Taming Complexity: From Network Science to Control

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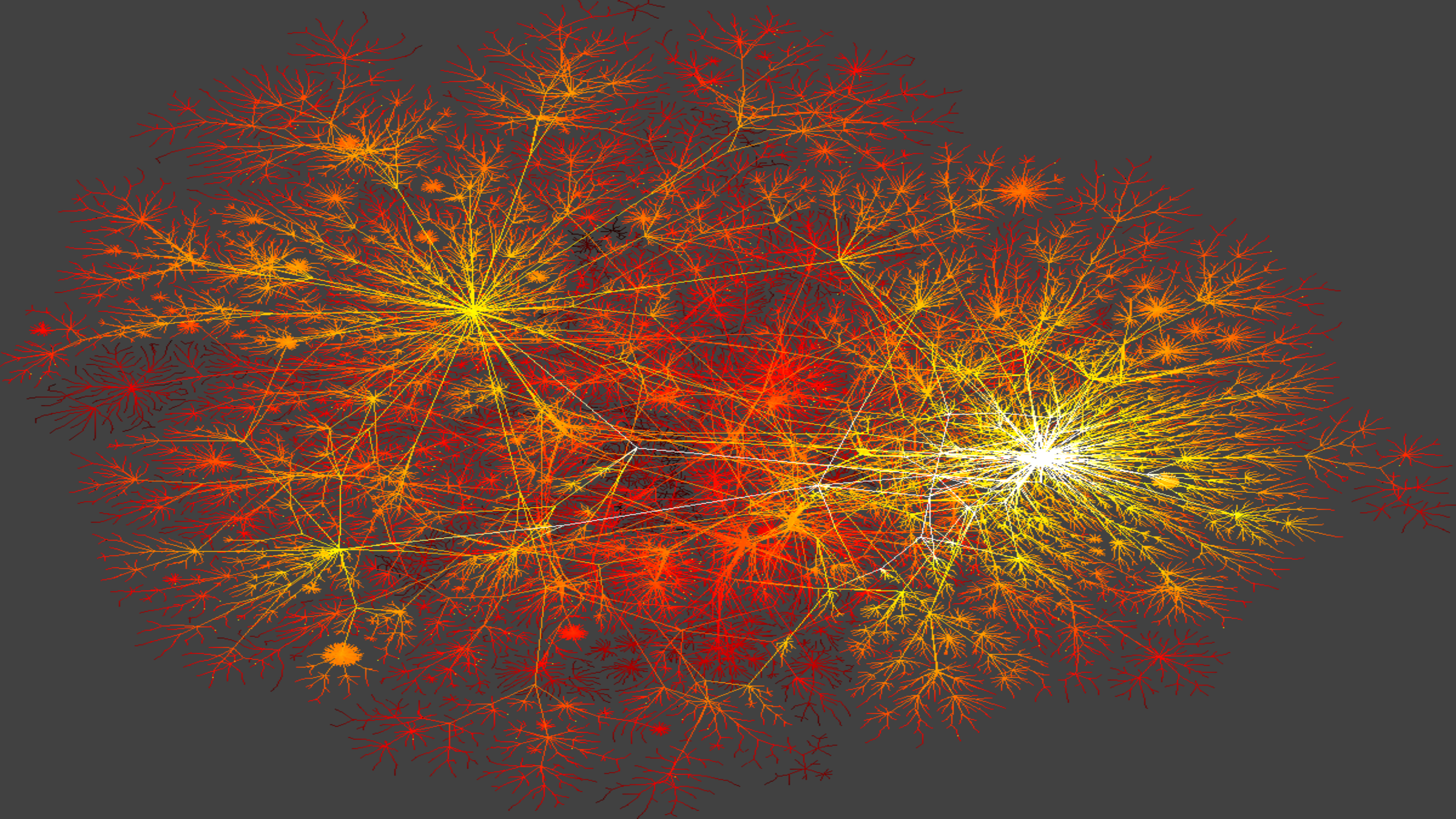
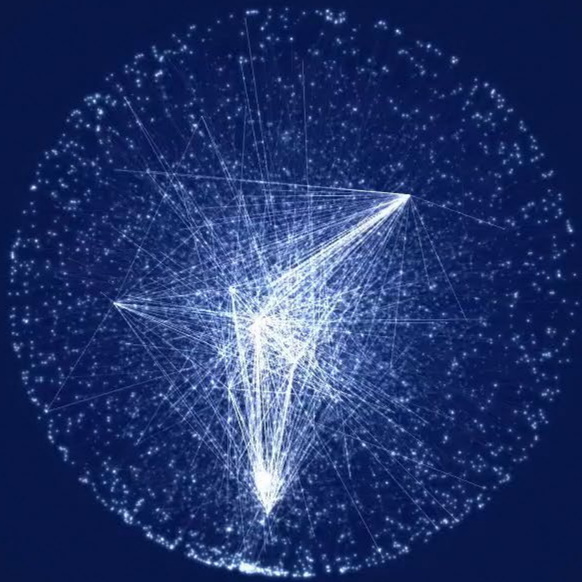




