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Comparison of the Operation of Fixed Point Iteration-based Adaptive and Robust VS/SM-type Solutions for Controlling Two Coupled Fluid Tanks

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Motivations and Aims

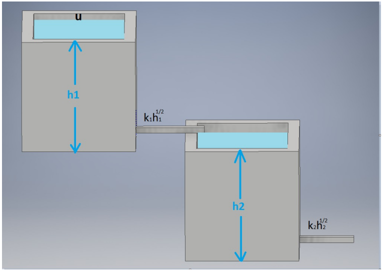
- In this study, controlling the fluid level in the lower fluid tank of two coupled ones by setting the ingress rate at the upper tank was considered
- For comparing the behavior of two solutions: the PD-type Robust VS/SM, and the adaptive, Fixed Point Iteration (FPI)-based controllers.
- The controllers were simulated by simple sequential codes written in the Julia programming language.
- It is shown that the adaptive controller can accommodate itself to subtle details that are not taken into consideration in the robust control.

Motivations (Cont.)

- The robust controller provides non-smooth, switching-type results while the adaptive one can provide smooth solution even in the presence of system modeling errors.
- In the case of the adaptive controller appropriate setting of the control parameters plays central role.
- While mathematically the FPI-based solution is as simple as the Robust VS/SM controllers, in contrast to the Lyapunov function-based adaptive solutions, it easily can be inserted in the mathematical structure of the optimal controllers, so studying its behavior in the case of strongly nonlinear systems may open new ways for its successful applications.

The Dynamic Model of the Coupled Fluid Tanks

- the fluid tanks to control the flow of their liquid is symbolically shown.



- The horizontal blue line indicates the water level which has to be controlled while the vertical line shows the depth of the water tank which for the **upper first tank** is h_1 and for the **second (below) tank** is h_2.

The Dynamic Model of the Coupled Fluid Tanks (Cont.)

- The variables $h_1, h_2\,[m]$ indicate the *state variable of the problem*.
- The *control signal* is the volumetric fluid rate input into the upper tank $u\,\left[m^3 \cdot s^{-1}\right]$.
- The equations of motion are given as:

$$\dot{h}_1 = -\frac{k_1}{S} h_1^{1/2} + \frac{1}{S} u \quad , \tag{1a}$$

$$\dot{h}_2 = \frac{k_1}{S} h_1^{1/2} - \frac{k_2}{S} h_2^{1/2} \quad , \tag{1b}$$

in which $k_1, k_2\,\left[m^{5/2} \cdot s^{-1}\right]$, and $S\,\left[m^2\right]$ are parameters of the outlet assuming turbulent egress.

The Dynamic Model of the Coupled Fluid Tanks (Cont.)

- For simulation purpose, the appropriate “approximate” and the “exact” parameters are:

Parameter	**App. value**	**Ex. value**
Tank cross-section $S\ \left[m^2\right]$	1.8	1.0
Valve parameter 1 $k_1\ \left[m^{5/2}\cdot s^{-1}\right]$	0.005	0.02
Valve parameter 2 $k_2\ \left[m^{5/2}\cdot s^{-1}\right]$	0.0058	0.03

The Dynamic Model of the Coupled Fluid Tanks (Cont.)

- The state variables and the control signal physically can be interpreted if they take non-negative values.
- For the sake of generality, the problem was restricted to the $u \geq 0$ case.
- It can be evidently observed that for each control value u where $u > 0$ from previous equation yields some stationary solution (i.e. $\dot{h}_1 \equiv \dot{h}_2 \equiv 0$) as

$$u = k_1 h_1^{1/2} = k_2 h_2^{1/2} \quad . \tag{2}$$

Control of Nonlinear System Models

- In the case of strongly nonlinear system models for controlling purposes the robust methods may use iterative solutions of the nonlinear systems.
- To track the trajectory in a convenient way this kind of solution may be quite good.
- However, there are methods to convert the nonlinear systems into linear and then proceed for the stable solution of the system with some limitation difficulties of convergence.
- One of the idea to control the nonlinear system models with iterative adaptive techniques in 2009 a *Robust Fixed Point Iteration (RFPT)*-based method

Explanation of The Robust Fixed Point Transformation Method

- The used RFPT-based method transforms the control task into a *“Fixed Point Problem”* then so solves it via iteration that during one digital control step only one step of iteration happens.
- For controlling purposes, this method uses an “available approximate system model” for the calculation of the control signal.
  On the basis of the system’s observed response it adaptively modifies this signal.

