TP Model Based $\mathcal{H}_\infty$ Control Design for the Heavy Vehicle Rollover Prevention Problem

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Abstract

This paper is focusing on rollover prevention to provide a heavy vehicle with the ability to resist overturning moments generated during cornering. A combined yaw-roll model including the roll dynamics of unsprung masses is studied. This model is nonlinear with respect to the velocity of the vehicle. In our model the velocity is handled as an LPV scheduling parameter. The Linear Parameter-Varying model of the heavy vehicle is transformed into a proper polytopic form by Tensor Product model transformation. The $\mathcal{H}_\infty$ gain-scheduling based control is immediately applied to this form for the stabilization. The effectiveness of the designed controller is demonstrated by numerical simulation.

1 Introduction

Roll stability is determined by the height of the center of mass, the track width and the kinematic properties of the suspensions. The problem with heavy vehicles is a relatively high mass center and narrow track width. When the vehicle is changing lanes or trying to avoid obstacles, the vehicle body rolls out of the corner and the center of mass shifts outboard of the centerline, and a destabilizing moment is created.

In the literature there are many papers with different approaches on the active control of the heavy vehicles to decrease the rollover risk. Three main schemes concerned with the possible active intervention into the vehicle dynamics have been proposed: active anti roll bars, active steering and active brake. The control design is usually based on linear time invariant models and linear approaches. The forward velocity is handled as a constant parameter in the yaw-roll model; however, velocity is an important parameter as far as roll stability is concerned [1–3].

In this paper, a combined yaw-roll model including the roll dynamics of unsprung masses is studied [1]. This model is nonlinear with respect to the velocity of the vehicle. Thus, in our model velocity is handled as an LPV scheduling parameter. The controller based on this Linear Parameter-Varying (LPV) model is adjusted continuously by measuring the vehicle velocity in real-time.

The control design is based on the following steps:

1. The Linear Parameter-Varying (LPV) dynamic model of the heavy vehicle model is given.

2. We apply the Tensor Product (TP) model transformation to transform the LPV model to a TP-type convex polytopic model form. The TP model transformation is a recently proposed automatically executable numerical method. It is developed for controller design involving LPV model representation and linear matrix inequality (LMI) based control design. It is capable of numerically generate different convex polytopic forms of LPV dynamic models, whereupon LMI-based

design is immediately be executable. It is important to emphasize that in many cases, the analytical derivation of these polytopic models needs very sophisticated and time consuming derivations

3. Then we apply the LMI theorems of $\mathcal{H}_\infty$ gain-scheduling to design the controller. The paper is organized as follows: Section 2 defines the LPV model form, its representation in TP model form. Section 3 introduces the LPV model of the heavy vehicle, Section 4 presents the TP model representation, then Section 5 describes the proposed controller design method. Section 6 shows the simulation results. Finally we give a short conclusion at the end of the paper.

2 Definitions

2.1 Nomenclature

- $\{a, b, \ldots\}$: scalar values;
- $\{\mathbf{a}, \mathbf{b}, \ldots\}$: vectors;
- $\{\mathbf{A}, \mathbf{B}, \ldots\}$: matrices;
- $\{\mathcal{A}, \mathcal{B}, \ldots\}$: tensors;
- $\mathbb{R}^{I_1 \times I_2 \times \cdots \times I_N}$:vector space of real valued $(I_1 \times I_2 \times \cdots \times I_N)$-tensors.
- Subscript defines lower order: for example, an element of matrix $\mathbf{A}$ at row-column number i, j is symbolized as $(\mathbf{A})_{i,j} = a_{i,j}$. Systematically, the ith column vector of $\mathbf{A}$ is denoted as $\mathbf{a}_i$, i.e. $\mathbf{A} = \begin{bmatrix} \mathbf{a}_1 & \mathbf{a}_2 & \cdots \end{bmatrix}$.
- $(\cdot)_{i,j,n}, \ldots$: are indices;
- $(\cdot)_{I,J,N}, \ldots$: index upper bound: for example: $i = 1..I$, $j = 1..J$, $n = 1..N$ or $i_n = 1..I_n$.
- $\mathbf{A}^+$: the pseudo inverse of matrix $\mathbf{A}$.
- $\mathbf{A}_{(n)}$: n-mode matrix of tensor $\mathcal{A} \in \mathbb{R}^{I_1 \times I_2 \times \cdots \times I_N}$;
- $\mathcal{A} \times_n \mathbf{U}$: n-mode matrix-tensor product;
- $rank_n(\mathcal{A})$: n-mode rank of tensor $\mathcal{A}$, that is $rank_n(\mathcal{A}) = rank(\mathbf{A}_{(n)})$;
- $\mathcal{A} \boxtimes_{n=1}^{N} \mathbf{U}_n$: multiple product as $\mathcal{A} \times_1 \mathbf{U}_1 \times_2 \mathbf{U}_2 \times_3 .. \times_N \mathbf{U}_N$;