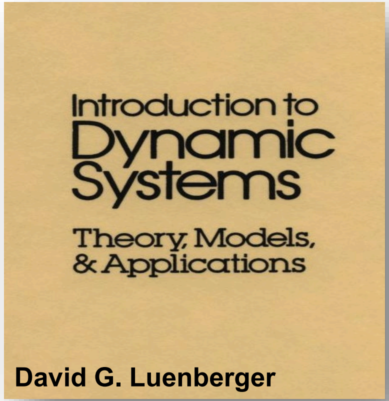
Understand

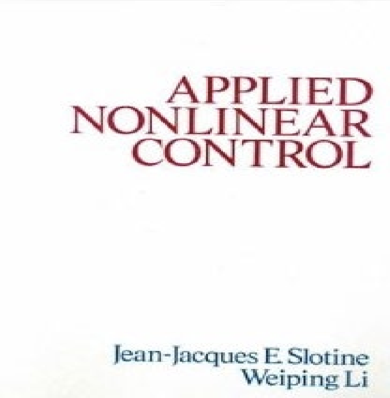
quantify

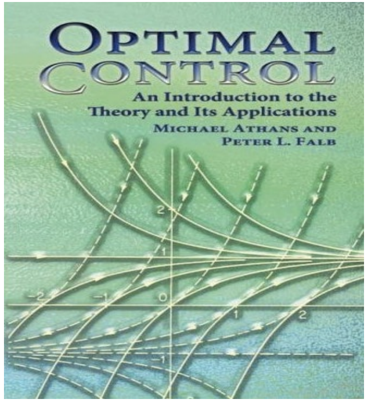
predict

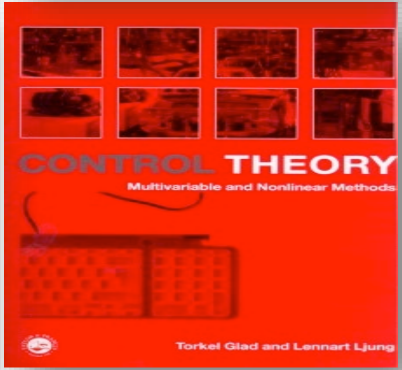
control

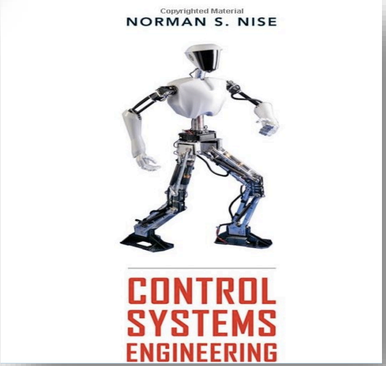
Control Theory

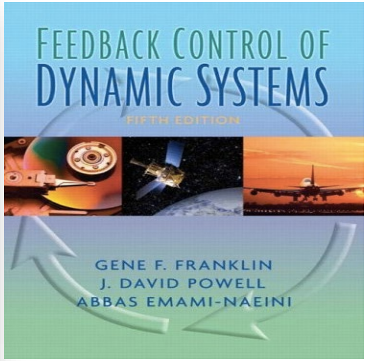


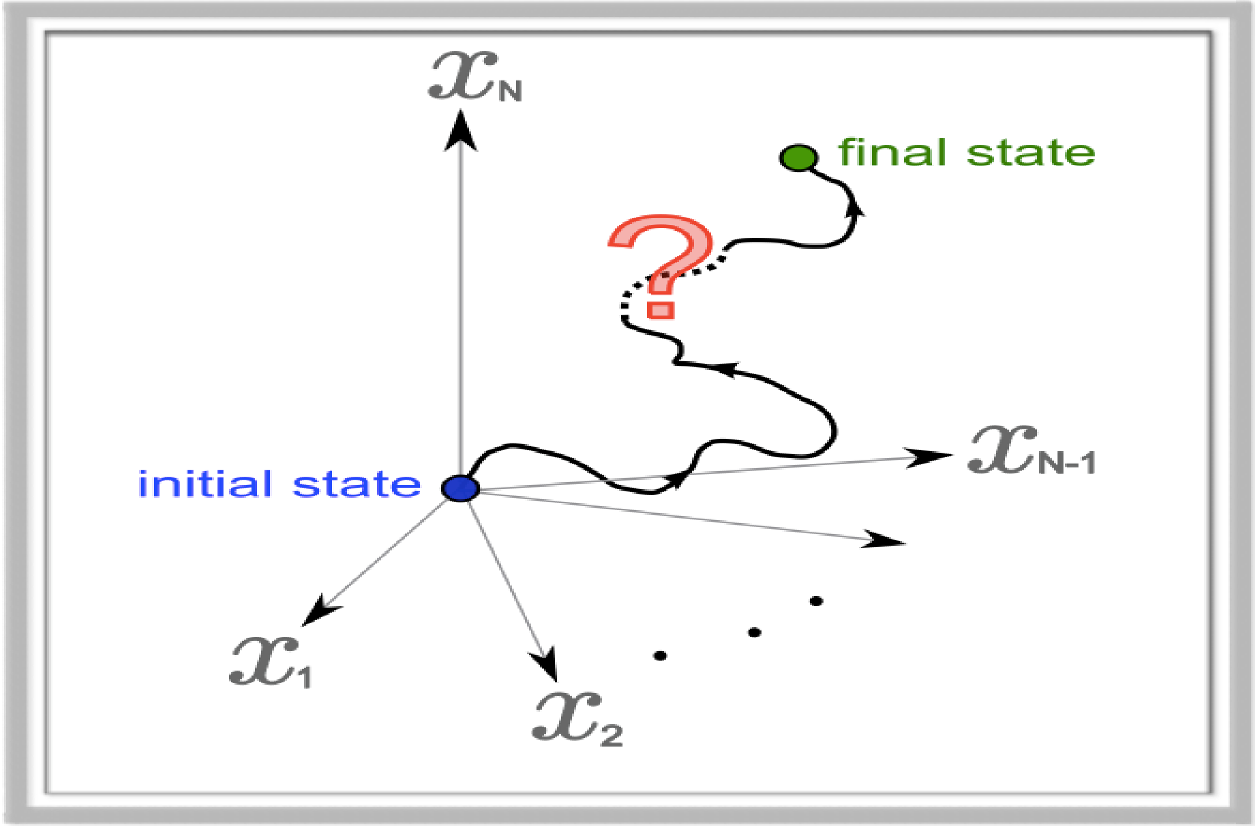




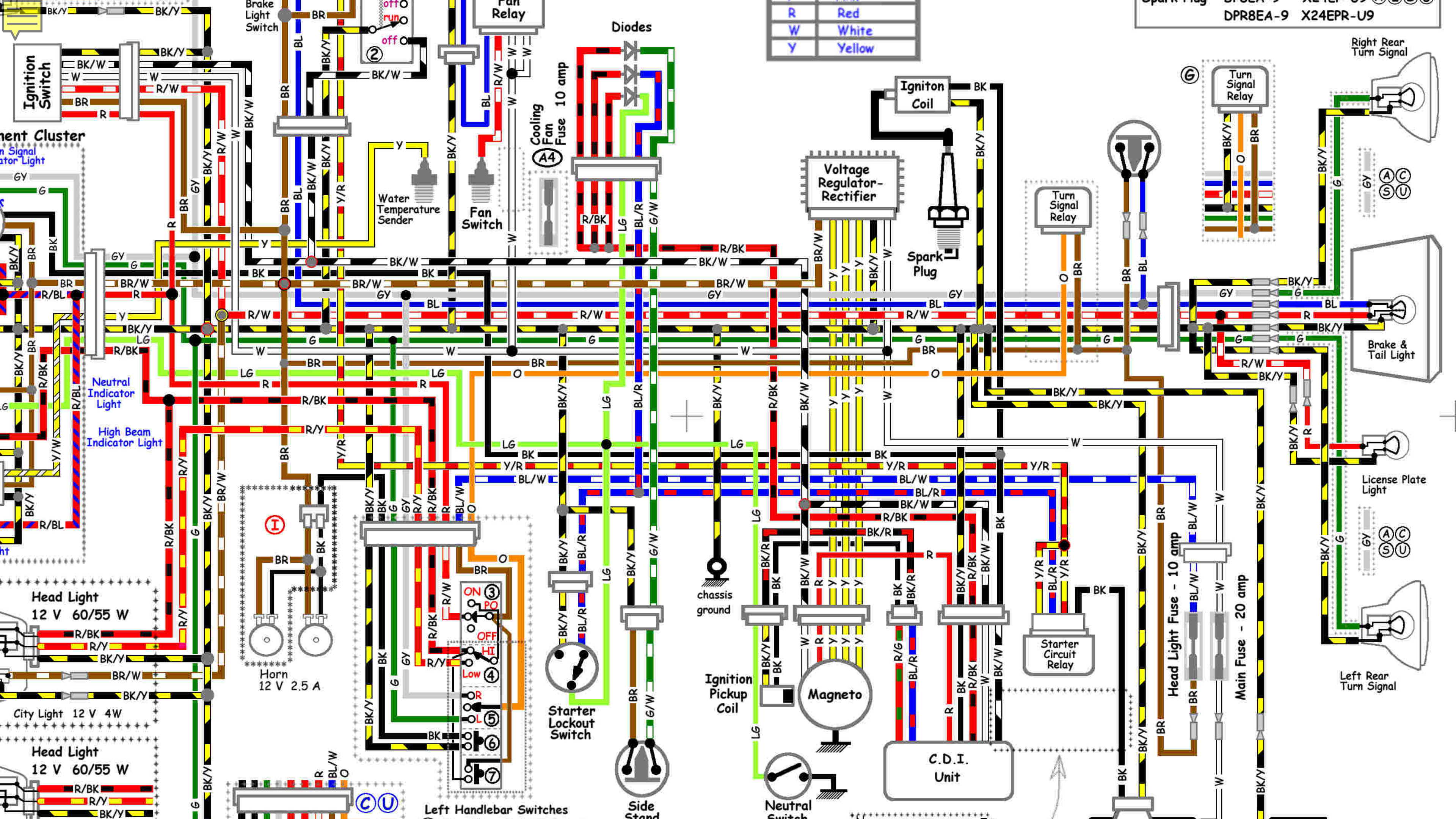








A system is controllable if it can be driven from any initial state to any desired final state in finite time.



Brake Light Switch
off
run
off
Fan Relay
Diodes
Spark Plug DPR8EA-9 X24EPR-U9
R Red
W White
Y Yellow
Ignition Switch
Instrument Cluster
Turn Signal Indicator Light
Water Temperature Sender
Fan Switch
Cooling Fan Fuse 10 amp
A4
Igniton Coil
Voltage Regulator-Rectifier
Spark Plug
Turn Signal Relay
Right Rear Turn Signal
Brake & Tail Light
Neutral Indicator Light
High Beam Indicator Light
License Plate Light
chassis ground
Head Light 12 V 60/55 W
City Light 12 V 4W
Horn 12 V 2.5 A
ON
PO
OFF
HI
Low
Starter Lockout Switch
Side Stand
Ignition Pickup Coil
Magneto
Neutral Switch
C.D.I. Unit
Starter Circuit Relay
Head Light Fuse - 10 amp
Main Fuse - 20 amp
Left Rear Turn Signal
Left Handlebar Switches

Controllability of complex networks

Yang-Yu Liu[1,2], Jean-Jacques Slotine[3,4] & Albert-László Barabási[1,2,5]

The ultimate proof of our understanding of natural or technological systems is reflected in our ability to control them. Although control theory offers mathematical tools for steering engineered and natural systems towards a desired state, a framework to control complex self-organized systems is lacking. Here we develop analytical tools to study the controllability of an arbitrary complex directed network, identifying the set of driver nodes with time-dependent control that can guide the system's entire dynamics. We apply these tools to several real networks, finding that the number of driver nodes is determined mainly by the network's degree distribution. We show that sparse inhomogeneous networks, which emerge in many real complex systems, are the most difficult to control, but that dense and homogeneous networks can be controlled using a few driver nodes. Counterintuitively, we find that in both model and real systems the driver nodes tend to avoid the high-degree nodes.

REVIEWS OF MODERN PHYSICS, VOLUME 88, JULY–SEPTEMBER 2016

Control principles of complex systems

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Albert-László Barabási

Center for Complex Network Research and D...
Computer Science and Biology, N...
Boston, Massach...

- **Linear Time-Invariant Dynamics**

$$\frac{d\mathbf{X}}{dt} = \mathbf{A}\cdot \mathbf{X}(t) + \mathbf{B}\cdot \mathbf{u}(t)$$

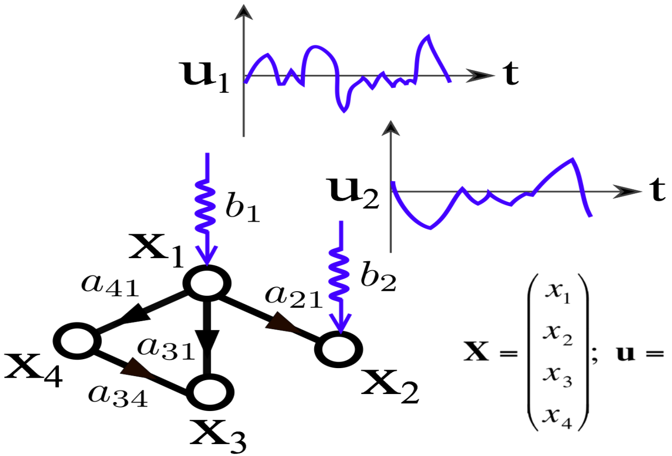
$\mathbf{A} \in R^{N\times N}$: weighted wiring diagram

$\mathbf{X}(t) \in R^{N\times 1}$: state vector.

$\mathbf{u}(t) \in R^{M\times 1}$: input vector $(M \leq N)$.

