Modelling Dataflow in Computer Network

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Abstract. Traffic behaviour in a large-scale network can be viewed as a complicated non-linear system. To describe the long-term network traffic behaviour is very difficult in case of large-scale network. The servers may become overloaded during temporary traffic peaks when more requests arrive than the server is designed for. Before implementing network equipments users wants to know capability of their computer network. According to the non-linear character of network traffic, a new system model is set up to exam behaviour of the network planned. This paper presents a non-linear simulation model that helps us to find bottleneck of the network whenever the traffic flow is too high. The model calculates density of data traffic along the network, which based on the estimation of servers' throughput.

Keywords: non-linear system analysis, computer networks

1 Introduction

The traffic behaviour and its tendency have bothered network managers for quite a long time, and is still a not fully understood problem for network management and planning. First, in order to research network behaviour, it is necessary to analyse the measured traffic and to find its statistical laws, such as the work done by Y. Bhole and A. Popescu [1]. Secondly, according to the statistical rules found, some traffic models were built, such as the establishment of a model for Novel network traffic by J. Jiang and S. Papavassiliou in [2]. If the time-scale of network traffic is considered, the network traffic behaviour will be different in different time-scale. Paxson and Floyd [3] showed that the traffic behaviour of millisecond-time scale is not self-similar by the influence of network protocol. Due to the influence of environment, the traffic behaviour whose time-scale is larger than ten minutes is not also self-similar and is a non-linear time-series. Only the traffic behaviour in second-time scale is self-similar. The third problem becomes known when transactions of the computer networks have to be described. As it was presented before, the system shows non-linear features, i.e. transactions. It means no tools to describe transactions by mathematics of linear control systems. In this paper, a new non-linear traffic behaviour model is set up and by the aid of this model, the non-linear features of dataflow are described.

2 Node Map Model

The computer network models known by the literature trace back the data traffic to the description of a communication happening between edges and nodes of a communication graph. The models handle the nodes as an important element in this description method. The outcome of model is the communication graph that faithfully imitates the physical arrangement of the computer network, where the nodes, the active elements of the system are the peaks of the graph, which are connected to each other by the transfer mediums called edges. This description gives back that view on a natural manner. In this model the central place are occupied by nodes as the peaks of the network graph and the edges of the network graph show transactions of the traffic in which the peaks communicate with each other along the data lines connecting them.

The most extensive and modern researches present directions of network parameter estimations [4], analyses of traffic generators [5a, 5b], elastic network nodes [6], etc. The non-linear analyses of network nodes [7a, 7b, 7c, 7d] is a separate important research area. Therefore, very important the optimal function of the nodes in the system. In terms of the data traffic that would either be the ideal if fewer nodes existed in the network, or in the case of huge number of nodes these all were linked to all the others. This apparently an absurd approach, already if we look at the economic sides of the solution only. A question becomes known, that necessary, that let the nodes be on the central place of the examinations. The correct answer is in this direction, that in terms of the data traffic, the whole network is necessary to put onto the examined central place.

Setting up our model, we have to take into consideration more viewpoints that the network's graph is unable to show:

Forming of a computer network several geometry features set conditions:

• Geometrical conditions:

- Distance of nodes
- Materials of the transfer medium
- Number of users

• Seasonal effects:

- The number of the users using the network actually moves on a very wide scale.

• Diverse external factors:

- Weather
- Electromagnetic storm

3 Structure of the Model

3.1 The Model's Elements

We may divide the elements of a real computer network into two big groups. On the one hand, the nodes imply the storage units; on the other hand, the transfer mediums connect the nodes to each other.