Detailed discussion of tensor notations and operations is given in [4].

2.2 Linear Parameter-Varying state-space model

Consider the following parameter-varying state-space model:

$$\begin{aligned} \dot{\mathbf{x}}(t) &= \mathbf{A}(\mathbf{p}(t))\mathbf{x}(t) + \mathbf{B}(\mathbf{p}(t))\mathbf{u}(t), \\ \mathbf{y}(t) &= \mathbf{C}(\mathbf{p}(t))\mathbf{x}(t) + \mathbf{D}(\mathbf{p}(t))\mathbf{u}(t), \end{aligned} \tag{1}$$

with input $\mathbf{u}(t)$, output $\mathbf{y}(t)$ and state vector $\mathbf{x}(t)$. The system matrix

$$\mathbf{S}(\mathbf{p}(t)) = \begin{pmatrix} \mathbf{A}(\mathbf{p}(t)) & \mathbf{B}(\mathbf{p}(t)) \\ \mathbf{C}(\mathbf{p}(t)) & \mathbf{D}(\mathbf{p}(t)) \end{pmatrix} \in \mathbb{R}^{O \times I} \tag{2}$$

is a parameter-varying object, where $\mathbf{p}(t) \in \Omega$ is time varying N-dimensional parameter vector, and is an element of the closed hypercube

$$\Omega = [a_1, b_1] \times [a_2, b_2] \times \cdots \times [a_N, b_N] \subset \mathbb{R}^N.$$

$\mathbf{p}(t)$ can also include some elements of $\mathbf{x}(t)$ in which case the model is a quasi-Linear Parameter-Varying model.

2.3 Finite element TP model form of quasi LPV models

$\mathbf{S}(\mathbf{p}(t))$ is given for any parameter $\mathbf{p}(t)$ as the combination of LTI system matrices $\mathbf{S}_r$, $r = 1, \ldots, R$. Matrices $\mathbf{S}_r$ are also called *vertex systems*. Therefore, one can define weighting functions $w_r(\mathbf{p}(t)) \in [0, 1] \subset \mathbb{R}$ such that matrix $\mathbf{S}(\mathbf{p}(t))$ can be expressed as parameter dependent weighted combination of system matrices $\mathbf{S}_r$. The explicit form of the TP model in terms of tensor product becomes:

$$\begin{pmatrix} \dot{\mathbf{x}}(t) \\ \mathbf{y}(t) \end{pmatrix} = \mathcal{S} \mathop{\boxtimes}_{n=1}^{N} \mathbf{w}_n(p_n(t)) \begin{pmatrix} \mathbf{x}(t) \\ \mathbf{u}(t) \end{pmatrix}. \tag{3}$$

Here, row vector $\mathbf{w}_n(p_n) \in \mathbb{R}^{I_n}$ $n = 1, \ldots, N$ contains the one variable weighting functions $w_{n,i_n}(p_n)$. Function $w_{n,j}(p_n(t)) \in [0, 1]$ is the j-th one variable weighting function defined on the n-th dimension of Ω, and $p_n(t)$ is the n-th element of vector $\mathbf{p}(t)$. I_n ($n = 1, \ldots, N$) is the number of the weighting functions used in the n-th dimension of the parameter vector $\mathbf{p}(t)$. The $(N + 2)$-dimensional tensor

$$\mathcal{S} \in \mathbb{R}^{I_1 \times I_2 \times \cdots \times I_N \times O \times I}$$

is constructed from LTI vertex systems $\mathbf{S}_{i_1 i_2 \ldots i_N} \in \mathbb{R}^{O \times I}$. Finite element TP model means that the LTI components of the model is bounded. For further details we refer to [5–7].

