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Improved Simple Noise Filtering for Fixed Point Iteration-based Adaptive Controllers

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Motivations and Aims

- The need for noise filtering generally occur in various fields of technical applications.
- *Example 1:* In communication technology, designing filters for suppressing signal under or over of certain frequency limits, or passing it only within a band.
- *Example 2:* Kalman Filters are used in the applications in which the main physical system under control has "causal dynamic behavior" and where the stochastic noises appear only as some not desirable additional terms

Motivations (Cont.)

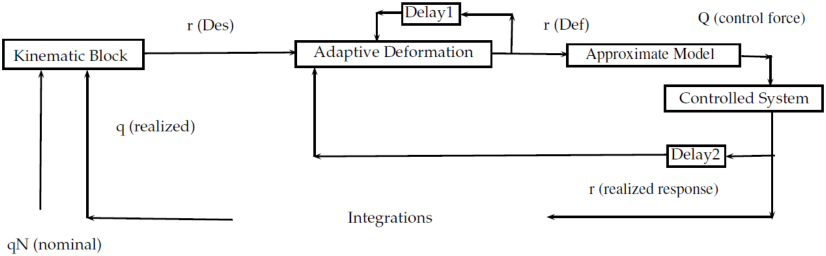
- The *Computed Torque Control (CTC)* invented about 1986 calculates the desired 2$^{\text{nd}}$ time-derivative of the generalized coordinates of the robots, and on the basis of the *available approximate system model* it calculates the generalized forces that have to be exerted by the robot's drives.
- However, it cropped up soon that it is hopeless to produce satisfactorily accurate dynamic models for manipulation robots , therefore these controllers needed further amendment.

Motivations (Cont.)

- For adaptive amendments purposes, many techniques were suggested.
- Through application of Lyapunov function (1892)-based control design the controller may meet various stability criteria.
  - However, this approach mathematically is not easy.
- For replacing it with a mathematically simpler, the ***FPI***-based approach was outlined in 2009.
  - Later was extended to the Model Reference Adaptive Controllers ***(MRAC)***.
    Which its operation was analyzed from the point of view of the Lyapunov function-based techniques.
  - Its starting point corresponds to the structure of the ***CTC*** controller

Motivations (Cont.)

- The schematic structure of the FPI-based Adaptive Controller



Motivations (Cont.)

- The idea of fitting an order n polynomial to the noisy coordinate signals in a *relative order* n control task in a time-window of fixed length was based on the
  observation that any $\mathbb{N} \ni m > n$ order polynomial corresponds to an "overfitting" manifesting itself in too "hectic signal variation" that has to be filtered out.
- In a primitive solution the filtered signal at the discrete time t, $\tilde{F}(t)$, is created from the noisy signal $F(t)$ as

$$\tilde{F}(t) = (1-\beta)\sum_{\ell=0}^{\infty}\beta^{\ell}F(t-\ell) \;\;,\;\; \beta \in (0,1) \;\;. \tag{1}$$

Motivations (Cont.)

- In this solution, small β causes "fast forgetting", greater one corresponds to "slower forgetting".
- Its software realization is very easy: a buffer has to be refreshed in the following manner:

```
BUFFER=beta*BUFFER+NEW_SIGNAL
OUTPUT=(1-beta)*BUFFER.
```

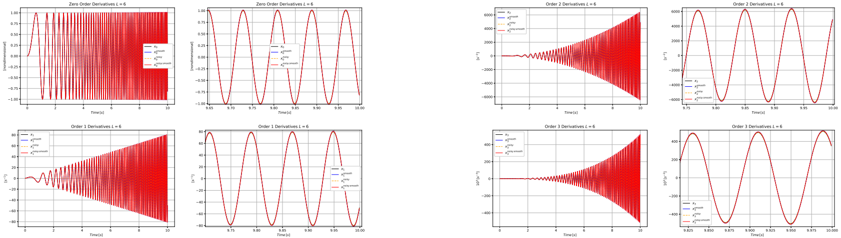
Motivations (Cont.)

- In the present paper the idea of the simple polynomial filter is utilized in a different manner:
  instead fitting some polynomial to recent discrete coordinate values $\{q(t), q(t-1), \ldots, q(t-L)\}$, in the first step the noisy derivatives as *extracted features* as $\dot{q}(t) \approx \frac{q(t)-q(t-1)}{\Delta t}$, $\ddot{q}(t) \approx \frac{q(t)-2q(t-1)+q(t-2)}{\Delta t^2}$, etc. are calculated, then for these signals, a first order polynomial approximation is fitted in a fixed length window.
- These signals are investigated from two points of view: the distortion of this estimation depending on the frequency of the signal to be filtered without any additional noise, then for the suggested method's noise filtering abilities.

Filtering Abilities and Limitations of the Suggested Method

- For simulation purposes, the following parameters were set:
  - ***chirp signal*** which is a "sinusoidal" signal with a frequency increasing in time was numerically simulated with the "step length" $\Delta t = 10^{-3}\,s$
  - ***digital time-resolution***, and the ***filtered time-derivatives*** at first were considered for a minimal discrete window-length $L = 6\,step$.
- In the following figure, a negligible signal distortion can be observed. It is a Noise-free derivative extraction with $L = 6\,step$ fixed window length.

Filtering Abilities and Limitations (Cont.)