Explanation of The Robust Fixed Point Transformation (Cont.)

- For the explanation let us assume the equation of motion of the controlled system as $\dot{h} = f(h,u)$,
- In which $h \in R^M$ denotes the *state variable*, $u \in R^K$ stands for the *control signal*, $h_0 \equiv h(t_0)$ corresponds to the *initial condition* of the motion that is given in advance.
- By the use of the *inverse of the approximate version* the *necessary estimated control force* $u^{Est}(t) = f_{Appr}^{-1}\left(h(t), \dot{h}^{Des}(t)\right)$ can be calculated.
- The realized state-drift will be $\dot{h}(t) = f_{Exact}\left(h(t), u^{Est}(t)\right)$.
- The exact and the inverse approximate models in this manner define a *response function* ψ, that in the above case yields $\dot{h} = \psi\left(h(t), \dot{h}^{Des}(t)\right)$.
- Evidently, due to the errors in the model, $\psi\left(h(t), \dot{h}^{Des}(t)\right) \neq \dot{h}^{Des}(t)$.

Explanation of The Robust Fixed Point Transformation (Cont.)

- The basic idea of this type of adaptive control is that we have to find a *deformed input argument* $r_\star$ for which $\psi\left(h(t), r_\star(t)\right) = \dot{h}^{Des}(t)$.
- This deformed value can be found as the limit of an iterative sequence $\left\{r_0 = \dot{h}^{Des}(t), \ldots, r_{k+1} = G\left(r_k, \psi\left(x(t), r_k\right), \dot{h}^{Des}(t)\right), \ldots\right\}$ in which the function G represents the *Robust Fixed Point Transformation*:

$$G\left(r, \psi\left(h(t), r\right), \dot{h}^{Des}(t)\right) \stackrel{def}{=} \left(r + K_c\right)\times \left[1 + B\tanh\left(A\left(\psi\left(h(t), r\right) - \dot{h}^{Des}(t)\right)\right)\right] - K_c \qquad (3)$$

- Where A, B, and K_c are *adaptive control parameters*.
- By substituting $r_\star$ (which is the fixed point of function G) into (4) it becomes evident that $G\left(r_\star, \psi\left(h(t), r_\star\right), \dot{h}^{Des}(t)\right) = r_\star$

Explanation of The Robust Fixed Point Transformation (Cont.)

- The task in this work is the controlling h_2 of two tanks via using the control variable u.
- According to the model , it seems that h_2 does not directly depend on the control signal u.
- However, its 2nd time–derivative depends on $\dot{h}_1$ that directly depends on the control variable u.
- For a *"desired 2nd time-derivative of h_2"* the model using the *available approximate parameters* is:

$$u = \frac{2S^2 h_1^{1/2}}{k_1} \ddot{h}_2^{Des} + k_1 h_1^{1/2} + \frac{k_2 S h_1^{1/2} \dot{h}_2}{k_1 h_2^{1/2}} \quad . \tag{4}$$

- In the sequel a VS/SM order 2 controller is developed for the control of $\ddot{h}_2$.