2.4 Convex TP model form of qLPV model

The convex combination of the LTI vertex systems is ensured by the conditions:

Definition 1 *The TP model (3) is convex if:*

$$\forall n \in [1, N], i, p_n(t) : w_{n,i}(p_n(t)) \in [0, 1]; \tag{4}$$

$$\forall n \in [1, N], p_n(t) : \sum_{i=1}^{I_n} w_{n,i}(p_n(t)) = 1. \tag{5}$$

This simply means that $\mathbf{S}(\mathbf{p}(t))$ is within the convex hull of the LTI vertex systems $\mathbf{S}_{i_1 i_2 \ldots i_N}$ for any $\mathbf{p}(t) \in \Omega$.

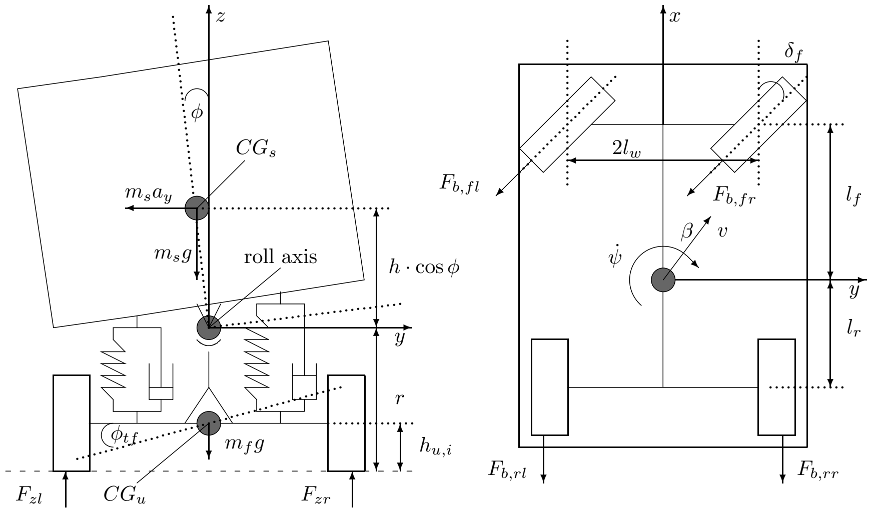

Figure 1: Rollover vehicle model

2.5 Link to the polytopic form

In order to have a direct link between the TP model form and the polytop formula, we define the following index transformation:

Definition 2 (Index transformation) *Let*

$$\mathbf{S}_r = \begin{pmatrix} \mathbf{A}_r & \mathbf{B}_r \\ \mathbf{C}_r & \mathbf{D}_r \end{pmatrix} = \mathbf{S}_{i_1,i_2,\ldots,i_N},$$

where $r = \text{ordering}(i_1, i_2, \ldots, i_N)$ *(*$r = 1 \ldots R = \prod_n I_n$*). The function "ordering" results in the linear index equivalent of an N dimensional array's index* $i_1, i_2, \ldots, i_N$*, when the size of the array is* $I_1 \times I_2 \times \cdots \times I_N$*. Let the weighting functions be defined according to the sequence of r:*

$$w_r(\mathbf{p}(t)) = \prod_n w_{n,i_n}(p_n(t)).$$

By the above index transformation one can write the TP model (3) in the typical polytopic form of:

$$\mathbf{S}(\mathbf{p}(t)) = \sum_{r=1}^{R} w_r(\mathbf{p}(t))\mathbf{S}_r. \tag{6}$$

Remark: Note that the LTI systems $\mathbf{S}_r$ and $\mathbf{S}_{i_1,i_2,\ldots,i_N}$ are the same, only their indices are modified, therefore the convex hull defined by the LTI systems is the same in both forms.

