Cooperative Systems and Their Control

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Cooperative control systems on graphs

Cooperative control: each system is endowed with its own state variable and dynamics.
A fundamental problem : in multi-agent dynamical systems on networks is the design of distributed protocols that guarantee consensus or synchronization in the sense that the states of all the systems reach the same value.

Outline

- Modeling Networked Control Systems
- Control of Networked and Hybrid Systems

Target areas

- high confidence medical devices, SW & HW
- aviation health management, design of certifiable systems,
- networked embedded control
  (beyond Supervisory Control And Data Acquisition (SCADA)),
- high confidence automotive systems,
- transportation systems (road, aerial, rail),
- robotics.

Quest for higher degree autonomy in systems.

Implication to Research in Systems And Control Theory

NIT Aspects:

- Control Over Network
- Control Of Networks

System Theory:

- Modeling Networked Control Systems
- Emphasis On Hybrid Systems

Control Theory:

- Control Of Networked And Hybrid Systems
- Detection And Reconfiguration
- Safety Critical Systems

Technical examples

Graphs and Network Abstraction

A networked system consists of

Nodes (vertices) physical entities with limited resources (computation, communication, perception, control)

Edges virtual entities that encode the flow of information between the nodes

The characteristics of the information flow is abstracted through the (possibly weighted and directed) edges of a **Graph**.

Degree matrix: $D = diag(deg(v_1), deg(v_2), \cdots deg(v_N))$

Adjacency matrix:

$$A = [a_{ij}]_{i,j=1,\dots,n} \quad a_{ij} = \begin{cases} 1 & (v_i, v_j) \text{ is an edge} \\ 0 & \text{otherwise} \end{cases}$$

Incidence matrix (directed graphs)

$$J = [j_{ik}]_{i,k=1,\dots,n} \quad j_{ik} = \begin{cases} 1 & (v_i, v_k) \text{ is an edge} \\ -1 & (v_k, v_i) \text{ is an edge} \\ 0 & \text{otherwise} \end{cases}$$

Graph Laplacian: $L = D - A = J^T J$

A simple Leader–Follower model

The leader is unaffected by the members whereas a member may be influenced by the leader as well as its neighbors:
$x_i(t+1) = x_i(t) - \sum_{j \in \mathcal{N}_i} w_{ij}(x_i(t) - x_j(t)) - \gamma_i w_{i0}(x_i(t) - x_0(t))$

$\gamma_i = 1$ if there is an edge from the leader and 0 otherwise.
$W = [w_{ij}]$ is the interaction matrix with nonnegative elements.
$x = (x_1, \dots, x_N)^T$ the stack vector of all the agent states:

$$x(k+1) = Fx(k) + rx_0(k), \quad F = \mathbb{I} - L - R$$

$$R = diag(r), \quad \text{with} \quad r = [\gamma_1 w_{10}, \cdots, \gamma_N w_{N0}]^T$$

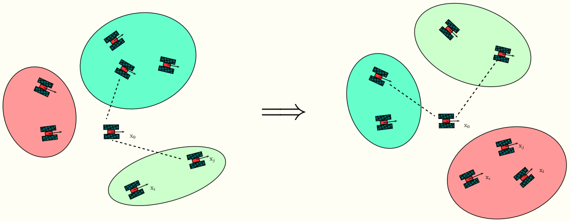
$$L = [l_{ij}], \quad \text{with} \quad l_{ij} = \begin{cases} -w_{ij} & j \neq i,\ j \in \mathcal{N}_i \\ \sum_{k \in \mathcal{N}_i} w_{i,k} & j = i \\ 0 & \text{otherwise} \end{cases}$$

Dynamic network with switching topology

The leader is fixed, but the interconnection graph $\mathcal{G}$ of the model is time varying: the model is a dynamic network with switching topologies:

$$x(t+1) = F_{\sigma(t)}x(t) + r_{\sigma(t)}x_0(t)$$

$x(t) \in \mathbb{R}^{nN}$ is the state, $x_0(t) \in \mathbb{R}^n$ is the effect of the leader.



Piecewise constant switching path: $\sigma(k) : \{0, 1, \dots\} \to \{1, \dots, K\}$
$\sigma(k) = i$ implies that the subsystem is chosen
K is the number of possible coupling patterns (switching topologies) of the network.