$\mathbf{B} \in R^{N\times M}$: input matrix

($\Rightarrow$ control configuration).



$$\mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix}; \quad \mathbf{u} = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}.$$

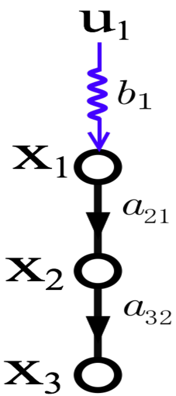
$$\mathbf{A} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ a_{21} & 0 & 0 & 0 \\ a_{31} & 0 & 0 & a_{34} \\ a_{41} & 0 & 0 & 0 \end{pmatrix}; \quad \mathbf{B} = \begin{pmatrix} b_1 & 0 \\ 0 & b_2 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}$$

- **Kalman's Rank Condition:**
  A system is controllable iff its controllability matrix has full rank.

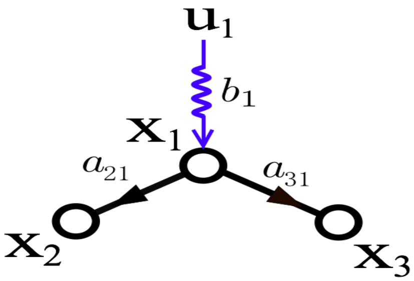
$$\text{rank } \mathbf{C} = N$$
$$\mathbf{C} = [\mathbf{B}, \mathbf{A}\cdot \mathbf{B}, \mathbf{A}^2\cdot \mathbf{B}, \ldots, \mathbf{A}^{N-1}\cdot \mathbf{B}]$$

R. E. Kalman, *J.S.I.A.M. Control* (1963)

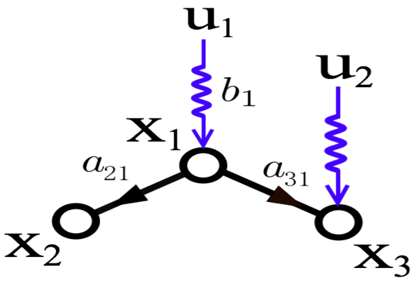
EXAMPLES: Controllable or not controllable?



Yes



No



Yes

Problem: The Kálmán condition does not identify the control nodes.

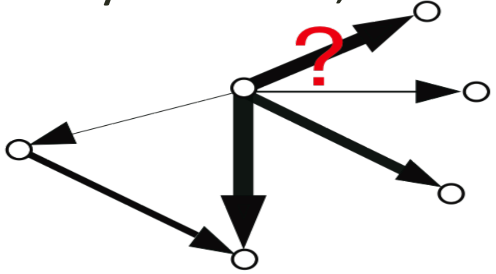
It only tells us if our guess is correct.

Finding the control nodes requires 2^N operations

Difficulties

1. Parameters (link weights): usually unknown.

e.g. gene regulatory network, Internet, etc.



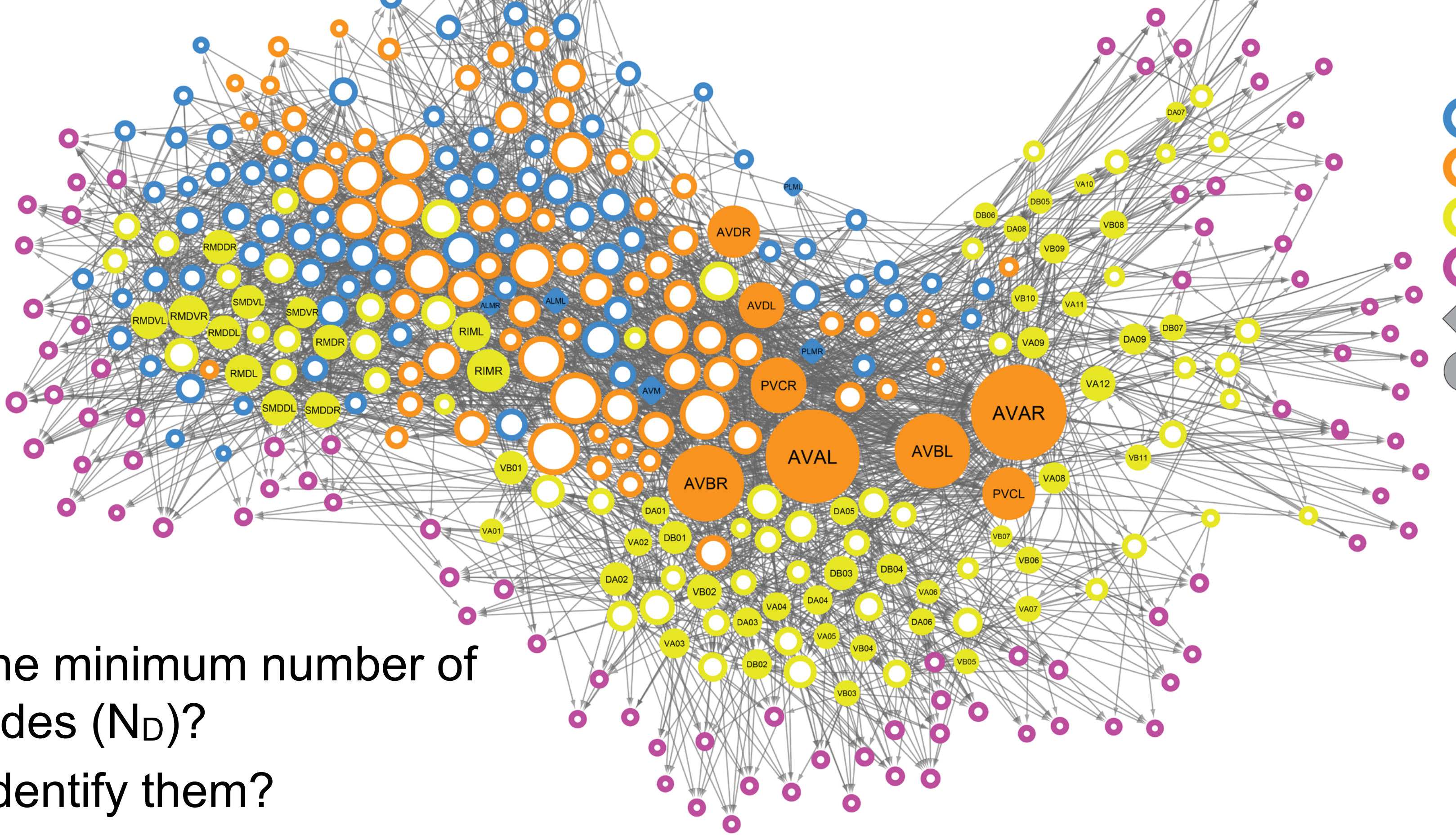
2. If brute-force search: (2^N-1) combinations.

$$\binom{N}{1} + \binom{N}{2} + \cdots + \binom{N}{N} = 2^N - 1$$

3. Kalman's rank condition is hard to check for large system.

$$\text{rank } \mathbf{C} = N$$

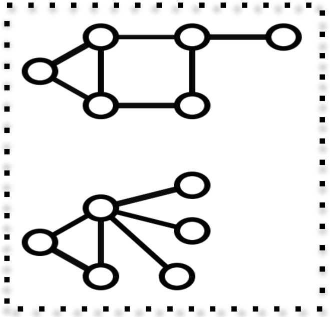
$$\mathbf{C} = [\mathbf{B}, \mathbf{A}\cdot\mathbf{B}, \mathbf{A}^2\cdot\mathbf{B}, \cdots, \mathbf{A}^{N-1}\cdot\mathbf{B}] \text{ has dimension } N \times NM.$$



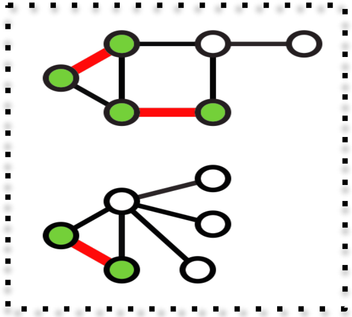
- What's the minimum number of driver nodes (N_D)?
- How to identify them?
- Which network characteristics determine N_D?

Matching

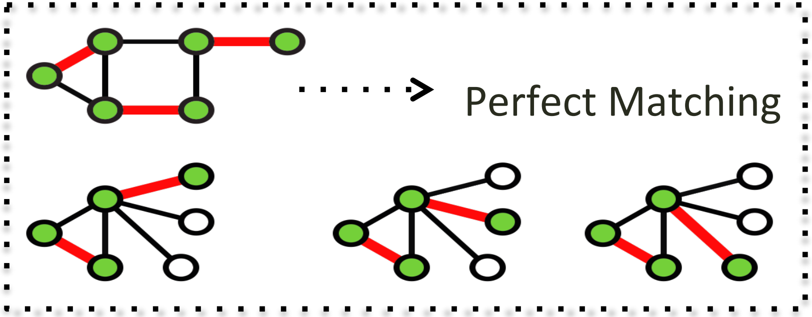
Network



Matching :
a set of edges without
common vertices.