3 LPV model of the heavy vehicles

Figure 1 illustrates the combined yaw-roll dynamics of the vehicle modeled by a three-body system, in which m_s is the sprung mass, $m_{u,f}$ is the unsprung mass at the

$$mv(\dot{\beta} + \dot{\psi}) - m_s h\ddot{\phi} = Y_\beta \beta + Y_{\dot{\psi}} \dot{\psi} + Y_{\delta_f} \delta_f \tag{7}$$

$$-I_{xz}\ddot{\phi} + I_{zz}\ddot{\psi} = N_\beta \beta + N_{\dot{\psi}} \dot{\psi} + N_{\delta_f} \delta_f + \frac{l_w}{2} \Delta F_b \tag{8}$$

$$\left(I_{xx} + m_s h^2\right)\ddot{\phi} - I_{xz}\ddot{\psi} = m_s g h \phi + m_s v h(\dot{\beta} + \dot{\psi}) - k_f(\phi - \phi_{t,f}) - b_f(\dot{\phi} - \dot{\phi_{t,f}}) - k_r(\phi - \phi_{t,r}) - b_r(\dot{\phi} - \dot{\phi_{t,r}}) \tag{9}$$

$$-r\left(Y_{\beta,f}\beta + Y_{\dot{\psi},f}\dot{\psi} + Y_{\delta_f}\delta_f\right) = m_{u,f} v(r - h_{u,f})(\dot{\beta} + \dot{\phi}) + m_{u,f} g h_{u,f} \phi_{t,f} - k_{t,f}\phi_{t,f} + k_f(\phi - \phi_{t,f}) + b_f(\dot{\phi} - \dot{\phi_{t,f}}) \tag{10}$$

$$-r\left(Y_{\beta,r}\beta + Y_{\dot{\psi},r}\dot{\psi}\right) = m_{u,r} v(r - h_{u,r})(\dot{\beta} + \dot{\psi}) - m_{u,r} g h_{u,r} \phi_{t,r} - k_{t,r}\phi_{t,r} + k_r(\phi - \phi_{t,r}) + b_r(\dot{\phi} - \dot{\phi}_{t,r}) \tag{11}$$

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front including the front wheels and axle, and $m_{u,r}$ is the unsprung mass at the rear with the rear wheels and axle.

The conditions of yaw-roll model used in control design are considered. It is assumed that the roll axis is parallel to the road plane in the longitudinal direction of the vehicle at a height r above the road. The location of the roll axis depends on the kinematic properties of the front and rear suspensions. The axles of the vehicle are considered to be a single rigid body with flexible tires that can roll around the center of the roll. The tire characteristics in the model are assumed to be linear. The effect caused by pitching dynamics in the longitudinal plane can be ignored in the handling behavior of the vehicle. The effects of aerodynamic inputs (wind disturbance) and road disturbances are also ignored. The roll motion of the sprung mass is damped by suspensions and stabilizers with the effective roll damping coefficients $b_{s,i}$ and roll stiffness $k_{s,i}$.

In the vehicle modeling the the lateral dynamics, the yaw moment, the roll moment of the sprung and the unsprung masses are taken into consideration. The symbols of the yaw-roll model are found in Table 1. The motion differential equations are the following.

Here, the tire coefficients are given by: $Y_\beta = -(C_f + C_r)\mu$, $N_\beta = (C_r l_r - C_f l_f)\mu$, $Y_{\dot{\psi}} = (C_r l_r - C_f l_f)\frac{\mu}{v}$, $N_{\dot{\psi}} = -(C_f l_f^2 + C_r l_r^2)\frac{\mu}{v}$, $Y_{\delta_f} = C_f \mu$, $N_{\delta_f} = C_f l_f \mu$. These equations can be expressed in a state space representation. Let the state vector be the following:

$$\mathbf{x} = \begin{bmatrix} \beta & \dot{\psi} & \phi & \dot{\phi} & \phi_{t,f} & \phi_{t,r} \end{bmatrix}^T. \tag{12}$$