Open-loop vs. Closed-loop switching

TIME-DEPENDENT SWITCHING

- (exogenous) switching signal $\sigma(t)$ that specifies the system that is active at time t
- piecewise constant function of time

STATE-DEPENDENT SWITCHING

- (endogenous) switching signal $\sigma(t)$ that specifies the system that is active when the state is x
- the state space $X = \cup_{i \in \mathcal{R}} \Omega_i$ is partitioned into operating regions Ω_i associated to a system $s_i \in S$

Graph Based Control

Based on variations of the consensus equation different control problems can be handled:

Formation control: drive the collection to a **predetermined configuration**

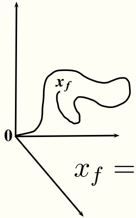
Coverage control: produce **prescribed structures**

Containment Control: ensure that the followers are contained in a **given set/area** during the maneuver.

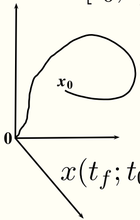
Controlled linear switched systems

$$\dot{x}(t) = A(\sigma(t))x(t) + B(\sigma(t))u(t).$$

A state $x \in \mathbb{R}^n$ is **reachable** (**controllable**) at time t_0, if there exist a time instant $t_f > t_0$, a switching function $\sigma : [t_0, t_f] \to \mathfrak{S}$, and a bounded measurable input function $u : [t_0, t_f] \to \mathfrak{U}$ such that



$$x_f = x(t_f; t_0, 0, u, \sigma)$$



$$x(t_f; t_0, x_0, u, \sigma) = 0$$

$\Sigma_{(\mathfrak{S},\mathfrak{U})}$ is said to be **globally controllable** if every point in the state space is reachable from any other point in the state space by using bounded measurable controls and a suitable switching function.

Unconstrained controllability ($\mathfrak{U} = \mathbb{R}^m$)

The unconstrained switching system is controllable if and only if the *multivariable Kalman rank condition*, **rank** $\mathcal{R}_{\mathcal{A},\mathcal{B}} = n$, holds, where

$$\mathcal{R}_{(\mathcal{A},\mathcal{B})} = \textbf{span}\left\{ \prod_{j=1}^{J} A_{l_j}^{i_j} B_k \,|\, k = 1, \cdots, s \right\}$$

where $J \geq 0$, $l_j \in \{0, \cdots, s\}$, $i_j \in \{0, \cdots, n-1\}$. The subspace $\mathcal{R}_{\mathcal{A},\mathcal{B}}$ is the minimal subspace invariant for all A_i containing $\mathcal{B} = \cup_{i=1}^{s} \textbf{Im}\ B_i$.

Considering the finitely generated Lie-algebra $\mathcal{L}(A_0, \ldots, A_s)$ which contains $A_0, \ldots, A_s$, and a basis $\hat{A}_1, \ldots, \hat{A}_K$ of this algebra

$$\mathcal{R}_{\mathcal{A},\mathcal{B}} = \sum_{k=0}^{s} \sum_{n_1=0}^{n-1} \cdots \sum_{n_K=0}^{n-1} \textbf{Im}\ (\hat{A}_1^{n_1} \ldots \hat{A}_K^{n_K} B_k).$$

Note, that $\mathcal{R}_{\mathcal{A},\mathcal{B}}$ is the minimal subspace invariant for all of the A_is containing $\mathcal{B} = \sum_{i=0}^{s} \textbf{Im} B_i$.

Example: simple four–agent network

The switching topology is described by graphs.
For simplicity, we let $w_{ij} = w_{i0} = 1 \quad i, j = 1, 2, 3.$

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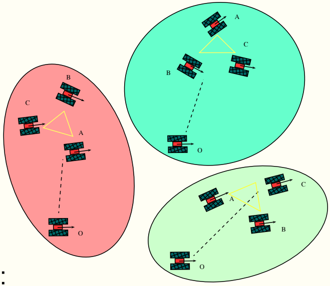


The switched linear system is defined by:

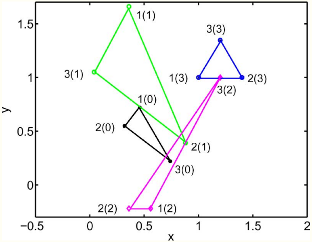
$$F_1 = \begin{bmatrix} -2 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{bmatrix} r_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad F_2 = \begin{bmatrix} -1 & 1 & 1 \\ 1 & -2 & 1 \\ 1 & 1 & -1 \end{bmatrix} r_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \quad F_3 = \begin{bmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -2 \end{bmatrix} r_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

Four-agent network controllability

Each subsystem (F_i, r_i) is uncontrollable however the switching system is completely controllable, since the Kalman rank condition holds. Hence, the system can be steered to realize any desired formation by the leader.

The switching sequence $1 \to 2 \to 3$ steers any $x(0)$ to a given x_f, i.e., $x_f = F_3F_2F_1x(0) + F_3F_2r_1x_0(0) + F_3r_2x_0(1) + r_3x_0(2)$.

The member agents(circles) move from a random initial configuration to the desired one: forming a regular triangle.
The indices in parenthesis indicate time steps.



