Applying Fuzzy Rule Interpolation for the Task of Controlling Guidance and Obstacle Avoidance Behaviour of a Robot

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Abstract: Several Fuzzy Rule Interpolation (FRI) techniques have limitations from the direct application point of view, for example their applicability is limited to the one dimensional case, or they can be defined only based on the two closest surrounding rules of the actual observation. This is the reason why relatively few FRI methods can be found among the practical fuzzy rule based applications. With the application of FRI methods sparse rule bases can be used, which substantially simplify the construction of fuzzy rule bases, because FRI methods can provide reasonable (interpolated) conclusions even if none of the existing rules fires under the gathered observation. Compared to the classical fuzzy CRI (compositional rule of inference), by eliminating the derivable rules, the number of the fuzzy rules needed in the rule base could be dramatically reduced. This paper provides a brief overview of several FRI methods and in more details an application oriented simple and quick FRI method "FIVE" will be introduced. For the demonstration of the benefits of the interpolation-based fuzzy reasoning as systematic approach, a robot guidance application is presented, where the robot is able to cycle through defined waypoints while avoiding collision with obstacles and walls. All of the controlling parts were accomplished with fuzzy rule bases of the "FIVE" FRI method.

Keywords: fuzzy rule interpolation, FIVE FRI, robot guidance

1 Introduction

Traditional fuzzy reasoning methods (e.g. the Zadeh-Mamdani compositional rule of inference (CRI) and the Takagi-Sugeno reasoning method) are demanding "complete rule bases", and hence the construction of a classical rule base requires extensive work to define all the possible rules. In contrary, the application of fuzzy rule interpolation (FRI) methods, where the derivable rules are missing on purpose (as FRI methods are capable of providing reasonable (interpolated) conclusions even if none of the defined rules fire under the current observation) allows to avoid a considerable amount of unnecessary work during construction of the rule bases, because the rule base of an FRI controller could contain the most significant fuzzy rules alone. On the other hand most of the FRI methods are sharing the burden of high computational demand, e.g. the task of searching for the two closest surrounding rules to the observation, and calculating the conclusion at least in some characteristic α-cuts. Additionally in some methods interpreting the gained fuzzy conclusion is also not straightforward [7] even if there have been a lot of efforts to rectify the interpretability of the interpolated fuzzy conclusion [17]. In [1] Baranyi *et al.* give a comprehensive overview of the recent existing FRI methods. Moreover some of the FRI methods need special extension for the multidimensional case (e.g. [2]-[3]) because they are originally defined for one dimensional input space. In [22] Wong et al. gave a comparative overview of the multidimensional input space capable FRI methods and in [2] Jenei introduced a way for axiomatic treatment of the FRI methods. In [14] Johanyák *et al.* introduces an automatic way for direct sparse fuzzy rule base generation based on given input-output data. Many of these methods are hardly suitable for real-time applications due to the high computational demand (notably the search for the two closest surrounding rules to an arbitrary observation in the multidimensional antecedent space). Some FRI methods, e.g. LESFRI [19] or the method introduced by Jenei *et al.* in [3], eliminate the search for the two closest surrounding rules by taking all the rules into consideration, and therefore speed up the reasoning process. Eliminating the searching process for the two closest surrounding rules by taking all the rules into consideration, hence speeding up the reasoning can be achieved by some FRI methods (e.g. LESFRI [19] or the method introduced by Jenei *et al.* in [3]). An application oriented aspect of the FRI emerges in the concept of "FIVE" (Fuzzy Interpolation based on Vague Environment). In the followings after a brief introduction of several FRI techniques, the method "FIVE" will be introduced in more details.