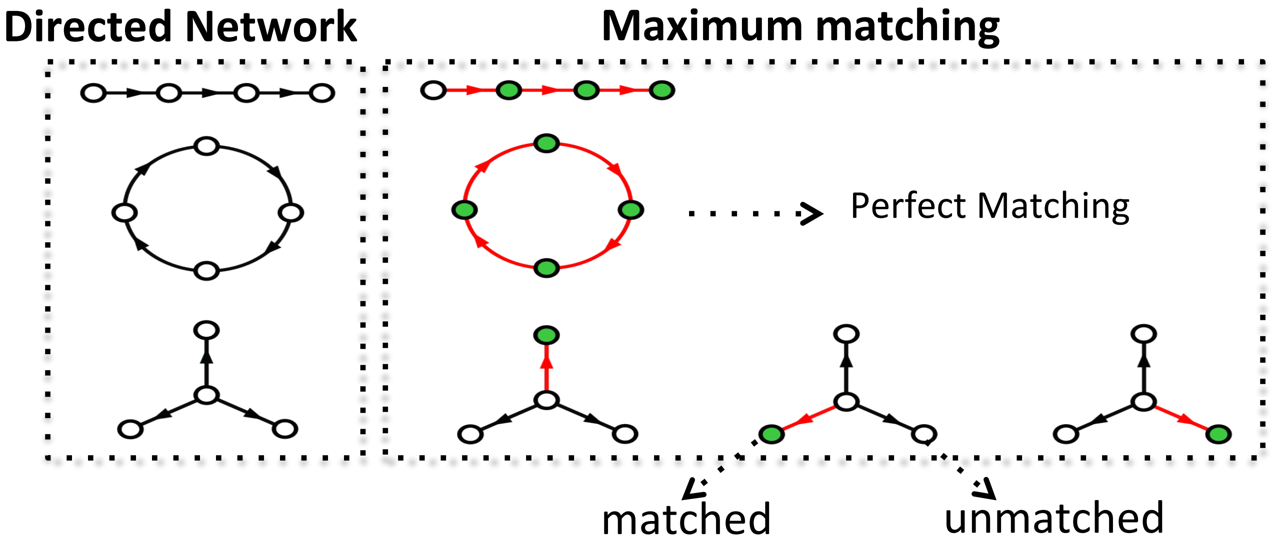
Maximum matching :
a matching of the largest size.



Lovász, L. & Plummer, M.D., *Matching Theory*

MATCHING IN DIRECTED NETWORKS:

Matching: a set of edges without common heads or tails.

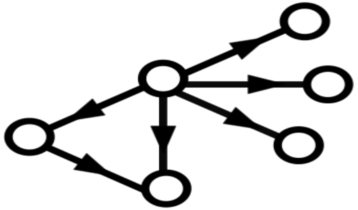


Minimum Input Theorem: Driver nodes = Unmatched nodes

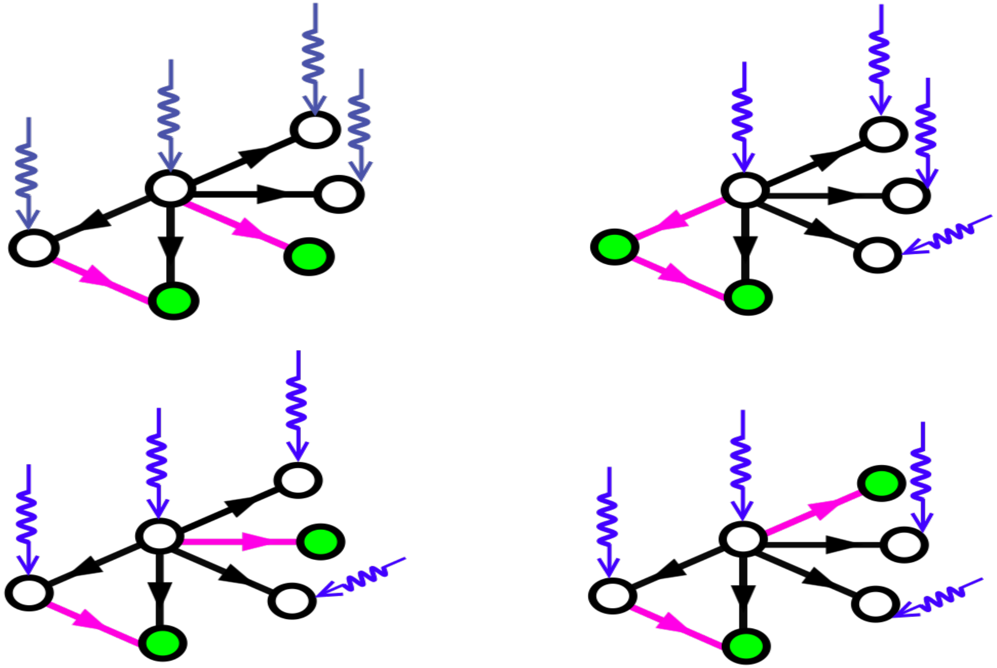
Y.-Y. Liu, J.-J. Slotine, A.-L. Barabási, Nature (2011)