Stability notions

The zero solution of $\dot{x} \in F(x)$ is (asymptotically)

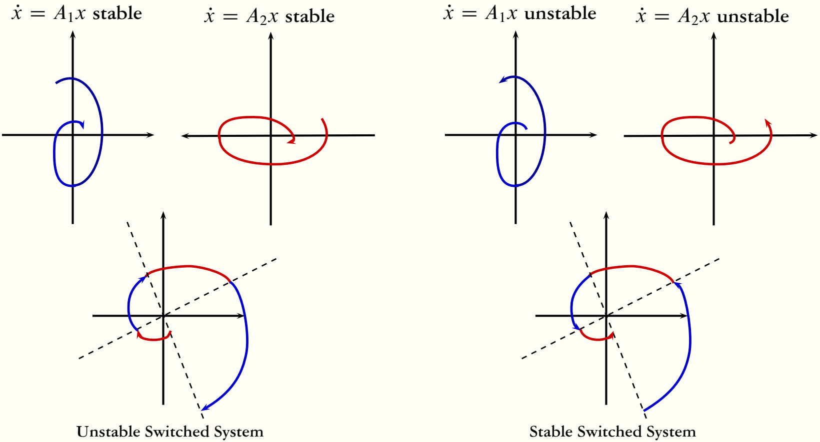
strongly stable if for each solution $x(t)$

weakly stable if there exists a solution $x(t)$

for any $\epsilon > 0$ there is a $\delta > 0$ and $\Delta > 0$ such that:

- if $||x(0)|| < \delta$ then $||x(t)|| < \epsilon$ holds for all $t \geq 0$
- if $||x(0)|| < \Delta$ then $\lim_{t \to \infty} x(t) = 0$ holds.

Switched stability



Switching introduces a new quality (behavior) which is not present in the individual components.

Strong stabilizability [Liu and Molchanov (2002)]

For the strong asymptotic stability of the zero solution of the differential inclusion $\dot{x} \in \{\, y = Ax \,|\, A \in \{A_1, \cdots A_s\}\}$ it is necessary and sufficient that there exist an $m \geq n$, a matrix $\mathcal{L} \in \mathbb{R}^{n \times m}$ with rank n and matrices $\Gamma_l \in \mathbb{R}^{m \times m}$ with $\gamma_{ii}^l + \sum_{j \neq i} |\gamma_{ij}^l| < 0$, $i = 1, \cdots, m$; $l = 1, \cdots s$, such that $A_l \mathcal{L} = \mathcal{L} \Gamma_l, \quad l = 1, \cdots s.$

$\Longrightarrow$ [Liberzon, Agrachev] when the Lie algebra generated by the matrices A_i is solvable, the existence of $\mathcal{L}$ is guaranteed with upper triangular matrices Γ_s.

If all matrices A_s of a switched linear system have a solvable Lie algebra, then a common Lyapunov function exists and the system is stable under arbitrary switching.

Weak stabilizability [Geromel & Colaneri (2006)]

If for a given $Q \geq 0$ there exist positive definite matrices $\{P_i\}$ and a Metzler matrix Π ($\pi_{ij} \geq 0$, $i \neq j$ and $\sum_{i=1}^{s} \pi_{ij} = 0$) such that the Lyapunov-Metzler inequalities holds

$$A_i^T P_i + P_i A_i + \sum_{j=1}^{s} \pi_{ij} P_j + Q < 0, \quad i \in \mathcal{S},$$

then the state switching control

$$\sigma(x) = arg \min_{i \in \mathcal{S}} x^T P_i x$$

make the system globally asymptotically stable.

Generalized Piecewise Linear Feedback Stabilizable Systems

Given a set of linear switched systems, i.e.,

$$\dot{x} \in \{\, A_i x + B_i u_i \,|\, i \in \mathcal{S} = \{1, \cdots, s\}\},$$

find a closed-loop switching strategy with

- suitable linear feedbacks $u_i = K_{l_i} x, \quad i \in \mathcal{S}$
- a switching law $\kappa(x) \in \mathcal{S}$, $x \in \mathbb{R}^n$

that (weakly)stabilizes the system.

Z. Szabó, European Control Conference, Budapest, 2009

Proposition

The completely controllable linear switched system is generalized piecewise linear feedback stabilizable.

Separate the task of finding a suitable switching strategy and that of finding suitable control inputs with low complexity that stabilizes the system in "closed–loop".

Networked and Distributed Sensing and Control

The effect of the network:

- information constraints
- shared communication channels
- delays, dropped packets, fading
- time varying throughput

Renewed emphasis on Distributed Control

- potential for superior performance
- loosely interconnected clusters of control systems

Thank You for Your Attention