The system states are the side slip angle of the sprung mass β, the yaw rate $\dot{\psi}$, the roll angle ϕ, the roll rate $\dot{\phi}$, the roll angle of the unsprung mass at the front axle $\phi_{t,f}$ and at the rear axle $\phi_{t,r}$ respectively. Then the state equation arises in the following form

$$\mathbf{E}(\mathbf{p})\dot{\mathbf{x}} = \mathbf{A}_0(\mathbf{p})\mathbf{x} + \mathbf{B}_{1,0}\delta_f + \mathbf{B}_{2,0}u, \tag{13}$$

where the matrices are defined by equations (18) and (19). The parameter of the system is the forward velocity

$$\mathbf{p} = v.$$

Equation (13) can be rewritten as

$$\dot{\mathbf{x}} = \mathbf{A}(\mathbf{p})\mathbf{x} + \mathbf{B}_1(\mathbf{p})\delta_f + \mathbf{B}_2(\mathbf{p})u, \tag{14}$$

$$\mathbf{E}(v)=\begin{bmatrix} mv & 0 & 0 & -m_s h & 0 & 0\\ 0 & I_{zz} & 0 & -I_{xz} & 0 & 0\\ -m_s vh & -I_{xz} & 0 & I_{xx}+m_s h^2 & -b_f & -b_r\\ m_{u,f}v(r-h_{u,f}) & 0 & 0 & 0 & -b_f & 0\\ m_{u,r}v(r-h_{u,r}) & 0 & 0 & 0 & 0 & -b_r\\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix},\quad \mathbf{B}_{1,0}=\begin{bmatrix} Y_{\delta_f}\\ N_{\delta_f}\\ 0\\ rY_{\delta_f}\\ 0\\ 0\end{bmatrix},\quad \mathbf{B}_{2,0}=\begin{bmatrix}0\\ l_w/2\\ 0\\ 0\\ 0\\ 0\end{bmatrix} \tag{18}$$

$$\mathbf{A}_0(v)=\begin{bmatrix} Y_\beta & Y_{\dot\psi}-mv & 0 & 0 & 0 & 0\\ N_\beta & N_{\dot\psi} & 0 & 0 & 0 & 0\\ 0 & m_s hv & m_s gh-k_f-k_r & -b_f-b_r & k_f & k_r\\ -rY_{\beta,f} & rY_{\dot\psi,f}-m_{u,f}v(r-h_{u,f}) & -k_f & -b_f & k_f+k_{t,f}-m_{u,f}gh_{u,f} & 0\\ -rY_{\beta,r} & -rY_{\dot\psi,r}-m_{u,r}v(r-h_{u,r}) & -k_r & -b_r & 0 & k_r+k_{t,r}-m_{u,r}gh_{u,r}\\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \tag{19}$$

where

$$\mathbf{A}(\mathbf{p})=\mathbf{E}^{-1}(\mathbf{p})\mathbf{A}_0(\mathbf{p}) \tag{15}$$

$$\mathbf{B}_1(\mathbf{p})=\mathbf{E}^{-1}(\mathbf{p})\mathbf{B}_{1,0} \tag{16}$$

$$\mathbf{B}_2(\mathbf{p})=\mathbf{E}^{-1}(\mathbf{p})\mathbf{B}_{2,0} \tag{17}$$

The δ_f is the front wheel steering angle. The control input is the difference of brake forces between the left and the right hand side of the vehicle.

$$u=\Delta F_b$$

The control input provided by the brake system generates a yaw moment, which affects the lateral tire forces directly. In our case it is assumed that the brake force difference ΔF_b provided by the controller is applied to the rear axle. This means that only one wheel is decelerated at the rear axle. This declaration is caused by an appropriate yaw moment. In our case the difference between the brake forces can be given $\Delta F_b = F_{b,rl} - F_{b,rr}$. This assumption does not restrict the implementation of the controller because it is possible that the control action be distributed on the front and the rear wheels at one of the two sides. The reason for distributing the control force to front and rear wheels is to minimize the wear of the tires. In this case a logic is required which calculates the brake forces for the wheels.

In the equation (14) the $\mathbf{A}(\mathbf{p})$ matrix depends on the forward velocity of the vehicle nonlinearly. In the linear yaw-roll model the velocity is considered a constant parameter. However, forward velocity is an important stability parameter so that it is considered to be a variable of the motion. Hence the throttle is constant during a lateral maneuver and the forward velocity depends on only the brake forces. The differential equation for forward velocity is

$$m\dot v = -F_{b,rl} - F_{b,rr}.$$

4 Convex TP model of the heavy vehicle model

In this section we derive the TP model of the LPV model (14) by TP model transformation. We execute the TP model transformation over M ($M = 137$) points grid net in the $v \in \Omega = [40\text{km/h}, 120\text{km/h}]$ domain. We have applied the MATLAB Tensor Product Model Transformation Toolbox (TPTool) (`http:\\tptool.sztaki.hu`) for