EXAMPLES: Identifying the driver nodes

network



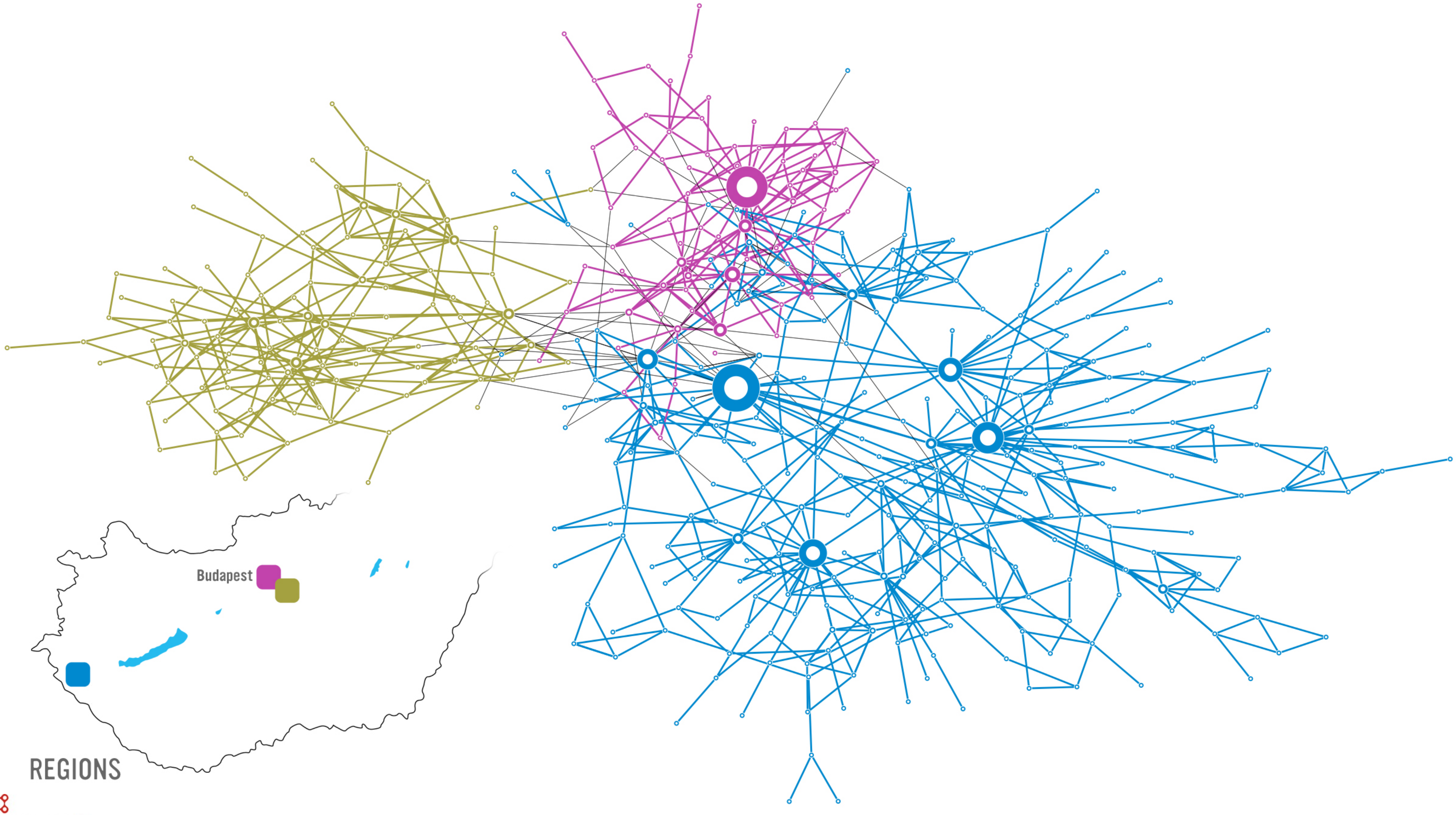
Maximum matching
controlled network



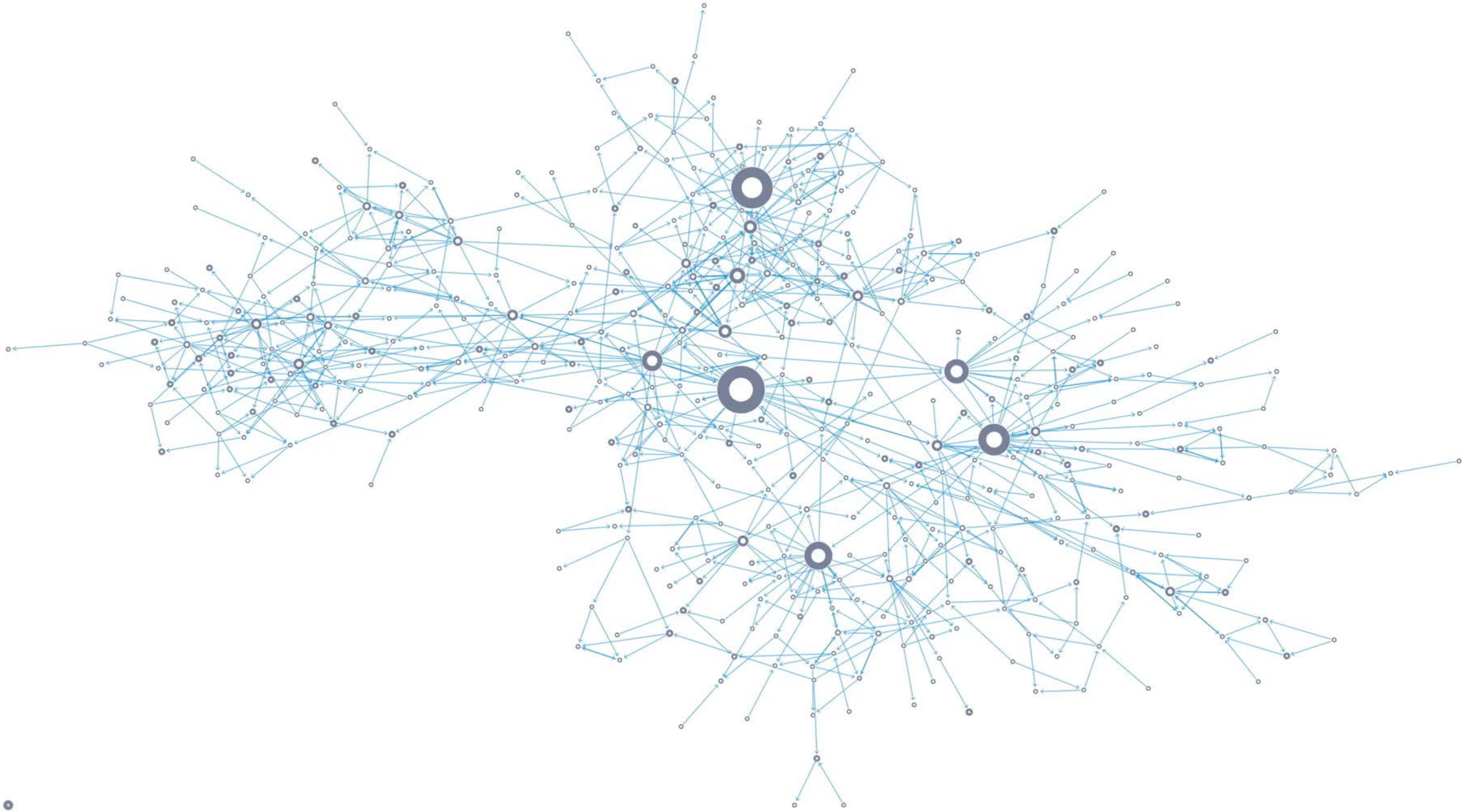
Brute-force search
$O(2^N)$~10^{30} for N=100.
Hopeless!

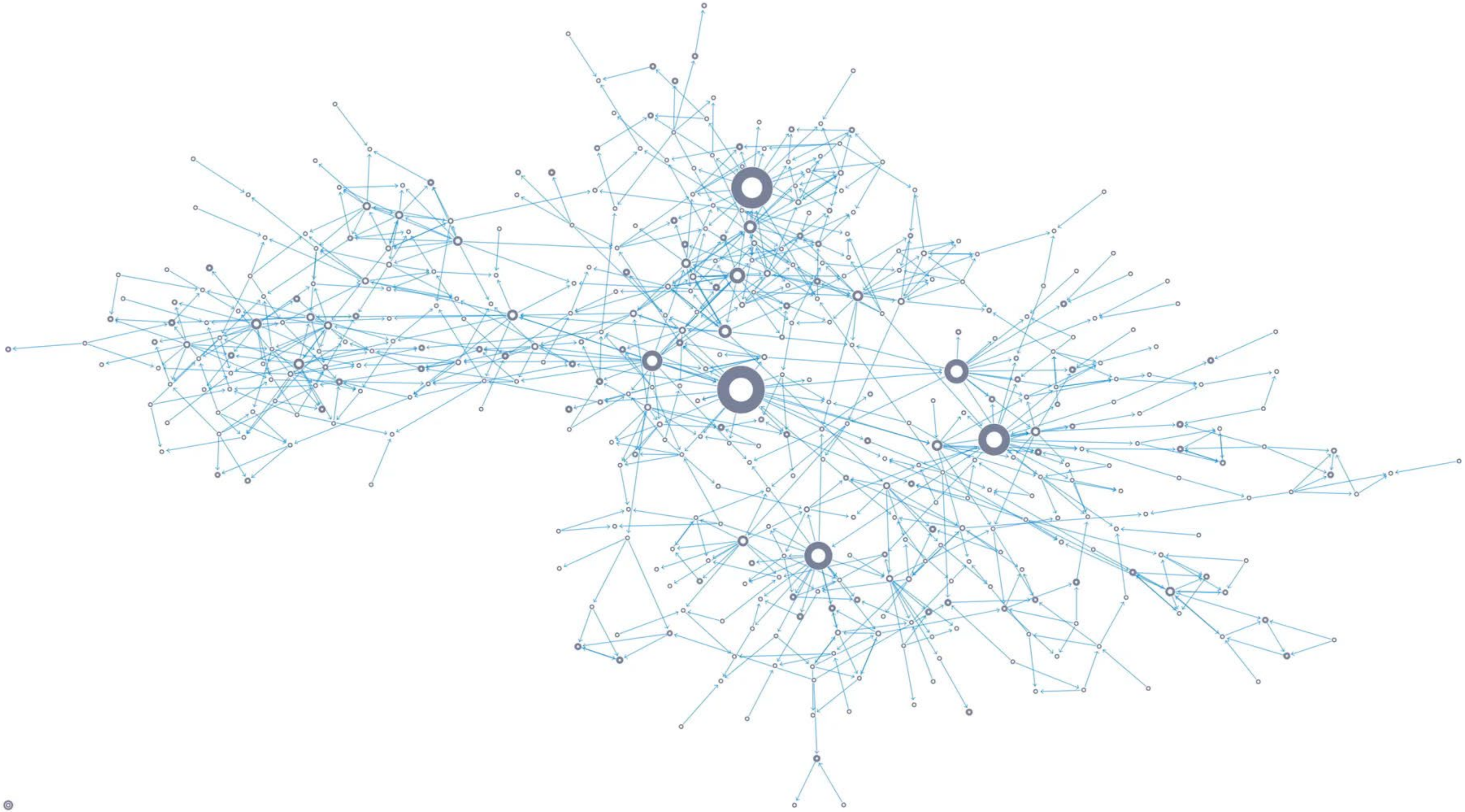
Hopcroft-Karp Algorithm
$O(N^{1/2}L)$ Polynomial!
Fast even for N~10^6.

Y.-Y. Liu, J.-J. Slotine, A.-L. Barabási, *Nature* (2011)