3.2 Nodes

That statement appears unambiguous, that in the network actually the nodes, the active elements of the network communicate with each other, and the nodes form the peaks of the network graph in the graph theory model of the computer network. Figure 1 shows a plain example as some are numbered nodes communicate:

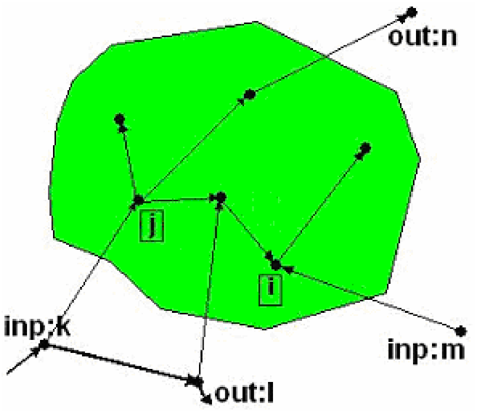

Figure 1
Internal and external elements of a network

In Figure 1 node j communicates with three internal nodes and one external one (inp k), while node i communicates with two internal nodes and one external one (inp m).

3.3 Transfer Medium

The process of the communication is implemented through the transfer medium. The communication happens with almost speed of light. This hilltop speed makes the transfer of a big amount of data possible already under short time. In order to avoid data collisions in the computer networks, the one-way communication is forced always along the network. Naturally, two-way communication may happen when dataflow is separated in time on the same medium. This means when a message splits into the packets to transfer them through the medium, in the same time only one data bit of the split message is crossed the transfer medium. For example, the message may represent a very long train crossing on an extraordinary short railway rail. Therefore, while the backside of the train still stays on the starting railway station and the engine has already arrived to the destination station only one railway carriage stays always on the examined railway rail. Similarly, although the network model's graph may imply one-way directed edges, in most case these mediums are used as two-ways channels since their communications are divided in time.

3.4 The Communication

Communication between the nodes of the computer network is always dynamic. Peak hours are usual between the nodes, but no broadcast periods also exist. The temporal changes of the dynamic message transmissions are modelled with the help of a communicational matrix. The task of the communicational matrix is to take into consideration all elements of the network as well as their features, which regulate the data traffic between the nodes. These rules prescribe the conditions for the usage of the data transfer between the nodes and the data transfer medium. During the communication of the network's active elements, the model examines full connection system of the network. To the faultless simulation of the communication, the model has to imply some important parameters yet namely, the lengths and numbers of data transfer sections, the physical parameters of the medium, the sizes of storage units in the nodes, the degree of the utilisation, etc.

3.5 Some Special Characteristics of the Computer Network in Case of Faultless Data Transfer

In the course of data transfer, although transfer velocity of one data bit is constant in time, transmission times of packets with the same longitude may be different. (During distribution the data density is constant.)

In the course of data transfer, the data transfer sections running in parallel with each other do not affect each other directly, but in the nodes, for example the appearance of multiplied packets makes disturbing effects.

Two-way traffic does not exist.

The intensity of inner communication changes in time between the nodes directly connected to each other. In case of wrongly chosen parameters for example this inner communication capable to create peak-load on the examined network without traffic arriving from the external network.

To control affecting message transmission, an inner communicational system works between the nodes connected to each other. For example, the receiver can receive a message vainly if the transmitter does not have a message to be sent on.

3.6 Relationship Between State Variables and Density

In our model, under the data density we mean a without dimension number between ($0 <= s <=1$). Traditionally, data density shows the rate of number of bits that are transferred in a time unit and the number of bits that can be transferred in a time unit through the transfer medium. Now data density is introduced as a rate of the length of recent messages and the length of the messages finding enough room in the node maximally. Since storage capacity of transfer medium is only one bit, these definitions are equivalents. The densities occurring in the inner network's nodes are the state variables of the system. The model describes examined part of a network with n nodes delimited with a closed curve. Data densities taking shape on the inner network in this case the state variables of the system, successively $x_1(t)$, $x_2(t)$, $x_3(t)$,…, $x_n(t)$. The model uses that part of the external network with m nodes that has direct connection to the inner nodes which are marked with $s_1(t)$, $s_2(t)$,…, $s_m(t)$. . These data densities are based on measurements. Our mathematical model describing the examined network takes into consideration the inner connection system of the examined network as well as their connections to the external network (Figure 1).

In function of the network, the data transfer lines work as generalized nodes. A data transfer medium can store one bit of a message. In case of data transfer s=1, in opposite case s=0, that is the condition of $0 <= x_i(t) <= 1$ comes true.