Table 1: Symbols of the yaw-roll model

Symbols	Description
h	height of CG of sprung mass from roll axis
$h_{u,i}$	height of CG of unsprung mass from ground
r	height of roll axis from ground
a_y	lateral acceleration
β	side-slip angle at center of mass
ψ	heading angle
$\dot{\psi}$	yaw rate
ϕ	sprung mass roll angle
$\phi_{t,i}$	unsprung mass roll angle
δ_f	steering angle
u_i	control torque
C_i	tire cornering stiffness
F_{zi}	total axle load
R_i	normalized load transfer
k_i	suspension roll stiffness
b_i	suspension roll damping
$k_{t,i}$	tire roll stiffness
I_{xx}	roll moment of inertia of sprung mass
I_{xz}	yaw-roll product of inertial of sprung mass
I_{zz}	yaw moment of inertia of sprung mass
l_i	length of the axle from the CG
l_w	vehicle width
μ	road adhesion coefficient

the TP model transformation to determine the LTI systems ($\mathbf{S}_i$) and the weightings ($\mathbf{w}_i$). The TP model transformation shows that the LPV model of the heavy vehicle model can exactly be given by the convex combination of 3 LTI vertex systems:

$$\mathbf{S}(\mathbf{p}(t)) = \sum_{i=1}^{3} w_i(\mathbf{p}(t))\mathbf{S}_i \tag{20}$$

This polytopic for is not unique and the type of the weightings can considerably influence the feasibility of LMI constraints and the resulting controller control performance [8]. In order to relax the feasibility of the LMI conditions, we define the tight convex hull of the LPV model via generating close to NO type weighting functions by the TP model transformation.

Definition 3 (NO – Normality) *Vector* $\mathbf{w}(p)$*, containing weighting functions* $w_i(p)$ *is NO if they satisfy conditions (4) and (5), and the maximum values of the weighting functions are one. We say* $w_i(p)$ *is close to NO if it satisfies conditions (4) and (5), and the maximum values of the weighting functions are close to one.*

Its geometrical meaning is that we determine a convex hull in such a way that as many of the LTI systems as possible are equal to the $\mathbf{S}(\mathbf{p})$ over some $\mathbf{p} \in \Omega$ and the rest of the LTIs are close to $\mathbf{S}(\mathbf{p})$ (in the sense of $\mathcal{L}_2$ norm). The resulting weightings are depicted in Figure 2.

5 Control design

The aim of the rollover prevention is to provide the vehicle with the ability to resist overturning moments generated during cornering. Roll stability is determined

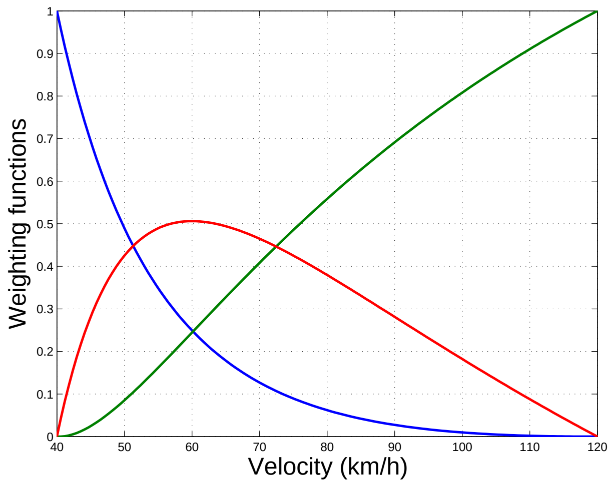


Figure 2: Close to NO weighting functions of the TP model

by the height of the center of mass, the track width and the kinematic properties of the suspensions. The problem with heavy vehicles is a relatively high mass center and narrow track width. When the vehicle is changing lanes or trying to avoid obstacles, the vehicle body rolls out of the corner and the center of mass shifts outboard of the centerline, and a destabilizing moment is created.