2 A Short Overview of Several FRI Techniques

Kóczy and Hirota [5] published one of the first FRI technique usually referred as the *KH method*, which is applicable to convex and normal fuzzy (CNF) sets. The conclusion is determined by its α-cuts in such a way that the ratio of distances between the conclusion and the consequents should be identical with the ones between the observation and the antecedents for all important α-cuts. It is shown in, e.g. in [7], [8] that the conclusion of the KH method is not always directly interpretable as a fuzzy set. Many alternative solutions emerged motivated by this drawback. Vass, Kalmár and Kóczy [20] proposed a modification (*VKK method*), where the conclusion is computed based on the distance of the centre points and the widths of the α-cuts, instead of the lower and upper distances. The VKK method decreases the applicability limit of KH method, but does not eliminate it completely. The technique cannot be applied if any of the antecedent sets is singleton, because the width of the antecedent's support must be nonzero. Another modification of KH is the modified α-cut based interpolation (*MACI*) method [17], which alleviates completely the abnormality problem. The main idea of MACI is to transform fuzzy sets of the input and output universes to such a space where abnormality is excluded, then computes the conclusion there, which is finally transformed back to the original space. MACI uses vector representation of fuzzy sets and originally applicable to CNF sets [23]. These latter conditions (CNF sets) can be relaxed, but it considerably increases the computational need of this interpolation method [18]. MACI is one of the most frequently used FRI methods [22], since it preserves advantageous computational and approximate nature of KH, while it excludes its abnormality. A fuzzy interpolation technique applicable to CNF sets was proposed by Kóczy *et al.* [6], which is called conservation of "relative fuzziness" (*CRF*) method, meaning that the left (right) fuzziness of the approximated conclusion in proportion to the flanking fuzziness of the neighbouring consequent should be the same as the (left) right fuzziness of the observation in proportion to the flanking fuzziness of the neighbouring antecedent. Another improved fuzzy interpolation technique for multidimensional input spaces (*IMUL*) was proposed in [21], and described in details in [22]. IMUL applies a combination of CRF and MACI methods, and mixes their advantages. The core of the conclusion is determined with using the MACI method, while its flanks are determined by using CRF. The main advantages of this method are its applicability for multi-dimensional problems and its relative simplicity. Conceptually different approaches were proposed by Baranyi *et al.* [1] as "General Methodology" (*GM*) (this notation refers to the feature that these methods are able to process arbitrary shaped fuzzy sets). The basic concept is to calculate the fuzzy conclusion in two steps. First a new fuzzy rule is interpolated at the reference point of the fuzzy observation, then a single rule reasoning method (revision function) is used to determine the final fuzzy conclusion based on the similarity of the fuzzy observation and the "interpolated" rule antecedent. Recently FRI methods have been successfully adapted in several practical application areas like fuzzy modelling of an anaerobic tapered fluidized bed reactor (Johanyák *et al.* [14]).

3 The FRI Method "FIVE"

The FIVE method was originally introduced in [9], [10] and [11] and it was developed to fulfill the speed requirements of direct fuzzy control, where the conclusions of the fuzzy controller are applied directly as control actions in a real-time system, so the concept of the FIVE method is an application oriented aspect of the FRI techniques. Most of the control applications serves crisp observations and requires crisp conclusions from the controller, the main idea behind the FIVE is based on the fact aforementioned. Adopting the idea of the vague environment (VE) [4], FIVE can handle the antecedent and consequent fuzzy partitions of the fuzzy rule base by scaling functions [4] therefore it can turn the task of fuzzy interpolation to crisp interpolation. The idea of a vague environment is based on the similarity or in other words the indistinguishability of elements. In a vague environment the fuzzy membership function $\mu_A(x)$ is indicating the level of similarity of x to a specific element a which is a representative or prototypical element of the fuzzy set $\mu_A(x)$, or it can be interpreted as the degree to which x is indistinguishable from a [4]. Therefore the α-cuts of the fuzzy set $\mu_A(x)$ are the sets which contain the elements that are $(1\text{-}\alpha)$-indistinguishable from a. Two values in a vague environment are ε-distinguishable if their distance is greater than ε, where the distances are weighted distances. The weighting factor or function is called scaling function (factor) [4]. The scaling function serves the purpose of describing the shapes of the fuzzy sets in the partition. After determining the vague environment of both the antecedent and consequent part universes (the scaling function or at least the approximate scaling function [9], [11]), every member set of the fuzzy partition can be characterized by points in that vague environment (for example see the approximated scaling function s shown on Fig. 2). Fig. 1 presents a one dimensional antecedent and consequent system with two fuzzy rules. Therefore if the observation is a singleton, by the concept of vague any crisp interpolation, extrapolation, or regression method can be adapted very simply for FRI [9], [11]. In method FIVE because of its simple multidimensional applicability, the Shepard operator based interpolation (first introduced in [15]) was adapted (see e.g. Fig. 1). The consequent and antecedent sides of the vague environment can be precalculated and cached, this provides the fastness of the method, since only the interpolation between the points defining the rule base should be performed real time. The applicability of the FIVE method can be limited whether an approximate universal scaling function can be found both for the antecedent and consequent partitions, which describes the whole partition even if the partition is not a Ruspini partition. For selecting the scaling function in case of triangular or trapezoid fuzzy sets a solution can be found in [13]. Provided that the fuzzy sets are triangles, each fuzzy term can be characterised by a triplet (three values, see Fig. 2): values of the left and the right scaling factors and the value of its core point. With this three cardinal points the scaling function can be simply interpolated as an approximate scaling function (see e.g. on Fig. 2). Beyond the simplicity and therefore the high reasoning speed, the original FIVE method has two obvious drawbacks: the lack of the fuzziness on the observation and conclusion side. The explanation is that this deficiency is inherited from the nature of the applied vague environment, which describes the indistinguishability of two points and therefore the similarity of a fuzzy set and a singleton only. The lack of the fuzziness on the conclusion side has a small influence on common applications where the next step after the fuzzy reasoning is the defuzzification. On the other hand, the lack of the fuzziness on the observation side can restrict applicability of the method. Moreover an extension of the original "FIVE" method was suggested in [16], where the question of the fuzzy observation is handled by merging vague environments of the antecedent universes and the fuzzy observation.