4 The Mathematical Model

4.1 Creating Mathematical Model

To create the mathematical model, we need some tools to describe structure of the computer network to be analyzed, then we need functions to describe processes of the network as well as we need state variables to examine internal and external status of the network.

Our mathematical model inherits from the general equations of non-linear differential equation system (1) and (2).

$$x'(t) = F(t, x(t), s(t))\, x(t) + G(t, x(t), s(t))\, s(t) \qquad (1)$$

$$y(t) = H(t, x(t), s(t))\, x(t) + J(t, x(t), s(t))\, s(t) \qquad (2)$$

where time is marked t, vectors of x(t) and x'(t) are the state variables and their derivates of the examined system, vector of s(t) and y(t) represents input and output signals. The connection between state variables and their derivates are given by F and G matrices, while output is inherited from the state variables and the input signals by the aid of H and J matrices.

In our case, (1) and (2) become to the following non-linear differential equation system:

$$x'(t) = C_{int}(x(t), s(t))*x(t) + C_{inp}(x(t), s(t))*s(t) \qquad (3)$$

$$y(t) = C_{out}(x(t), s(t))*x(t) + C_{ext}(x(t), s(t))*s(t) \qquad (4)$$

where our task is to establish C = [C_{int} C_{inp}; C_{out} C_{ext}] Communication matrix of the network and define both s(t) input signals and x(t) state variables. As it can be seen in equations (3) and (4), the Communication matrix is an hypermatrix showing relationship of communication of inside and an external network. Matrix C_{int} presents the examined network (see node i and j in figure 1) with n nodes. Size of C_{int} is nxn. Matrix of C_{inp} with nxm elements handles the incoming communication in the network. Task of C_{out} is to describe the output system. It size is mxn. Since no data traffic measured in our examined network among external nodes, matrix C_{ext} is a zero matrix with mxm elements. Elements of the communication matrix are shown in Figure 2.

$$\underline{\underline{C}} = \begin{bmatrix} \underline{\underline{C_{\text{int}}}} & \vdots & \underline{\underline{C_{inp}}} \\ \cdots & \cdots & \cdots \\ \underline{\underline{C_{out}}} & \vdots & \underline{\underline{C_{ext}}} \end{bmatrix}$$

Figure 2
Parts of the Communication matrix

4.2 Creating State Variables and Input Signals

State variables show status of the examined system. In our system, the data density is elected as state variables. This quantity shows the change of data volume in the nodes of the computer network. Data density (5) is a rate representing rates of recent data quantity in time and the maximal data quantity in node i.

$$x_i(t) = \frac{\text{number of recent data bits in buffer of node i}}{\text{maximal number of data bits in buffer of node i}} \quad (5)$$

Input signals (6) of our system s(t) are also data densities. In an input node we simply measure the output density of the node and it become input density one of our examined nodes.

$$s_m(t) = \text{defined by measurements} \quad (6)$$

4.3 Creating Communication Matrices

An element of the Communication matrix grants the connection when node j communicates with node i. Creating Communication matrix is done column by column. We go through each element of column j, and if a connection exists between nodes j and node i, that is node j works to node i, the communication function marked C_{ij} - is created, where ($i \neq j$, $1 \leq i,j \leq n$). All features are necessary to be taken into consideration at the time of the forming of the communicational matrix by the aid of connection functions C_{ij}. The most important function is the model parameter of c_{ij} showing properties of the connection. Inside traffic regulations are also necessary to be taken into consideration at the time of the forming of the communicational matrix (i.e. data link mechanisms depending on the density of the traffic). In our model, the inner traffic regulations depend on density three functions of $S_i(t)$, $R_j(t)$ and $v_{ij}(t)$, where $S_i(t)$ defines properties of sender node, $R_j(t)$ declares features of the receiver, while $v_{ij}(t)$ is the speed of the data transfer between nodes I and j. So in this manner C_{ij} are defined by product of these four factors $C_{ij} = c_{ij}(t) \bullet S_i(t) \bullet R_j(t) \bullet v_{ij}(t)$.