In this section we utilize the above obtained convex model for the stabilization control of the heavy vehicle model. We seek an LPV controller of the form

$$\begin{aligned} \dot{\mathbf{x}}_K &= \mathbf{A}_K(\mathbf{p}(t))\mathbf{x}_K + \mathbf{B}_K(\mathbf{p}(t))y \\ u &= \mathbf{C}_K(\mathbf{p}(t))\mathbf{x}_K + \mathbf{D}_K(\mathbf{p}(t))y \end{aligned}$$

where

$$\begin{pmatrix} \mathbf{A}_K(\mathbf{p}(t)) & \mathbf{B}_K(\mathbf{p}(t)) \\ \mathbf{C}_K(\mathbf{p}(t)) & \mathbf{D}_K(\mathbf{p}(t)) \end{pmatrix} = \mathbf{K}(\mathbf{p}(t)) = \sum_{i=1}^{3} w_i(\mathbf{p}(t))\mathbf{K}_i$$

with the same $w_i(\mathbf{p}(t))$ weighting functions as in the model representation (20), $\mathbf{x}_K$ is the internal state of the controller, the measured output of the model is the yaw rate so $y = \dot{\psi}$ and $u = \Delta F_b$ is the control signal. To design a suitable $\mathbf{K}(\mathbf{p})$ for the given polytopic model the self-scheduled $\mathcal{H}_\infty$ controller design method [9, 10] was used.

The closed-loop interconnection structure, which includes the feedback structure of the model $\mathbf{P}$ and controller $\mathbf{K}$, is shown in Figure 3. In the diagram, $\mathbf{d}$, $\mathbf{u}$, $\mathbf{y}$ and $\mathbf{z}$ are the disturbance, the control input, the measured output and the performance output, respectively.

A standard feedback configuration with weights strategy is illustrated in Figure 4. In the diagram $\mathbf{u}$ is the control input, $\mathbf{y}$ is the measured output, $\mathbf{z}_p$ is the performance output, $\mathbf{z}_u$ and $\mathbf{z}_y$ are performances at the input and the output, $\mathbf{w}$ is the disturbance, $\mathbf{n}$ is the measurement noise. The aim of the weighting function $\mathbf{W}_p$ is to define the performance specifications. They can be considered as penalty functions, i.e. weights should be large in a frequency range where small signals are desired and small where large performance outputs can be tolerated. $\mathbf{W}_u$ and $\mathbf{W}_y$ may be used to reflect some restrictions on the

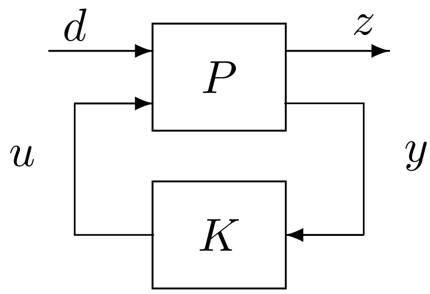


Figure 3: The general $P - K$ structure for control design

actuator and on the output signals. The purpose of the weighting functions $\mathbf{W}_w$ and $\mathbf{W}_n$ is to reflect the disturbance and sensor noises. The disturbance and the performances in the general $\mathbf{P} - \mathbf{K}$ structure are $\mathbf{d} = \begin{bmatrix}\mathbf{w} & \mathbf{n}\end{bmatrix}^T$ and $\mathbf{z} = \begin{bmatrix}\mathbf{z}_u & \mathbf{z}_y & \mathbf{z}_p\end{bmatrix}^T$.

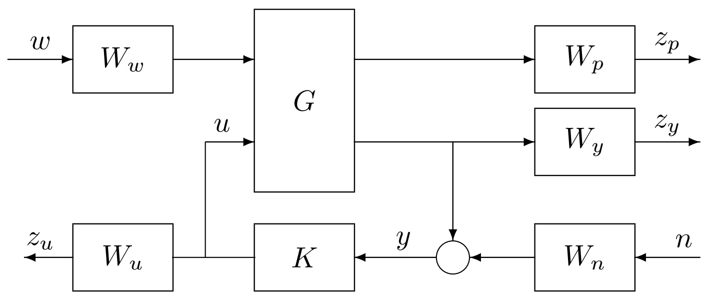


Figure 4: The standard feedback configuration with weights

The augmented plant includes the parameter dependent vehicle dynamics and the weighting functions, which are defined in the following form:

$$\begin{bmatrix}\mathbf{z}\\ \mathbf{y}\end{bmatrix} = \mathbf{P}(\mathbf{p})\begin{bmatrix}\mathbf{d}\\ \mathbf{u}\end{bmatrix}. \tag{21}$$

In a Linear Parameter Varying (LPV) model $\mathbf{p}$ denotes the scheduling variable.