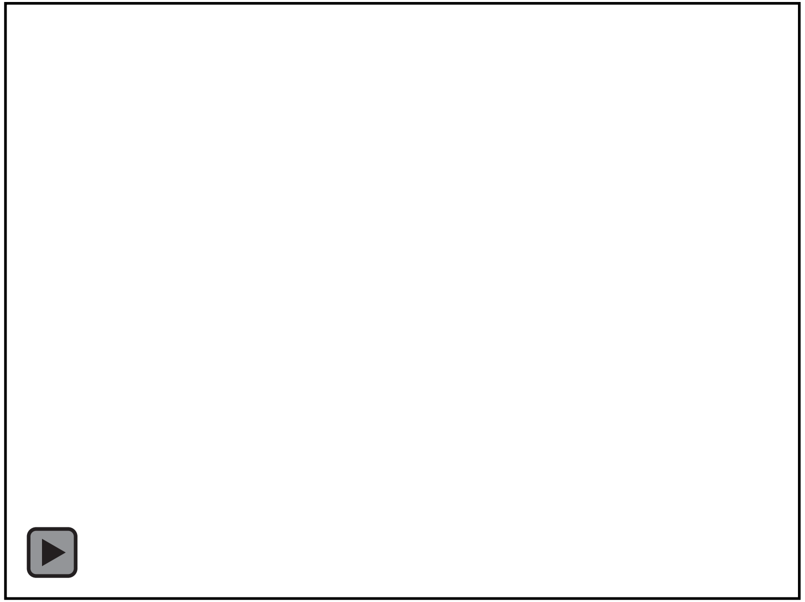


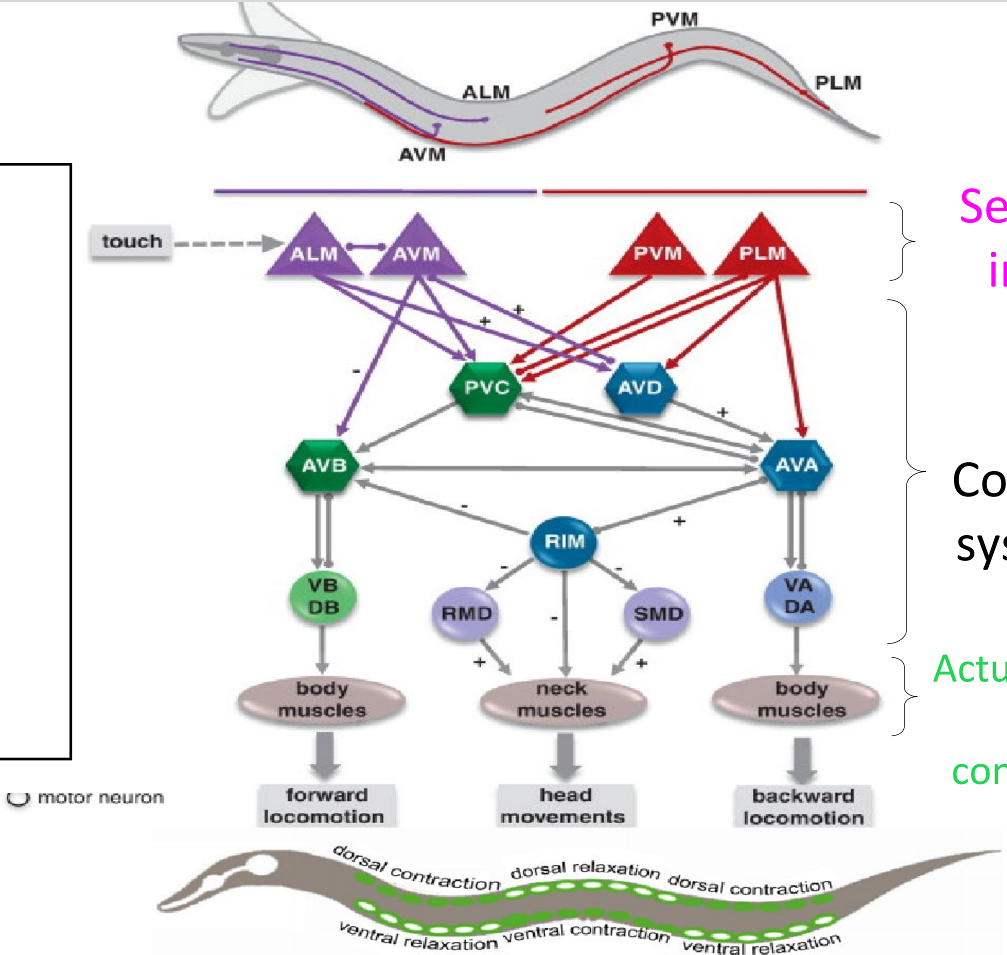
REGIONS





C Elegans: Light Body Touch Circuit





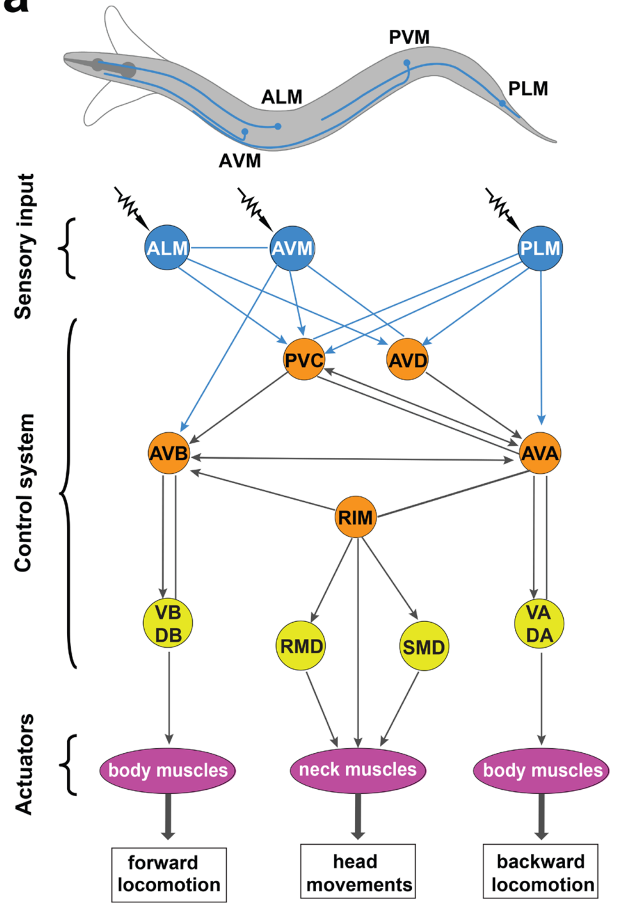
Sensory input

Control system

Actuators to be controlled

C Elegans: Light Body Touch Circuit

a

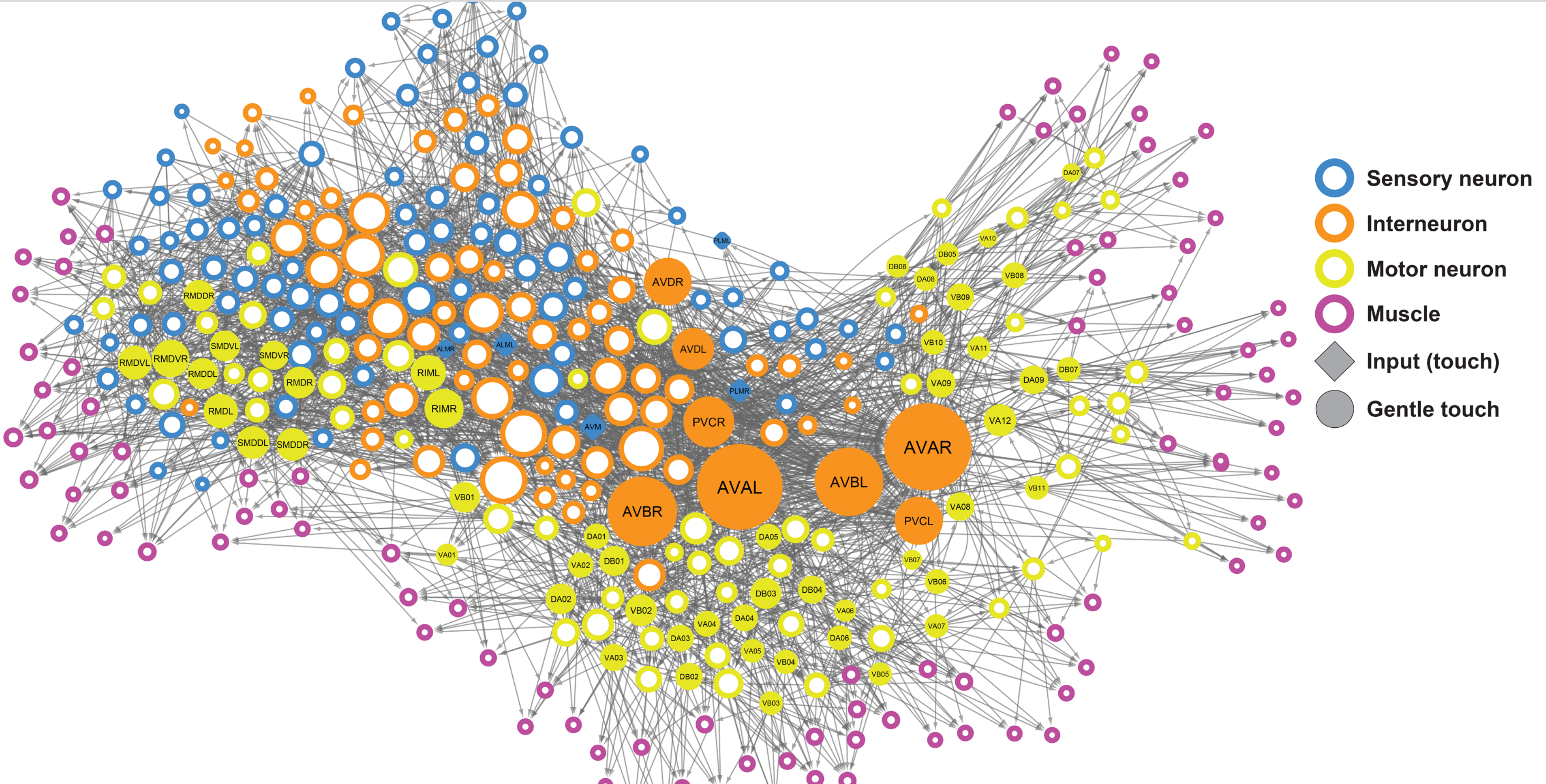


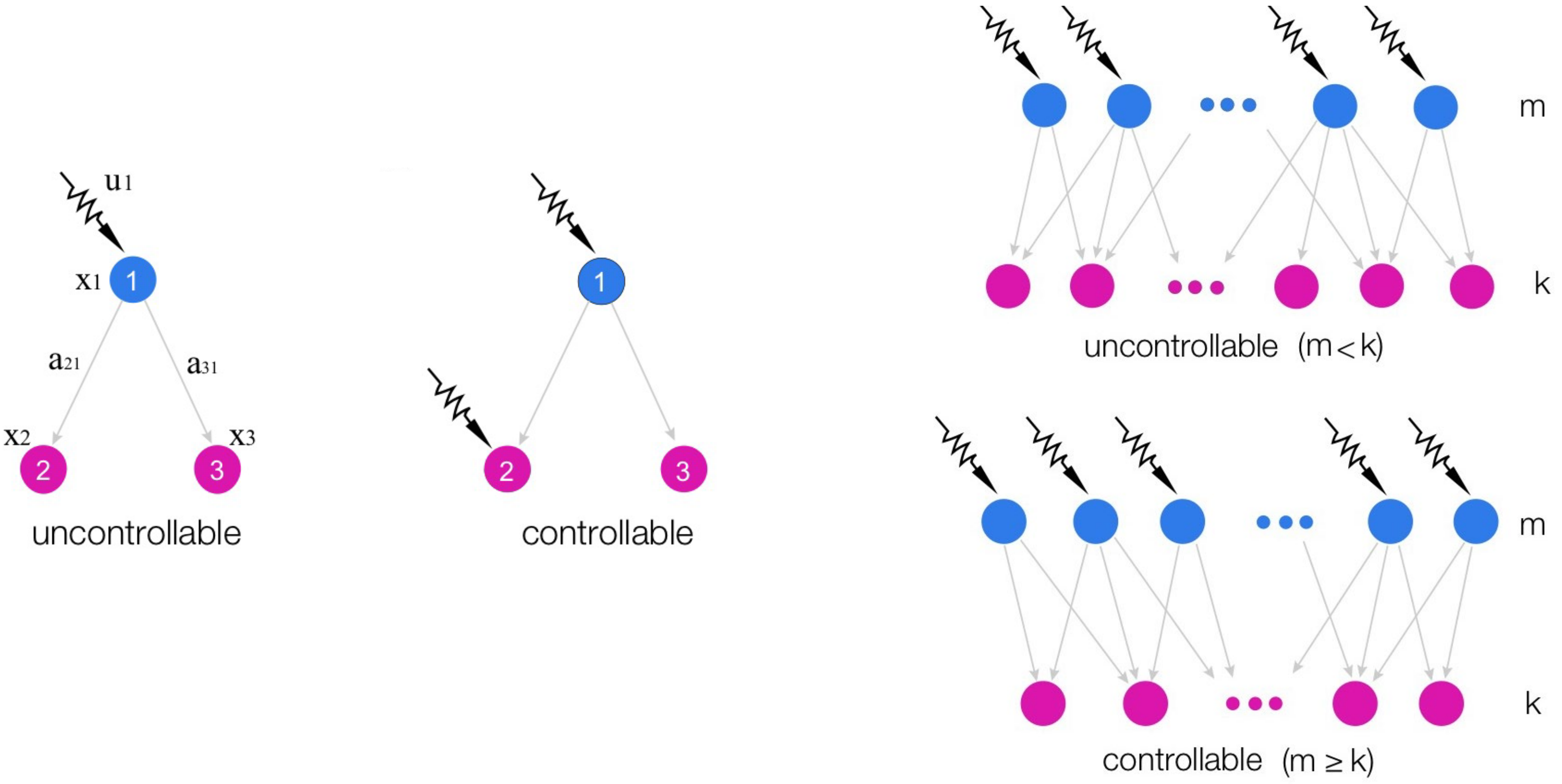
b

Control	Predicted Neuron Classes	Experimental Facts
Control Muscles	DA, DB	loss of backward/forward locomotion
	DD	uncoordinated motion
	AVA	uncoordinated motion
	VA, VB, VD, AS	likely loss of locomotion
Control Motor Neurons	AVA, AVB	uncoordinated motion
	AVD	loss of reversal response