Structure of $C_{ij}(t)$:

• If there is the opportunity of the permanent communication between two nodes, and the node j section works for node i, then $c_{ij}=1$, if there is not a physical connection between these nodes, then $c_{ij}=0$.

• If distributed traffic information exist in our system, and the node j works for more (m) than one node (multipath routing), then

All nodes treat a table, in which all of its rows are reserved for a possible (existing) destination. A $p_{ij}(t)$ grants the distribution proportion of given routes belonging to a node with relative weighting, where $0 <= p_{ij}(t) <= 1$.

Before sending a message, a random value is created by the recent node. Then the node sorts out a route from the possible ones based on weightings used for this value as the probabilities. In this case - for each j - $\Sigma p_{ij}(t) = 1$ in each column, that is the probability of sending equals to 1.

• If distributed traffic information exist or not in our system, and the node j works for more (m) than one node with broadcasting technique, then each message is transferred to all (altogether m) nodes being in a connection with node j. Therefore, node j does not have to treat a table, because the relative weighting value equals to 1 in each direction, that is for column j $\Sigma p_{ij}(t) = m$. In this case the data traffic may be restricted only by the receiving node (see it $S_i(t)$).

• If distributed traffic information do not exist in our system and the node j works for more (m) than one node, then the system uses the following relative weighting values:

- $p_{ij}(t) = p_{ij} = 1/m$ in case of multipath routing
- $p_{ij}(t) = p_{ij} = 1$ in case of broadcasting

• If the communication is disturbed, i.e. broadcast time or a packet grew as a result of electronic circuit broadcasts repeated because of mistakes and then this is taken into consideration by a disturbing factor. The distribution factor marked d and $0<d_{ij}(t)<1$.

• Values of p_{ij} and dij may be constant values, but in the reality rather $p_{ij} = p_{ij}(t)$ and $d_{ij} = d_{ij}(t)$ functions depending on time.

• $R_i(t)$ is an automatic inner self-regulation function (7) of the receiver node with values of 1 or 0. Connection is enable if density of node i ($s_i(t)$) smaller, than 1, anyway 0. This means in the practice, that in case of overloading, node i closes its communication port to direction of node j, therefore node i does not receive any message from node j.

$$R_i(t)= \begin{cases} 1 & , \quad s_i(t) < 1 \\ o & , \quad s_i(t) = 1 \end{cases} \tag{7}$$

• $S_j(t)$ is another automatic inner self-regulation function (8) of transmitter node with values of 1 or 0. It shows whether node j has message(s) to send or not. Connection is disable if density of node j ($s_j(t)$) less than 1, anyway 0.

$$S_j(t)= \begin{cases} 1 & , \quad s_j(t) > o \\ o & , \quad s_j(t) = o \end{cases} \tag{8}$$

Transfer speed between node i and j is the function of the data densities of the nodes $v_{ij}(t) = f(x_i(t), x_j(t))$, that is v_{ij} transfer speed of nodes connected to each other in a time moment depends on collective data densities of nodes. To description this type of the connections the literature [8] recommends several type of function type calculated as the result of measurements and regression methods. All procedures imply the substantive context that the transfer velocity of the messages is monotonous and in case of growing data density however it decreases.

C_{jj} communication function is the j^{th} element of main diagonal in the communication matrix. This value is the negative value of the resultant of j^{th} column of the internal and output communication matrix, because during a realized data traffic of j^{th} column distraction happens from internal node j. This paper does not deal with that case when external nodes work for each other.