The closed-loop system $\mathbf{M}(\mathbf{p})$ is given by a lower linear fractional transformation (LFT) structure:

$$\mathbf{M}(\mathbf{p}) = \mathcal{F}_\ell(\mathbf{P}(\mathbf{p}), \mathbf{K}(\mathbf{p})), \tag{22}$$

where $\mathbf{K}(\mathbf{p})$ also depends on the scheduling variable $\mathbf{p}$. The goal of the control design is to minimize the induced $\mathcal{L}_2$ norm of an *LPV* system $\mathbf{M}(\mathbf{p})$, with zero initial conditions, which is given by

$$||\mathbf{M}(\mathbf{p})||_\infty = \sup_{\mathbf{p}\in\Omega} \sup_{||\mathbf{w}||_2\neq 0, \mathbf{w}\in\mathcal{L}_2} \frac{||\mathbf{z}||_2}{||\mathbf{w}||_2}. \tag{23}$$

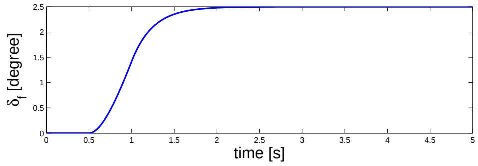
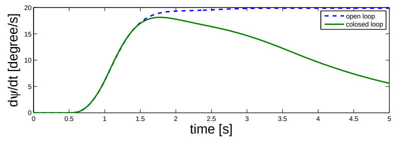
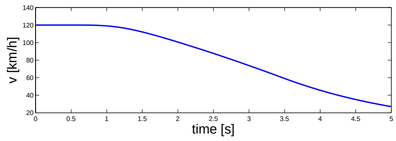


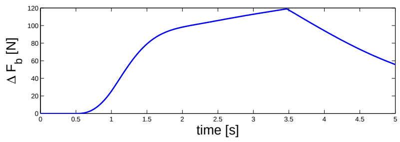
Figure 5: Disturbance signal: Steering angle



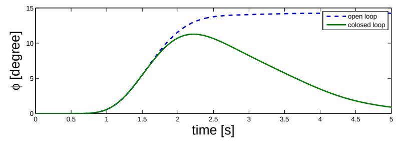
(a) Measured signal: yaw rate



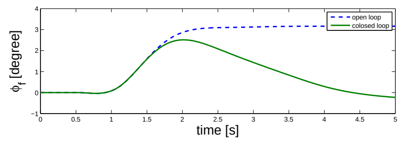
(b) Forward velocity



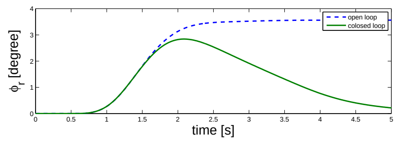
(c) Control signal: difference of left and right brake forces



(d) Sprung mass roll angle



(e) Front unsprung mass roll angle



(f) Rear unsprung mass roll angle

Figure 6: Simulation results

6 Simulation results

At the initial configuration of the simulation the system had a velocity of $v =$ 120km/h and all the state variables were set to zero. Then a sharp maneuver was simulated as seen in Figure 5 which describes the situation when the truck performs an obstacle avoidance. The goal is to stabilize the truck by braking the rear wheels.

The results can be seen on Figure 6. From the results it can be seen that the rollover likelihood can be reduced efficiently with active breaking.

7 Conclusion

In this paper we investigated the rollover prevention problem for the heavy vehicle model. The novelty of this paper is that the convex polytopic representation of the vehicle model was generated by Tensor Product Model Transformation. An $\mathcal{H}_\infty$ gain-scheduling method was used to determined the controller to solve the rollover problem. Simulations present the performance of this controller design method.

Acknowledgement

This research was supported by the Hungarian National Found OTKA No. F 049838, Dr. Baranyi is supported by János Bolyai Postdoctoral Scholarship.

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