix} v \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} R/2 & R/2 \\ R/2b & -R/2b \end{bmatrix} \begin{bmatrix} \omega_r \\ \omega_l \end{bmatrix} \tag{10}$$

In this case, we now consider the mobile robot motion as a nonholonomic mechanical system, where three kinematics constraints exist:

$$\begin{aligned} &\dot{x}\sin\theta - \dot{y}\cos\theta = 0 \\ &\dot{x}\cos\theta + \dot{y}\sin\theta = R\omega_r - b\dot{\theta} \\ &\dot{x}\cos\theta + \dot{y}\sin\theta = R\omega_l + b\dot{\theta} \end{aligned} \tag{11}$$

The constraints can be written in the form (1), where matrix A $\in R^{mxn}$ (m=3, n=5) can be described as:

$$A = \begin{bmatrix} \sin\theta & -\cos\theta & 0 & 0 & 0 \\ \cos\theta & \sin\theta & b & -R & 0 \\ \cos\theta & \sin\theta & -b & 0 & -R \end{bmatrix} \tag{12}$$

In this paper the angular velocities of the two wheels of the mobile robot are independently controlled.

3 The Control Strategies of Wheeled Mobile Robots Using Feedforward Compensator

In this section a control strategy of wheeled mobile robots using feedforward compensator is proposed (Figure 2). The controller for each motor includes the simple PI feedback controller and the feedforward compensator which has the inverse vehicle dynamics. The inputs to the feedforward compensator are both sides of the desired wheel angular velocity (ω_r and ω_l – angular velocity of the right and left wheel).

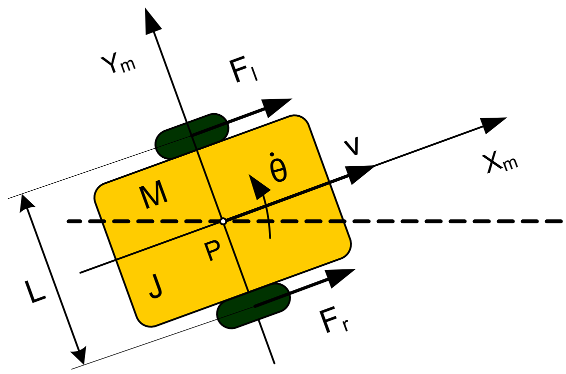

Figure 2
Parameters of a vehicle in motion

The vehicle motion equations are:

$$M\frac{dv}{dt} = F_r + F_l \tag{13}$$

$$J\frac{d\dot{\theta}}{dt} = L(F_r - F_l) \tag{14}$$

Where the parameters of the vehicle motion are:

F_r – driven force at the right wheel,

F_l – driven force at the left wheel,

M – mass of the vehicle,

J – inertia of the vehicle.

The following equations can formulate the torque (the vehicle dynamics) at each side of the driving wheel:

$$\tau_r = A\dot{\omega}_r + B\dot{\omega}_l + C\omega_r \tag{15}$$

$$\tau_l = D\dot{\omega}_l + E\dot{\omega}_r + F\omega_l \tag{16}$$

Where: A, B, C, D, E and F are the parameters of the motion of the vehicle.
The proposed controller is as follows:

$$\tau_r = k_p(\omega_{rd} - \omega_r) + k_i\int_o^t(\omega_{rd} - \omega_r)dt + A\dot{\omega}_{rd} + B\dot{\omega}_{ld} + C\omega_{rd} \tag{17}$$

$$\tau_l = k_p(\omega_{ld} - \omega_l) + k_i\int_0^t(\omega_{ld} - \omega_l)dt + A\dot{\omega}_{ld} + B\dot{\omega}_{rd} + C\omega_{ld} \tag{18}$$

Where: k_p – is the proportional gain, k_i – is the integral gain, ω_{rd} – is the desired value of the angular velocity of the right wheel, ω_{ld} - is the desired value of the angular velocity of the left wheel. The structure of the proposed controller is presented in Figure 3:

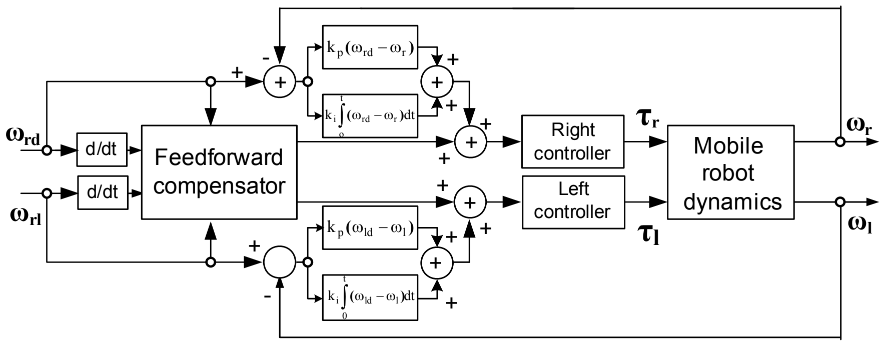

Figure 3

Block diagram of the proposed controller using feedforward compensator

4 Fuzzy Reactive Control of a Mobile Robot Motion in an Unknown Environment with Obstacles

4.1 Reactive Navigation Strategy of Collision-Free Motion in an Unknown Environment with Obstacles

In this section fuzzy control is applied to the navigation of the autonomous mobile robot in an unknown environment with obstacles [2], [3]. We supposed that the autonomous mobile robot has two wheels driven independently and groups of ultrasonic sensors to detect obstacles in the front, to the right and to the left of the vehicle. The reactive strategies are based on ultrasonic sensory information and only the relative interactions between the mobile robot and the unknown environment have to be assessed. In this case, a structural modeling of the environment is unnecessary.

When the vehicle is moving towards the target and the sensors detect an obstacle, an avoiding strategy is necessary. While the mobile robot is moving it is important to compromise between avoiding the obstacles and moving towards the target position. With obstacles present in the unknown environment, the mobile robot reacts based on both the sensed information of the obstacles and the relative position of the target (Fig. 4) [4].

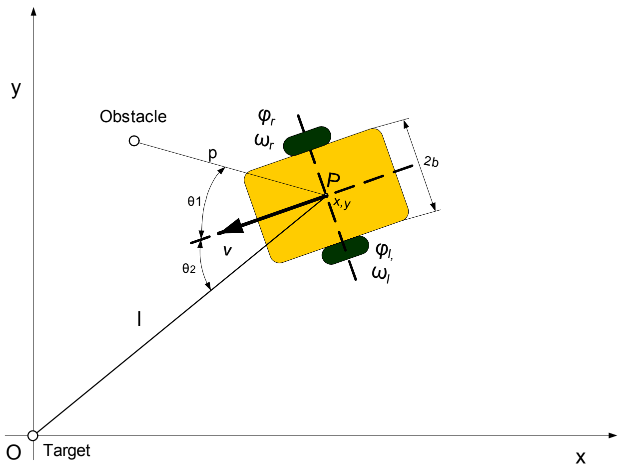

Figure 4
Definition of obstacle angle θ_1 and obstacle distance p

In moving towards the target and avoiding obstacles, the mobile robot changes its orientation and velocity. When the obstacle in an unknown environment is very close, the mobile robot slows down and rapidly changes its orientation. The navigation strategy is to come as near to the target position as possible while avoiding collision with the obstacles in an unknown environment. The intelligent mobile robot reactive behavior is formulated in fuzzy rules.

4.2 Fuzzy Implementation of Obstacle Avoidance

Fuzzy-logic-based control is applied to realize a mobile robot motion in an unknown environment with obstacles. Inputs to the fuzzy controller are:

- the obstacle distances p,
- the obstacle orientation θ_1 (which is the angle between the robot moving direction and the line connecting the robot center with the obstacle).
- the target distances l,
- the target orientation θ_2 (which is the angle between the robot moving direction and the line connecting the robot center with the target).

Output of the fuzzy controller is the angular speed difference between the left and right wheels (wheel angular speed correction) of the vehicle: $\Delta\omega = \omega_r - \omega_l$. The block diagram of the fuzzy inference system is presented in Fig. 5.

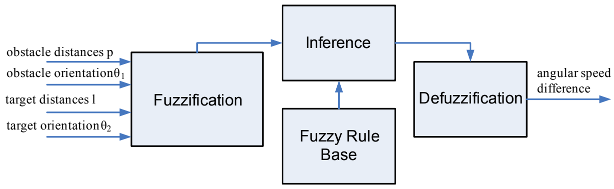

Figure 5

Block diagram of the fuzzy inference system

The obstacle orientation θ_1 and the target orientation θ_2 are determined by the obstacle/target position and the robot position in a world coordinate system, respectively. The obstacle orientation θ_1 and the target orientation θ_2 are defined as positive when the obstacle/target is located to the right of the robot moving direction; otherwise, the obstacle orientation θ_1 and the target orientation θ_2 are negative [2]. For the proposed fuzzy controller the input variables for the obstacle distances p are simply expressed using two linguistic labels *near* and *far* (p є [0, 3 m]). Fig. 6 shows the suitable Gaussian membership functions.