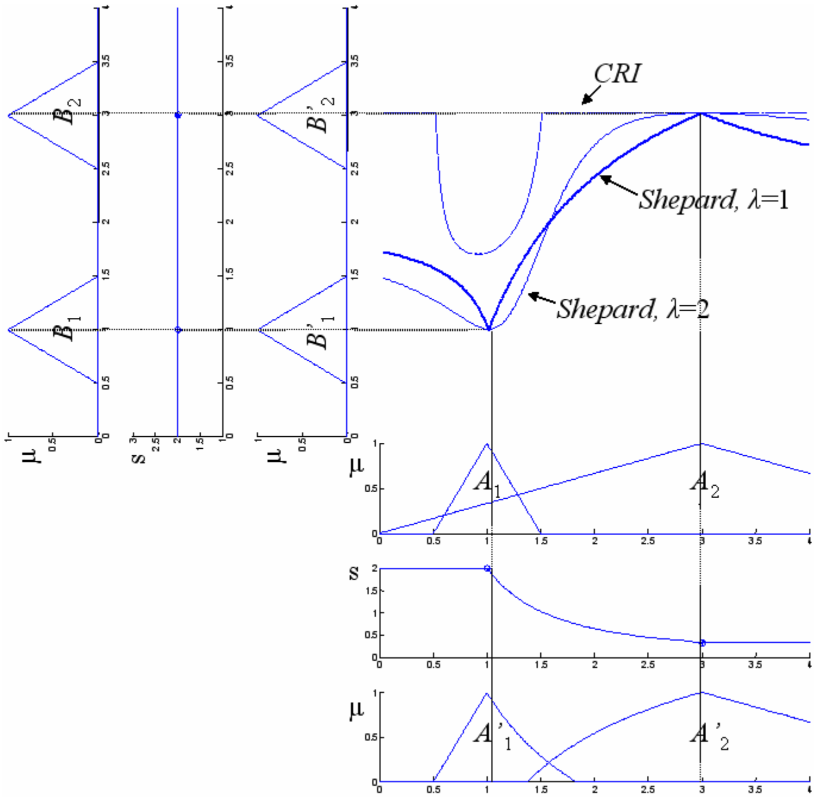

Figure 1
Interpolation of two fuzzy rules (R_i: $A_i \rightarrow B_i$), by the Shepard operator based *FIVE*, and for comparison the min-max *CRI* with COG defuzzification

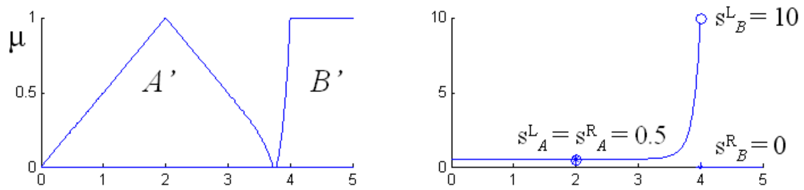

Figure 2
Approximate scaling function *s* generated by non-linear interpolation, and the partition as the approximate scaling function describes it (A', B')

An implementation of FIVE as a component of the FRI Matlab Toolbox [12] can be downloaded from [24] and [25].