The PID-type Variable Structure / Sliding Mode Controller

- The idea of the *Robust VS/SM Controller* was developed in the sixties of past century in the Soviet Union.
- It can be used for the control of $n \in \mathbb{N}$ order systems in which the n^{th} order time-derivative of q can be instantaneously set by the control action (force, torque, etc.).
- For prescribing the “desired trajectory tracking error relaxation”

$$e(t) \stackrel{def}{=} q^N(t) - q(t), \quad e_{int}(t) = \int_{t_0}^{t} e(\xi) \mathrm{d}\xi \tag{5}$$

The PID-type Variable Structure / Sliding Mode Controller (Cont.)

- the tracking error can be introduced and with a constant $\Lambda > 0$ parameter the error metrics can be defined as

$$S(t) \overset{def}{=} \left(\Lambda + \frac{\mathrm{d}}{\mathrm{d}t}\right)^2 e_{int}(t) \tag{6}$$

for which the requirement can be defined

$$\dot{S}(t) = -K \tanh\left(S/w\right) \quad , \tag{7}$$

in which the term of the right hand side corresponds to an *approximate expectation* for the relaxation of the *"error metrics"* $S(t)$ with $K > 0$ and $w > 0$ "strength" and "width" parameters.

The PID-type Variable Structure / Sliding Mode Controller (Cont.)

- For large $|S|$ the function $\tanh(S/w)$ is saturated at ± 1 that means that the error metrics decreases with constant rate, therefore during finite time it has to achieve 0, and following that, it will be kept near to 0.
- The parameter w smooths the abrupt jump of the function between ± 1.
- If $S \equiv 0$, $e(t) \to 0$ is guaranteed.
- The initial values introduced are $h_{10} = 3.0\,[m]$ and $h_{20} = 0.5\,[m]$.
- The main control task is maintaining of the constant value $h_2^N \equiv 0.5\,[m]$.
- For getting rid of numerical problems instead of $h^{1/2}$ the Julia function *"sqrt(abs(h))"* can be used, but the results are believable only if $h_1, h_2 \geq 0$.

Simulation Results for The Coupled Tank

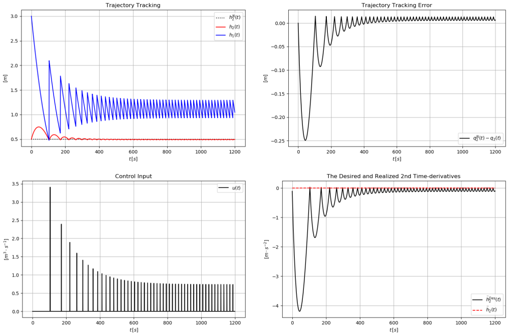
- In the process of flow of the liquid we assume that the flow is a slow process.
- In case of the exactly known parameters and the initial state does not correspond to the "stationary state":
  it is expected that the controller will be able to achieve or at least well approach this stationary state and remains "smooth" in this state.
- When the available model is not precise, in this case it is expected that the solution will keep "jumping" around the stationary solution.

Simulation Results for VS/SM Control Using Exact Model

- For the simulation purpose the proposed control parameters were: $\Lambda = 4.0\ \left[s^{-1}\right]$, $K = 0.1\ \left[m \cdot s^{-2}\right]$, $w = 1.0\ \left[s \cdot m^{-1}\right]$, the time-resolution was $\delta t = 10^{-2}\,[s]$.
- Following a considerable positive ingress, in the next session u must be kept 0 until the fluid level in tanks decreases through its exit valve because there is no possibility for applying negative u values.
- There are huge jumps in the initial state but with time its amplitude decreases, too, and gets to the minimum value moving forward.
- The second time-derivatives of the second state variable h_2 that has small fluctuation near 0. The desired result in red color is “almost stationary”.

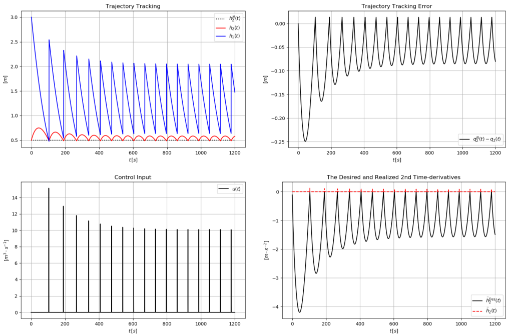
Simulation Results for VS/SM Control Using Exact Model (Cont.)