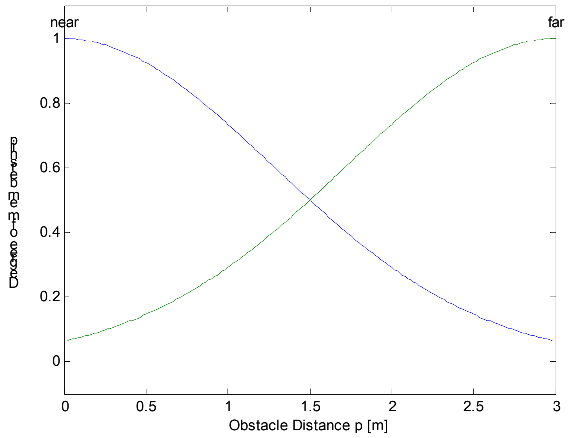

Figure 6

Membership functions of obstacle distances p

The input variables for the obstacle orientation θ_1 are simply expressed using two linguistic labels *left* and *right* (θ_1 є [-3.14, 3.14 rad]). Fig. 7 shows the suitable Gaussian membership functions.

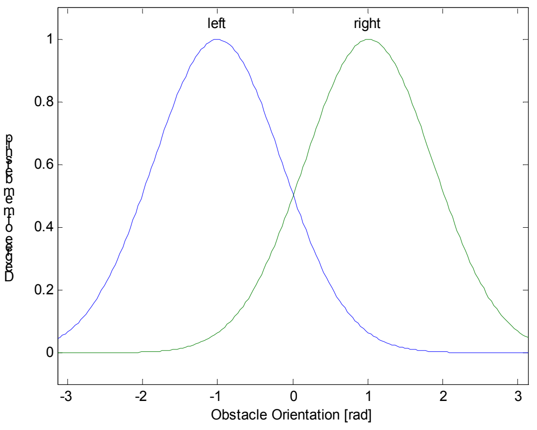

Figure 7
Membership functions of obstacle orientation θ_1

The input variables for the target distances l are simply expressed using two linguistic labels *near* and *far* (l ∊ [0, 3 m]). Fig. 8 shows the suitable Gaussian membership functions.

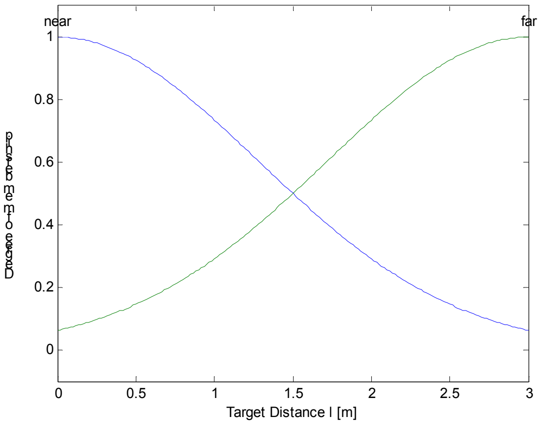

Figure 8
Membership functions of target distances l

The input variables for the target orientation θ_2 are simply expressed using three linguistic labels *left, targetdirection* and *right* (θ_2 Є [-3.14, 3.14 rad]). Fig. 9 shows the suitable Gaussian membership functions.

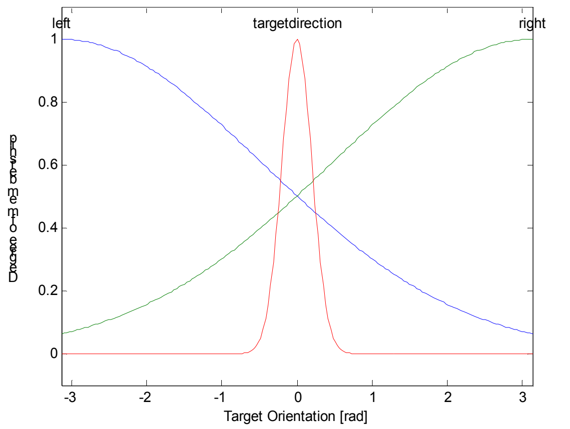

Figure 9

Membership functions of target orientation θ_2

The fuzzy sets for the output variables - the wheel angular speed correction $\Delta\omega = \omega_r - \omega_l$ (*turn-right, zero* and *turn-left*) of the mobile robot - are shown in Fig. 10. The output variables are normalized between: $\Delta\omega \in [-20, 20$ rad/s$]$.

