Structure and Randomness

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Probabilistic methods are very important and useful tools in many domains, from theoretical mathematics to applied engineering. In this – mostly mathematical – talk I will list several examples where randomness and quasi-randomness helps to work with mathematical and physical structures.

For example the key of data compression is taking smartly selected random samples from the data. Random variables are very useful, but extracting random bits is an "expensive" process.

Purely random systems are relatively easy to work with. For example if a large graph is random, where the edges are selected independently at random with probability $1/2$, then we know (almost) exactly the size of the largest complete and empty subgraph. On the other hand, constructing graphs with similarly small complete and empty subgraphs deterministically is a very difficult problem.

Most systems we are working with are not random, but not completely deterministic either. To illustrate a general method let us suppose that a function $y = f(x)$ is given. In order to work with it or to understand its behaviour, we would like to write it as

$$f(x) = g(x) + h(x) + r(x)$$

where $g(x)$ is very simple (like a step function), $h(x)$ is a random function and $r(x)$ is the error term which is hopefully small in the range we are working in.

The talk is intended for a general audience, no advanced mathematical background is expected.