- Trajectory tracking, tracking errors, control input and $\ddot{h}_2$ of the VS/SM controller for the exact model



Simulation Results for VS/SM Control Using Imprecise Model

- Trajectory tracking, tracking error, control input, the control signal, and $\ddot{h}_2$ for the VS/SM controller using the imprecise model

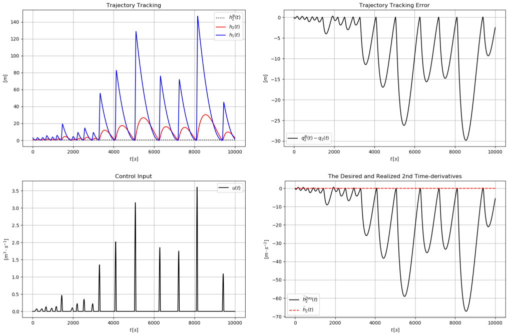


Simulation Results for VS/SM Control Using Imprecise Model (Cont.)

- In the case of the imprecise model no "smooth" solution is achieved, though the prescribed fluid level is well approximated.
- It must be stressed that the jumps in $h_1(t)$ do not correspond to the usual "chattering" of the VS/SM controllers that may occur in the case of smooth system models, too.
- It is strictly related to the truncation of the control action u at 0 for the negative region.

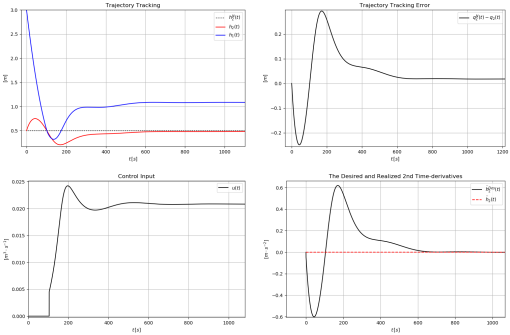
Simulation Results for the The RFPT Method

- For the simulations the $\Lambda = 1.5\ \left[s^{-1}\right]$, $K_c = 40.0\ \left[m \cdot s^{-2}\right]$
- The value of $A = 10^{-8}/K_c$ was too big for guaranteeing the appropriate convergence for the RFPT-based method.



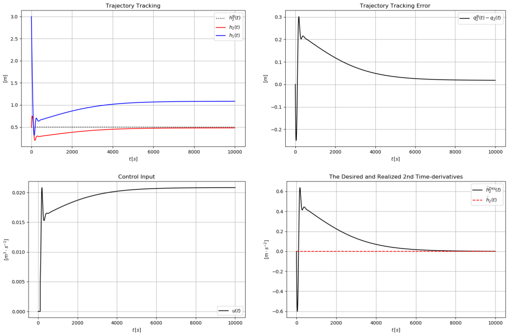
Simulation Results for the The RFPT Method (Cont.)

- In the case of $A = 10^{-9}/K_c$, “fast adaptation” was obtained, and a “smooth control” was achieved.



Simulation Results for the The RFPT Method (Cont.)

- Further decrease in A causes slower adaptation belongs to $A = 10^{-10}/K_c$.



Conclusions

- The operations of the order 2 PID-type robust VS/SM controller and a fixed point iteration-based adaptive, RFPT-based PD-type controller were investigated via numerical simulations.
- The control purpose was to maintain the desired liquid level h_2 in the lower tank at a predefined value 0.5 $[m]$.
- The simulation results were obtained by a sequential code written in Julia language with Euler integration with a discrete time resolution of $\delta t = 10–2\,[s]$.
- Both methods were found to provide useful results, however, the adaptive controller was able to provide “smooth solution”.

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Thank you for your attention!!