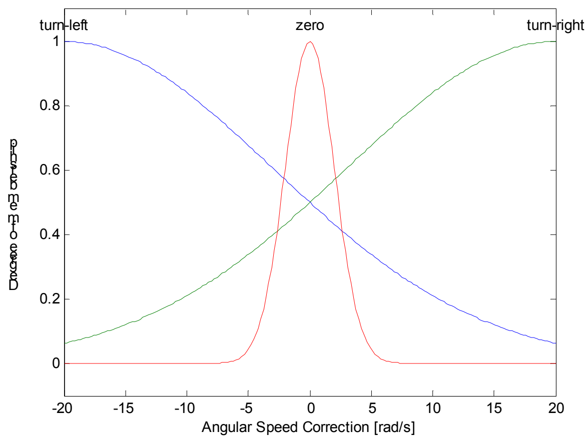

Figure 10

Membership functions of the angular speed difference between the left and right wheels f

The rule- base for mobile robot fuzzy reaction are:

R1: If θ_2 is right then $\Delta\omega$ is turn-right

R2: If θ_2 is left then $\Delta\omega$ is turn-left

R3: If p is near and l is far and θ_1 is left then $\Delta\omega$ is turn-right

R4: If p is near and l is far and θ_1 is right then $\Delta\omega$ is turn-left

R5: If θ_2 is targetdirection then $\Delta\omega$ is zero

R6: If p is far and θ_2 is targetdirection then $\Delta\omega$ is zero

In the present implementation of the fuzzy controller the Center of Area method of defuzzification is used.

4.3 Simulation Results

Now, we applied the proposed fuzzy controller to the mobile robot moving in an unknown environment with obstacle. The results of the simulation are shown in Fig. 11.

The mobile robot compares the nearness of the obstacles of two sides. Fig. 11 shows the obstacle avoidance mobile robot paths. The fuzzy reactive controller is powerful in view of the short reaction time and rapid decision-making of the obstacle avoidance process.

The simulation results show the effectiveness and the validity of the obstacle avoidance behavior in an unknown environment of the proposed control strategy.

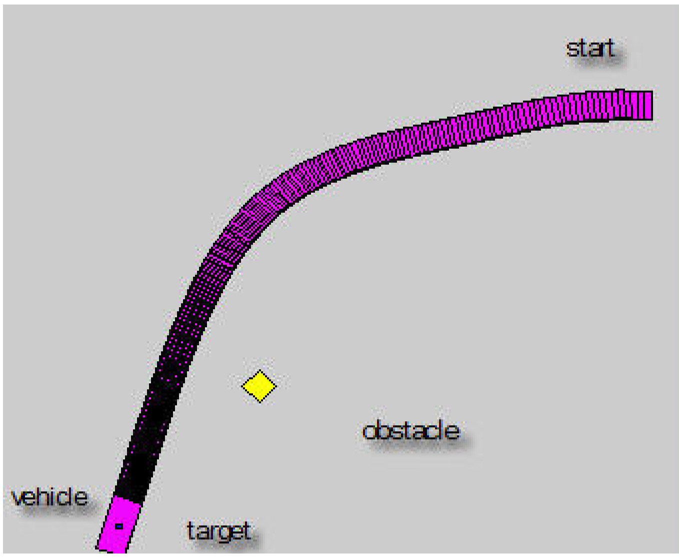

Figure 11
Obstacle avoidance trajectory of mobile robot

Conclusions

The paper deals with the modeling and control strategies of the motion of wheeled mobile robots. The model of the vehicle has two driving wheels and the angular velocities of the two wheels are independently controlled. First, the vehicle kinematics model and the control strategies using a feedforward compensator are analyzed. Second, fuzzy reactive control of a mobile robot motion in an unknown environment with obstacles is proposed. When the vehicle is moving towards the target and the sensors detect an obstacle, an avoiding strategy is necessary. We proposed fuzzy reactive navigation strategy of collision-free motion in an unknown environment with obstacles.

Finally, the mobile robot simulation is illustrated. The implemented controller is computationally simple. The fuzzy reactive controller is powerful in view of the short reaction time and rapid decision-making of the obstacle avoidance process. The simulation results show the effectiveness and the validity of the obstacle avoidance behavior in an unknown environment of the proposed fuzzy control strategy.

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