5 Non-Linear Model of Network's Data Flow

Consider the network is known in a time t and its data density is marked with N(t). Now let us examine status of the network in time moment (t+Δt). During time Δt if transfer speed of vij and the data density is sj, the data density changes as follow: $\Delta N = v_{ij}\, s_j\, \Delta t$ in node j. So change of data densities of the nodes based on this t+Δt time period can be described as follows (9):

$$N(t+\Delta t) \;=\; N(t) + [\,C_{int}\,,\,C_{inp}\,] * \begin{bmatrix} N_{int} \\ \cdots\cdots\cdots \\ N_{inp} \end{bmatrix} \qquad (9)$$

In case of transfer speed of 1bit/s, change of the internal data density in the network are shown as $\underline{N}_{int}=\underline{x}(t)\cdot\Delta t$ and change of the external region are implemented as follow: $\underline{N}_{inp}=\underline{s}(t)\cdot\Delta t$. The changes of data density flowing out of the inner nodes of the region are taken into consideration in the main diagonal of C matrix. Remember that $C=C(c_{ij}(t), R_i(t), S_j(t), v_{ij}(t))$.

If (7) is done, using equation of $N(t+\Delta t) = N(t) + C_{int}\,[x_j\,(t)]\,\Delta t + C_{inp}\,[s_j\,(t)]\,\Delta t$ and do $\Delta t \to 0$, the result is shown by (10):

$$\lim_{\Delta t \to 0} \frac{N(t+\Delta t) - N(t)}{\Delta t} = \underline{\underline{A}}\cdot x'(t) = \underline{\underline{C}}_{int}\cdot\underline{x}(t) + \underline{\underline{C}}_{inp}\cdot\underline{s}(t) \qquad (10)$$

Equation (11) shows the dimensions of the elements of (10)

$$\underline{x}'_{(n \times 1)} = \underline{\underline{A}}^{-1}_{(n \times n)}\,[\,\underline{\underline{C}}_{(n \times n)}\,\underline{x}_{(n \times 1)} + \underline{\underline{C}}^{inp}_{(n \times m)}\,\underline{s}^{inp}_{(m \times 1)}\,] \qquad (11)$$

or the same is in a shorter form (12)

$$\mathbf{x'} = \mathbf{A}^{-1}\cdot[\mathbf{C_{int}}(\mathbf{x},\mathbf{s})\cdot\mathbf{x} + \mathbf{C}_{inp}(\mathbf{x},\mathbf{s})\cdot\mathbf{s}] \qquad (12)$$

where C_{int}, and C_{inp} are elements of the Communication matrix. These elements contain communication functions as well as the functions depending on the density. The structure of (12) is equivalent to (3). For this reason, this model is applicable to the simulation, planning or regulation of computer networks. If (12) is discretized the equation of the computer simulation (13) is received.

$$x'_{k+1} = A^{-1}\cdot[\,C_{int}\cdot x_k + C_{inp}\cdot s_k] \qquad (13)$$

6 A Sample Model

Let us examine a node given in Figure 3, with s(0) initial inner density value, where $0 \leq s(0) \leq 1$. Mathematically simply reasonable, that if constant input delivering and constant output delivery features are given, then the input-output velocity and density become a stationary balance state after a time when the effect of the initial values disappears.

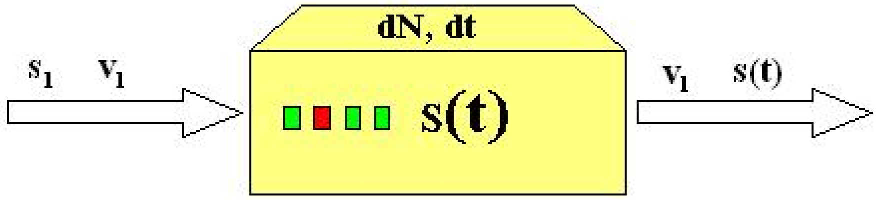

Figure 3
Examined node with constant input and output

So, if input and output delivering are constants, change of density dN can be calculated in a differentially little time dt (14):

$$ds(t)= (v_1 \cdot s_1 - v_1 \cdot s(t)) \cdot dt \qquad (14)$$

where ds(t) is change of density, s1 is data density of incoming bits, v_1 is speed of the transmission and s(t) is the data density in the examined node at time t.