4 An Application Example: Room Surveillance with Obstacle and Wall Avoidance with a Mobile Robot

In the demonstrative example of this paper, the FRI method FIVE was chosen, because it is application oriented, i.e. it is quick and simple, and hence it can be easily embedded into a direct robot navigation control. The example application of the paper is a room surveillance guidance control of a mobile robot. The goal is to control an unmanned robot capable of room surveillance by cycling through given waypoints within a room (exploration) with walls and moving obstacles avoidance. When the way of the robot seems to be blocked by an obstacle or by a wall, then the robot is capable of turning around and head in the opposite direction as a last resort. The order of the waypoints is a fixed sequence. This example configuration has four waypoints which correspond to the four corners of the room (see Fig. 3).

The guidance control is built up from three separate controlling components: the *selection of the next waypoint to approach*, the *wall and obstacle avoidance*, and the *heading direction change*.

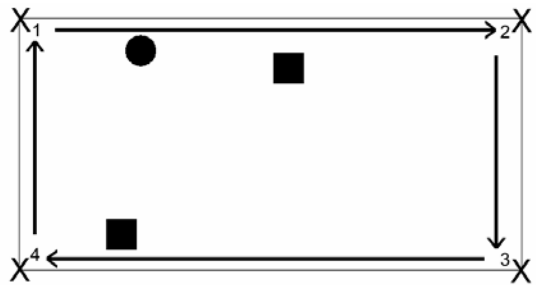

Figure 3
The room with the waypoints and obstacles (the two rectangles) where the robot (circle shaped object) can move. The shape of the room is a rectangle having a 4:3 side to side ratio.

The *selection of the next waypoint to approach component* is determined according to the followings: the current position of the robot, the heading weights of the four waypoints, the need for direction changing and the current direction of the robot. The method for the selection is simple: assign the waypoint in the predefined sequence which follows the nearest waypoint to the robot. The observations needed for this component are the measured distances of the robot from each of the defined waypoints (dw_1, dw_2, dw_3, dw_4) and the selection state, which waypoints (sw_1, sw_2, sw_3, sw_4) is the robot headings towards. States characterize that whether the waypoint is actively selected or not. For expressing the distance from an arbitrary waypoint in the fuzzy rule base, the linguistic terms for the antecedent universes are given as the following: zerus (*Z*), large (*L*) and for expressing the state of the heading waypoint, there are only two antecedent linguistic terms: true (*T*), false (*F*). The consequent of the rule base expressing the weight of the selected next waypoint (*WW*) it has only two linguistic terms: zerus (*Z*), large (*L*). The selection of the next waypoint to approach component has as many separate rule bases as the count of the pre-defined waypoints, and they have similar structure. In the example case this means four rule bases (having four waypoints), each needs to be evaluated with the same measured distances and state variables. Every conclusion is a normalized weight which is used to scale a vector pointing towards the corresponding waypoint. When these four scaled vectors are summarized the result will be the movement vector of the *exploration controller* component.

Having the required observations and the strategy described above, the fuzzy rule bases can be constructed. As mentioned, four rule bases are required in this particular case: first to calculate the weight needed to take the robot towards the second waypoint, second to direct the robot to the third waypoint, third to take the robot to the fourth waypoint, and a fourth rule base to navigate the robot back to the first waypoint. In Table 1. four rule bases are shown merged together, rule base for the first waypoint selection has the normal typeset, rule base of the second waypoint selection has a bold typeset, the rule base for the third waypoint has an italic typeset, and the last rule base for the fourth waypoint selection is underlined.

Table 1
Waypoint selection weight rule base

RW	dw_1	dw_2	dw_3	dw_4	sw_1	sw_2	sw_3	sw_4	*WW*
Rule 1	Z	**Z**	*Z*	<u>Z</u>					Z
Rule 2	L	**L**	*L*	<u>L</u>	T	**T**	*T*	<u>T</u>	L
Rule 3	<u>L</u>	L	**L**	*L*	<u>T</u>	T	**T**	*T*	Z
Rule 4	**L**	*L*	<u>L</u>	L	**T**	*T*	<u>T</u>	T	Z
Rule 5	*L*	<u>L</u>	L	**L**	*T*	<u>T</u>	T	**T**	Z
Rule 6	**Z**	*Z*	*Z*	Z					L

The rules are defined in the following form:

RW_i: **If** $dw_1 = A_{1,i}$ **and** $dw_2 = A_{2,i}$ **and** $dw_3 = A_{3,i}$ **and** $dw_4 = A_{4,i}$
and $sw_1 = A_{5,i}$ **and** $sw_2 = A_{6,i}$ **and** $sw_3 = A_{7,i}$ **and** $sw_4 = A_{8,i}$
Then $WW = B_i$

The rules in the first rule base (the normal typesetted values in Table 1) have the meanings as follows: the first rule means that when the corresponding waypoint is reached by the robot then that waypoint should be abandoned, hence the weight of the waypoint will be zerus (*Z*). The second rule keeps the robot coming to the waypoint if it has been selected earlier. The third rule serves the purpose of keeping down the weight when the robot is going to the next waypoint, so do the fourth and fifth rules, but for the remaining two waypoints. The sixth, last rule means that when the robot had reached the previous waypoint in the sequence, it should head to this very waypoint. The remaining three rule bases are for the same purpose but for the other three waypoints. With these rule bases the robot can cycle around the given waypoints, but when obstacles stand in its way further rule bases are required to handle the situation.

The applied collision avoidance strategy consists of two parts: *wall avoidance* and *obstacle avoidance*. By the definition walls are the borders of the room and the obstacles are objects which can move freely inside the room. Avoiding walls is a simple procedure. Based on the distance from the four walls, a repulsion rate is calculated, which then can be used to compute a vector perpendicular to the corresponding wall. Observations of the *wall and obstacle avoidance component* are the measured distances from each of the walls (d_w), and from each of the objects inside the room (d_o). The linguistic terms of the antecedent universes are: zerus (*Z*), small (*S*), medium (*M*), large (*L*), and for the consequent universe (*AV*): zerus (*Z*), small (*S*), large (*L*). Obstacle avoidance follows the same strategy. Summarizing the normalized wall and obstacle avoidance repulse vectors the result can overrun the maximum. In this case the length of the vector should be cut to the maximum allowed value.

Based on the above described technique a simple fuzzy rule base can be built (see Table 2). The *wall and obstacle avoidance component* uses the same rule base for all the required conclusions only the input distances differ within every evaluation. The rules are defined in the following form:

$RColl_i$: **If** $d_w = A_i$ **Then** $AV = B_i$

Table 2
Wall and obstacle avoidance weight rule base

RColl	d_w, d_o	*AV*
Rule 1	Z	L
Rule 2	S	S
Rule 3	M	Z
Rule 4	L	Z

In the case if the way of the robot seems to be blocked in the current exploration direction, the robot can change its heading, by assigning the waypoints in the reverse order. This direction change decision is made by the *heading direction change component*. The observations needed for this component are the sum of movement rate of the robot and the collision avoidance vector (*mr*), the summarized rate of the length of the wall and obstacle avoidance vectors (*ar*), and finally a rate of exploration is added (*er*). It serves as a movement component weight. Since the robot could do some other types of movements than exploring (as part of another application), an exploration rate value could be also considered. The linguistic terms of the two antecedent universes of the *heading direction change component* are: zerus (*Z*) and large (*L*). The conclusion universe (*DC*), which tells whether to change the direction of the robot, the linguistic terms are: false (*F*) and true (*T*). The rule base consists only of three rules, which can be seen in Table 3. The rules are defined in the following form:

RDirCh$_i$: **If** $er = A_{1,i}$ **and** $mr = A_{2,i}$ **and** $ar = A_{3,i}$ **Then** $DC = B_i$

Table 3
Direction changing decision rule base

RDirCh	*er*	*mr*	*ar*	*DC*
Rule 1	Z			F
Rule 2	L	Z	L	T
Rule 3	L	L		F

Another rule base can be used to determine the new heading direction for the robot. For this subcomponent two observations are required: a value which tells whether a direction heading change is necessary (*dirchg*) (the conclusion above, see Table 3.) and the current heading direction (*currdir*). The linguistic terms for the antecedent universes are the following: for expressing the need of direction changing: true (*T*), false (*F*), for expressing the current direction and also for the consequent universe, which gives the new direction (*ND*): clockwise (*C*), counter clockwise (*CC*). The rule base is simple: when *dirchg* is *F* then *ND* will be the same as *currdir*, but when *dirchg* is *T* then *ND* will be the opposite of *currdir*.