Simulations for the Adaptive Control of the Duffing Oscillator

- The Duffing oscillator is a popular paradigm of nonlinear system in various mathematical investigations
- Duffing oscillator is an example of a periodically forced oscillator with a nonlinear elasticity
- In our paper it was modified with a nonlinear, quadratic damping term as

$$\ddot{q} = -\frac{k}{m}q - \frac{c}{m}q^3 - \frac{b}{m}\dot{q} - \frac{d}{m}\operatorname{sign}(\dot{q})\dot{q}^2 + \frac{1}{m}F \quad , \tag{2}$$

- In which
  q , $\dot{q}$ are the state variables
  F is the control force.
  The “approximate” and “exact” model parameters are given as following

Simulations for the Adaptive Control (Cont.)

- The "approximate" and "exact" model parameters are given as following

Parameter	App. value	Ex. value
Mass m $[kg]$	2.0	1.0
Linear stiffness k $\left[N \cdot m^{-1}\right]$	130.0	100.0
Nonlinear stiffness c $\left[N \cdot m^{-3}\right]$	1.8	1.0
Viscous damping b $\left[N \cdot s \cdot m^{-1}\right]$	2.8	2.0
Nonlinear damping d $\left[N \cdot s^2 \cdot m^{-2}\right]$	2.5	1.0
Time resolution Δt $[s]$	10^{-3}	10^{-3}

Simulations for the Adaptive Control (Cont.)

- In the *"kinematic block"* the *"desired 2nd time-derivative* as the desired system response was

$$
\begin{aligned}
&e(t) \equiv q^N(t) - q(t), \;\; e_{int}(t) = \int_{t_0}^{t} e(\xi)\mathrm{d}\xi, \\
&r^{Des}(t) \equiv \ddot{q}^{Des}(t) = \\
&= M \tanh\left(\frac{\Lambda^3 e_{int}(t) + 3\Lambda^2 e(t) + 3\Lambda \dot{e}(t) + \ddot{q}^N(t)}{M}\right)
\end{aligned}
\tag{3}
$$

Simulations for the Adaptive Control (Cont.)

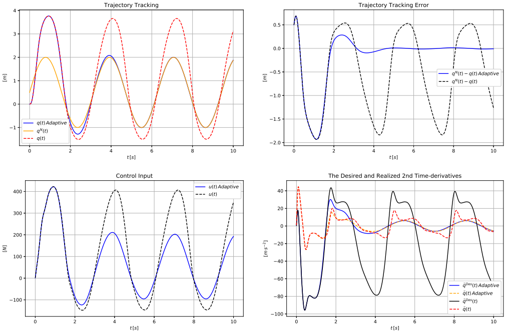
- In which
  $q^N(t)$ is the nominal trajectory to be tracked
  $q(t)$ is the realized trajectory
  $\Lambda = 6.0\,[s^{-1}]$ time constant
  $M = 10^3\,[m \cdot s^2]$ maximizing value were used for a sinusoidal nominal signal of amplitude $1.5\,[m]$ and circular frequency $\omega = 2.0\,rad \cdot s^{-1}$.
- The adaptive fixed point transformation given by:

$$r_{n+1} = (K_c + r_n) \times \quad \times \left[1 + B_c \tanh\left(A_c\left(f(r_n) - r^{Des}_{n+1}\right)\right)\right] - K_c \qquad (4)$$

with $K_c = 10^4\,[rad \cdot s^{-2}]$, $B_c = -1$, and $A_c = 0.1/K_c$.

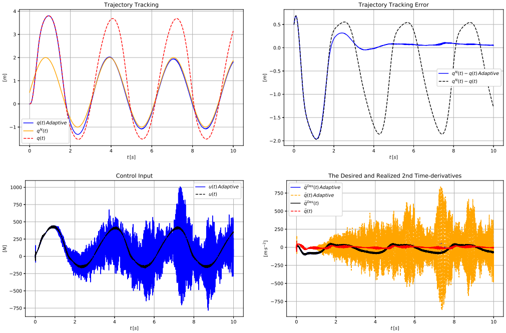
Simulations for the Adaptive Control (Cont.)

- In the following Fig, the adaptive and the non-adaptive results are depicted for the fixed window width $L = 25\,step$.



Simulations for the Adaptive Control (Cont.)

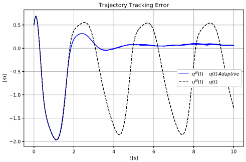
- In the following figures, a noise distribution in the range $[-5 \times 10^{-3}, 5 \times 10^{-3}]$ was applied, and the adaptive controller was able track the signal with a little bias.

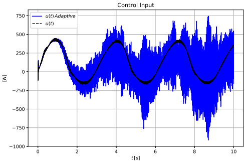


Simulations for the Adaptive Control (Cont.)

- An alternative solution is the use of PD-type feedback as follows

$$
\begin{aligned}
&r^{Des}(t) \equiv \ddot{q}^{Des}(t) = \\
&= M \tanh\left(\frac{\Lambda^2 e(t) + 2\Lambda \dot{e}(t) + \ddot{q}^N(t)}{M}\right)
\end{aligned}
\tag{5}
$$





Conclusion

- The main point is extracting the derivatives as ”features“ from the noisy coordinate signal, then smoothing the extracted derivatives by fitting to them a first order polynomial in a fixed window width that allows to reveal some variation tendency in the smoothed signals.
- Since at higher frequencies this filtering deteriorates the filtered quantities, its application has some frequency limit.
- Simulations conducted for the adaptive control of a Duffing oscillator revealed that good noise filtering effects can be achieved through some little bias remains in the trajectory tracking in the case of noisy signals.

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Thank you for your attention!!