Form the equation (14) and we get (15):

$$\frac{ds(t)}{dt} = v_1 s_1 - v_1 s(t) \qquad (15)$$

Solving differential equation of (15), the result is (16)

$$s(t) = s_1 (1-\exp(- v_1 * t)) + s(0) * \exp(-v_1 * t) \qquad (16)$$

Naturally, this deduction is true if and only if v_1 and s_1 are constants, because if $v_1(t)$ and $s_1(t)$ are continuous functions, then in case of $t \rightarrow T$ v_1 and s_1 put on the constant values of $v_1(T)$ and $s_1(T)$. We notice it, that in real processes the stationarity in a node means, that any t_1, t_2, … , tn in a time moment the distribution of the data density is the same, that is the process is a first order stationary one.

Conclusion

This paper looks for possibilities to describe dataflow model of large-scale computer networks. A model was presented that was applicable to simulation, planning and regulation of computer network’s traffic. At the time of the model's establishment, the partial differential equations were avoided in the mathematical model because of the specially chosen state variables. The nodes have honoured roles in this non-linear model because storage capacities of the transmission medium are practically zero. Nodes either communicate to each other or not. In our model, the mean of the data density is the proportion of the size of data stored in the single node and the data quantity, which can be stored maximally. Our model examines change of data density occurred by data flow among the nodes in a region demarcated by close curve. Input and output data densities are regarded as known. At first sight, these processes are the inputs and outputs of the model. Effectively, these processes together form the actual inputs of the mathematical model. State variables present data densities arising in the internal nodes of the system. Our system applies a data traffic model involving n internal and m external nodes. To create the mathematical model the communication matrices defining the network has fundamental importance. Our model applies four communication matrices. Finally, a simply exam was presented to demonstrate usage of the model and how this model is applicable to the simulation, planning or regulation of computer networks.

References

[1] Y. Bhole, A. Popescu: Measurement and Analysis of HTTP Traffic. Journal of Network and Systems Management, Vol. 13, No. 4 (2005) pp. 357-371

[2] J. Jiang and S. Papavassiliou: Detecting Network Attacks in the Internet via Statistical Network Traffic Normality Prediction. Journal of Network and Systems Management, Vol. 12, No. 1, (2004) pp. 51-72

[3] Paxson, V., Floyd, S.: Wide Area Traffic: The Failure of Poisson Modelling, IEEE/ACM Transactions on Networking 3(3), pp. 226-244 (1995)

[4] A. M. Galkin, O. A. Simonina, G. G. Yanovsky: Multiservice IP Network QoS Parameters Estimation in Presence of Self-similar Traffic, NEW2AN 2006, LNCS 4003, pp. 235-245, 2006

[5a] J. Sommers, H. Kim, P. Barford: Harpoon: A Flow Level Traffic Generator for Router and Network Tests, SIGMETRICS/Performance ’04, 2004, pp. 392-393

[5b] S. Avallone, A. Pescape, Giorgio Ventre: Analysis and Experimentation of Internet Traffic Generator, NEW2AN 2004, pp. 1-6

[6] M. A. Mansour, L. Scrimali: Hölder Continuity of Solutions to Elastic Traffic Network Models, J. Glob. Optim. (2008) 40:175-184

[7a] H. F. Du, Y. Xiao, D. Y. Xu: Nonlinear Stability Analysis for Active Queue Management, Proceedings of the IEEE International Symposium on Communications and Information Technology (ISCIT’2005), 2005, Vol. 2, 1314-1317

[7b] T. Peter, Analysis of Large Scale Transportation Networks, MMA Conference, 2007

[7c] Kihl, M., Robertsson, A., Wittenmark, B.: Performance Modelling and Control of Server Systems Using Non-Linear Control Theory. In: Proceedings of the 18th International Teletraffic Congress (2003)

[7d] Kihl, M., Robertsson, A., Wittenmark, B.: Control Theoretic Modelling and Design of Admission Control Mechanisms for Server Systems. In: Proceedings of IFIP Networking (2004)

[8] L. Yu and Z.-K. Shi: Density Waves in Traffic Flow Model with Relative Velocity, The European Physical Journal B 57, 115-120 (2007)