Having the rule bases for collision avoidance, direction changing decision and new heading direction, the original waypoint selection rule bases (Table 1) should be extended. New observations will be added: the current heading direction (*dir*) and a parameter expressing whether the heading direction was changed (*dirchg*). The newly added antecedent linguistic terms for the necessity of reversing the direction are: true (*T*), false (*F*). For the current direction: clockwise (*C*), counter clockwise (*CC*). The extended rule bases are shown on Table 4. The rules are defined in the following form:

RWX$_i$: **If** $dw_1 = A_{1,i}$ **and** $dw_2 = A_{2,i}$ **and** $dw_3 = A_{3,i}$ **and** $dw_4 = A_{4,i}$
and $sw_1 = A_{5,i}$ **and** $sw_2 = A_{6,i}$ **and** $sw_3 = A_{7,i}$ **and** $sw_4 = A_{8,i}$
and $dir = A_{9,i}$ **and** $dirch = A_{10,i}$
Then $WW = B_i$

Table 4
Waypoint selection weight with direction changing rule base

RW	dw_1	dw_2	dw_3	dw_4	sw_1	sw_2	sw_3	sw_4	*dir*	*dirch*	*WW*
Rule 1	Z	**Z**	*Z*	Z							Z
Rule 2	L	**L**	*L*	L	T	**T**	*T*	T		F	L
Rule 3	L	**L**	*L*	L	T	**T**	*T*	T		T	Z
Rule 4	**L** *L*	*L* L	L L	L **L**	T	T	**T**	*T*	CC	T	L
Rule 5	*L* L	L L	L **L**	*L* **L**	**T**	*T*	T	T	C	T	L
Rule 6	L	L	**L**	*L*	T	T	**T**	*T*		F	Z
Rule 7	**L**	*L*	L	L	**T**	*T*	T	T		F	Z
Rule 8	*L*	L	L	**L**	*T*	T	T	**T**			Z
Rule 9	**Z**	*Z*	Z	Z					C	F	L
Rule 10	Z	Z	**Z**	*Z*					CC	F	L

Compared to the original waypoint selection rule base, four new rules were added. In the following the new rules are explained, according to the first rule base (see Table 1). Rule 3 stops the robot when a direction change is necessary. The fourth rule changes the direction if needed and if the previous heading was towards the next waypoint in the defined sequence. Rule 5 is similar to Rule 4, it changes the direction if required and the previous heading was the previous waypoint in order. Rule 6, 7 and 8 are the same as the third, fourth and fifth rule in the original waypoint selection rule base. Rule 9, 10 means that when the robot reaches the previous waypoint in the sequence, then it should head to the next waypoint.

It is practical to arrange the evaluation of these rule bases and observation calculations in a loop. First the waypoint selection conclusions should be calculated, the result vector should be added to the current position of the robot. With this new position the distances from the walls and obstacles should be computed, then the wall and obstacle avoidance fuzzy rule bases should be evaluated, these results should be summarized with the current position also. This will be the next valid position of the robot. Finally we have all the required data to get the conclusion for the direction changing. If the direction has to be changed, the direction state variable should be inverted and in the next iteration it should take effect. Following this procedure gives a working model of surveillance guidance and obstacle avoidance.

Acknowledgement

This research was partly supported by the Intelligent Integrated Systems Japanese Hungarian Laboratory and the National Scientific Research Fund Grant OTKA T049499.

Conclusion

Applying the FIVE fuzzy rule interpolation method and sparse fuzzy rule bases described as a demonstration example of this paper, a room surveillance guidance control strategy was implemented. (The source code of the working example can be freely downloaded from [25].) Compared to the classical complete fuzzy rule base solutions, the main benefit of this approach is the reduced rule base size. For building a complete fuzzy rule base with the same strategies, $2^{(2n+2)}+8+4+4$ (where n is number of the defined waypoints) fuzzy rules would be needed, which is 1040 with the four waypoints of the given example. But the fuzzy rule interpolation and sparse fuzzy rule base solution of this paper has only $n*(6+n)+3+4+4$, which is only 51 with four waypoints. This rule base size is easily implementable even in embedded FRI fuzzy logic controllers. Therefore the main conclusion of the paper, that there are application areas, where FRI methods and the corresponding sparse rule bases turns strategies to be tractable sizes even with numerous input dimensions.

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[25] Some FRI applications are available at: http://www.iit.uni-miskolc.hu/~szkovacs/