	PVC	loss of reversal response

Yan, Vertes, Towlson, Chew, Walker, Schafer, Barabási, Nature (2017)

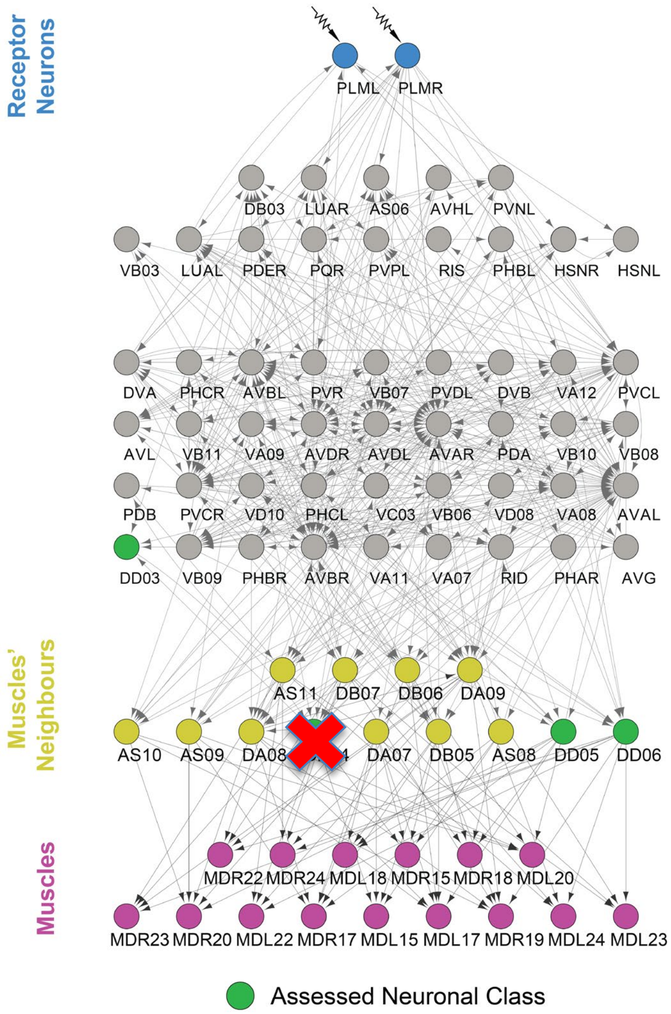
C Elegans: Light Body Touch Circuit





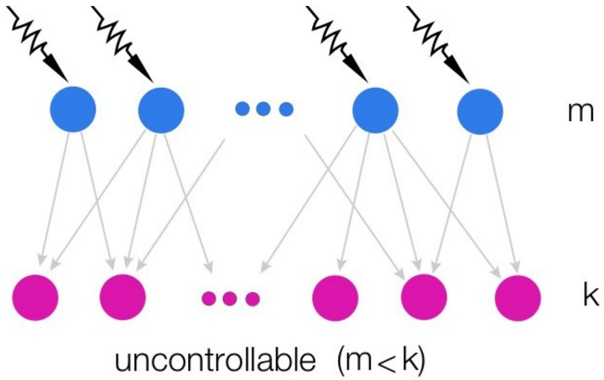
Yan, Vertes, Towlson, Chew, Walker, Schafer, Barabási, Nature (2017)

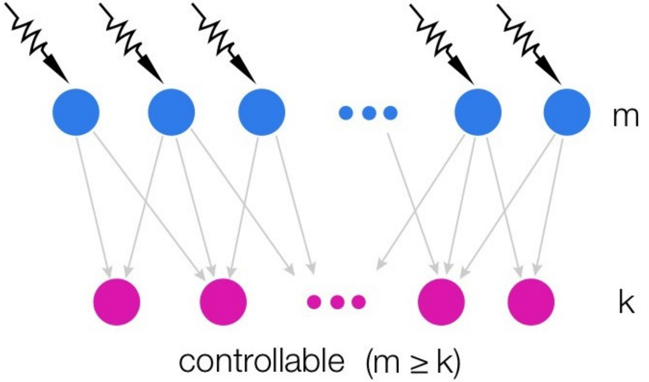
DD Class



9 → 8

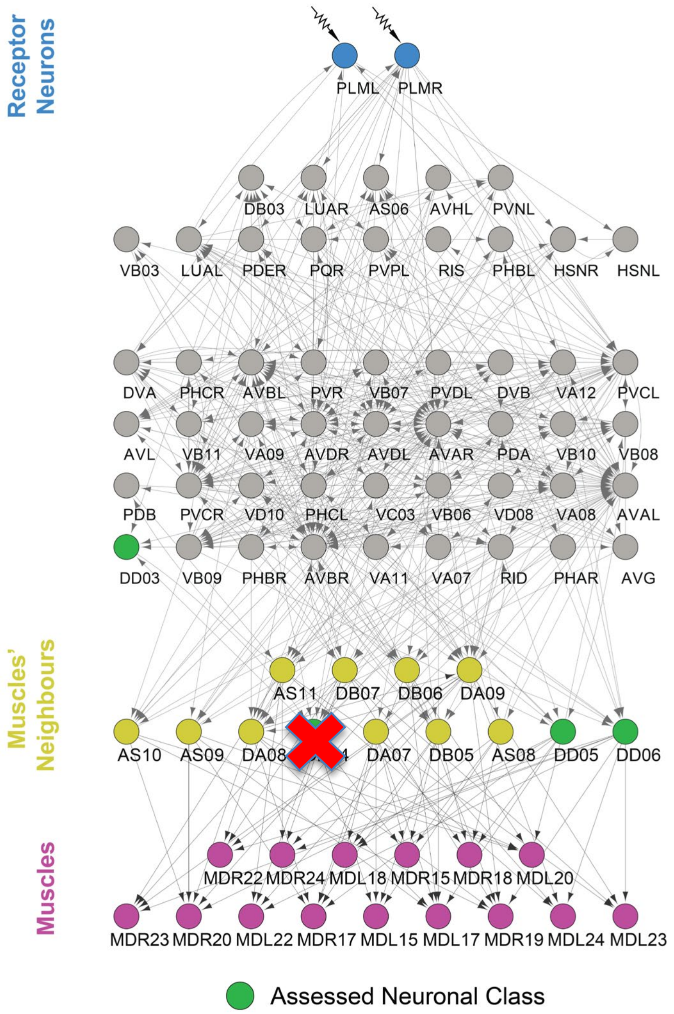
15 → 14





Yan, Vertes, Towlson, Chew, Walker, Schafer, Barabási, Nature (20

DD Class



The vast majority of neurons, if 'ablated' do not alter muscle controllability.

Of 279 neurons, only 12 neuronal classes affect muscle control.

9 → 8

15 → 14

Yan, Vertes, Towlson, Chew, Walker, Schafer, Barabási, Nature (20

Neuron classes indispensable for network control (predictions)

Control	Predicted Neuron Classes	Experimental Facts
Control Muscles	DA, DB	loss of backward/forward locomotion
	DD	uncoordinated motion
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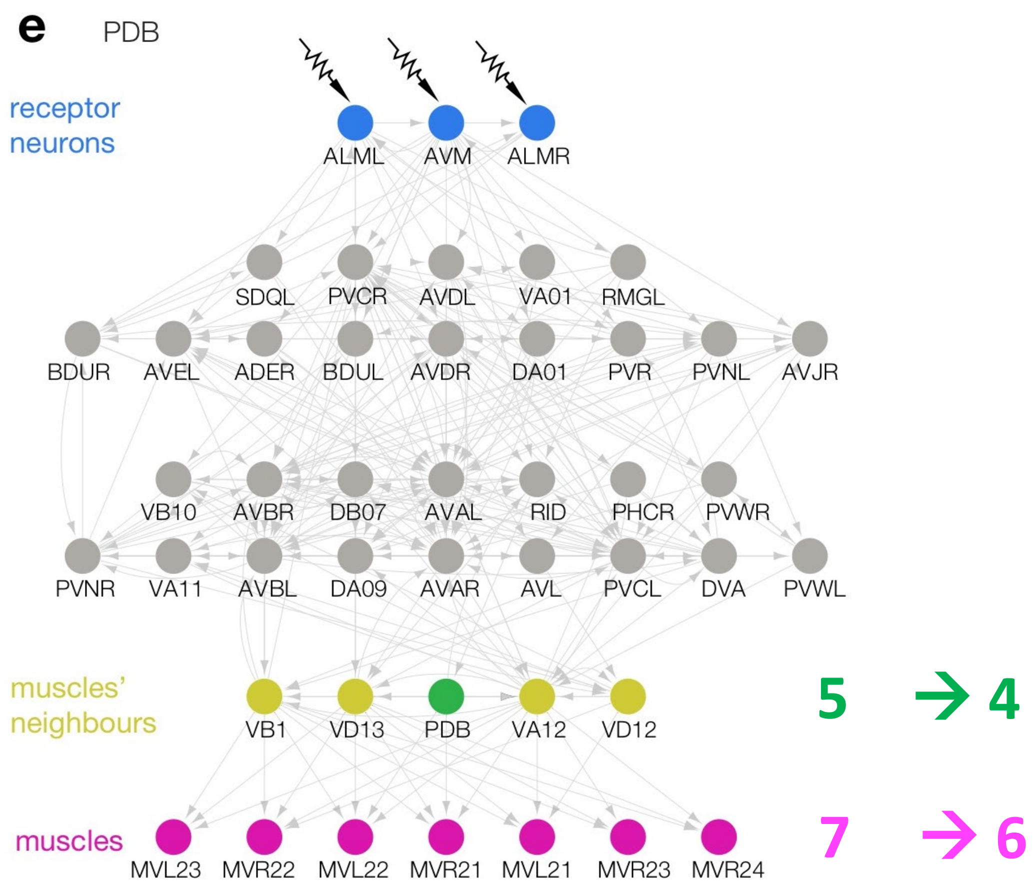
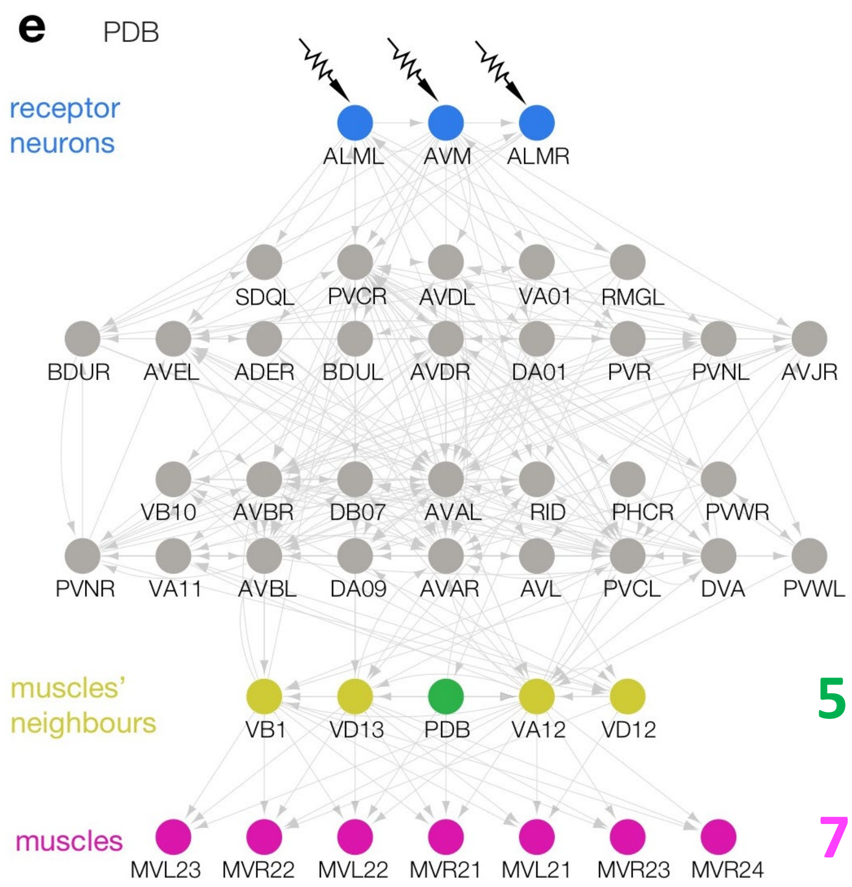
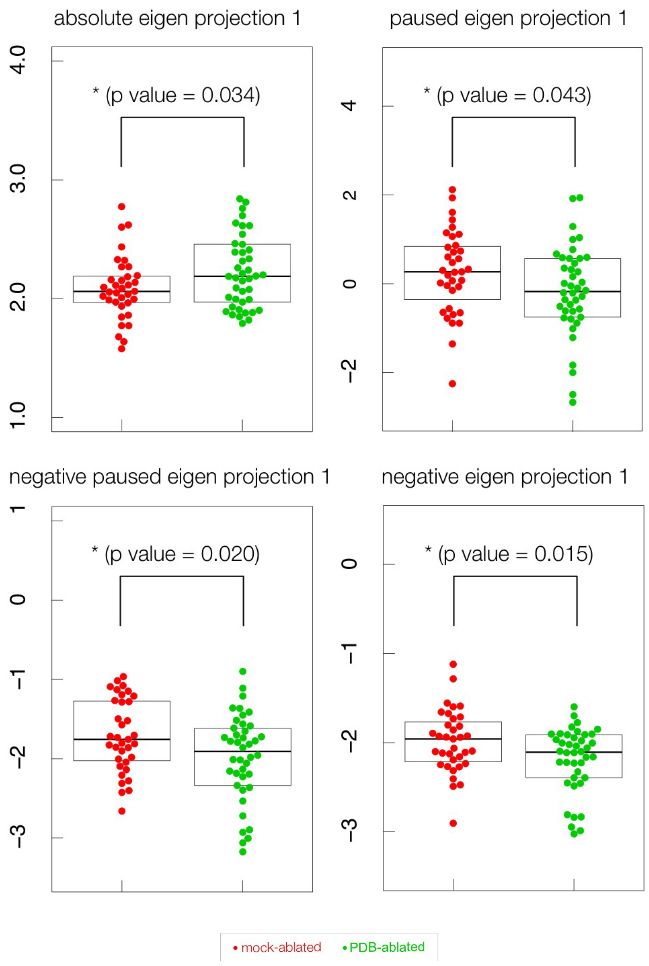


Yan, Vertes, Towlson, Chew, Walker, Schafer, Barabási, Nature (2017)



5 → 4

7 → 6



Yan, Vértes, Towlson, Chew, Walker, Schafer, Barabási, Nature (201

Ablation effects of neuron classes on locomotion

Control	Predicted Neuron Classes	Experimental Facts
Control Muscles	DA, DB	loss of backward/forward locomotion
	DD	uncoordinated motion
	AVA	uncoordinated motion
	VA, VB, VD, AS	likely loss of locomotion
	PDB	verified by new experiments
Control Motor Neurons	AVA, AVB	uncoordinated motion
	AVD	loss of reversal response
	PVC	loss of reversal response

Network Science: Structure determines function.

Structure matters:

Robustness

Spreading processes

Controllability

Observability

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Boston

Yang-Yu Liu
Harvard

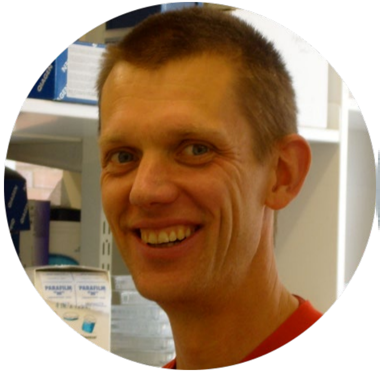
Jean-Jacques Slotine
MIT

Gang Yan

Emma Towlson

Univ. of Cambridge

Petra Vértes



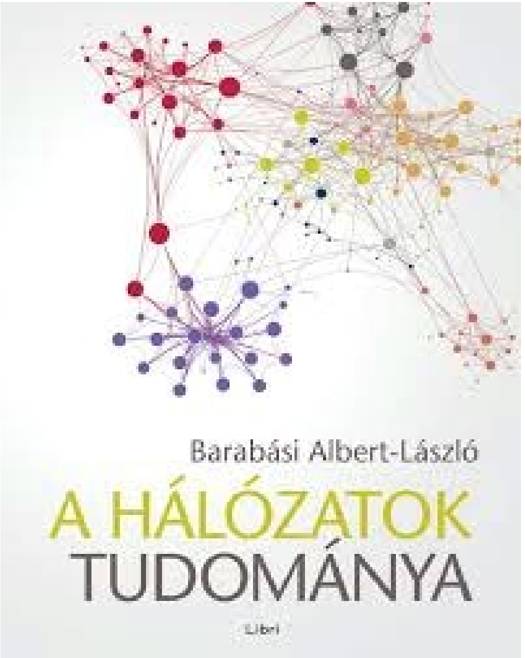
William R. Schafer

Yee Lian Chew

Denise S. Walker

We are looking for new lab members!

Talk to me if interested, or email: barabasi@gmail.com



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