Decision system theory model of operating U-tube source heat pump systems

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Abstract - In this paper we set up a decision system theory model for existing heat pump installations with U-tube heat exchanger. Our models describe the relation and connection between input, output and decision variables of the whole system. For every "stage" of the system we show the expectable return and then we demonstrate the optimal return function. Only the system theory modeling is able to provide an exact definition of the various working points, besides it enables decision-making required to ensure optimal management.

I. INTRODUCTION

U-tube source heat pump systems are complex systems. Their precise energetic and economic analysis can only be conducted with the help of system theory modeling.

The basics of the system theory modeling are demonstrated in our previous papers [3], [4], [5], [6]. In these studies we introduced the "basic" system theory schemes, the decision variables and the transformation equations, which describe the connection between the input and output variables in every stage of the model.

In the development of these models we refer to research and work by G. L. Nemhauser [1] and R. Bellmann [2].

II. DECISION SYSTEM THEORY MODEL'S OF HEAT PUMP SYSTEMS WITH U-TUBES

The energetic U-tube source heat pump systems are complex, consisting of numerous components, each influencing the system's operation.

We are only able to determine the optimal operation of such complex systems and optimal costs of their installation, if we decompose the system to several stages and we define the output accordingly. While describing the decision model we decompose the entire system to a number of sub-systems – stages -; then for certain stages we define input, output and decision variables together with transformation correlations, describing the relationship between inputs and outputs.

In both of the models we consider consumer's heat demand as basis. In existing systems we are looking for those operation conditions, which by certain installation conditions, in case of lower heat demand than the sizing by which parameters can the system's electricity use be minimized. Provided that we deal with systems under design or installation we have to inquire from what kind of elements (accessible on the market) we have to construct our system to achieve high COP value so we can minimize the investment and operation costs.

We performed this task by recursive function equations and optimization theory by Bellman. We decomposed both types of systems to stages, set certain variables and by stages we defined the optimization objective function.

The optimization theory is as follows. We perform optimization from back to the front base on the backward model. We define the optimum of the first stage, which is in our case the stage of consumers. Then we move to the next stage, which comes after the consumers' stage. Hereby we define the optimum of the stage and we add to this value the optimum of the previous stage. We continue this process until we reach the end of the system.

Optimization decision model of the serial mechanical system can be described accordingly "Fig. 1". In this paper we refer to input and output variables in the modeling of exact mechanical systems as Z, to transformation equations as g and result variables (costs) as f.

Objective function of the system [12]:

$$h = f_1(Z_1, U_1) + \mathrm{f}_2(\mathrm{Z}_2, \mathrm{U}_2) + \ldots + \mathrm{f}_\mathrm{M}(\mathrm{Z}_\mathrm{M}, \mathrm{U}_\mathrm{M}) + \mathrm{f}_{\mathrm{M}+1}(\mathrm{Z}_{\mathrm{M}+1}, \mathrm{U}_{\mathrm{M}+1}) + \ldots + \mathrm{f}_{\mathrm{N}-1}(\mathrm{Z}_{\mathrm{N}-1}, \mathrm{U}_{\mathrm{N}-1}) + \mathrm{f}_\mathrm{N}(\mathrm{Z}_\mathrm{N}, \mathrm{U}_\mathrm{N}) \rightarrow \text{Extreme.}$$

Function equation of recursive optimization of the system [12]:

$$O(Z_M) = \min_{U_m} \{ f_M(Z_M, \mathrm{U}_\mathrm{M}) + O(Z_{M+1}) \}, \qquad (1)$$

We take into consideration the transformational correlation between Z_{M+1} and Z_M state variables (input and output), which can be described as

$$Z_{M+1} = g_M(Z_M, U_M). \qquad (2)$$

If we substitute this to the function equation (1) it is only the function of the stage's Z_M input's as parameter, and of U_M decision variable (3).

$$O(Z_M) = \min_{U_m} \{ f_M(Z_M, \mathrm{U}_\mathrm{M}) + O(g_M(Z_M, \mathrm{U}_\mathrm{M})) \}. \qquad (3)$$

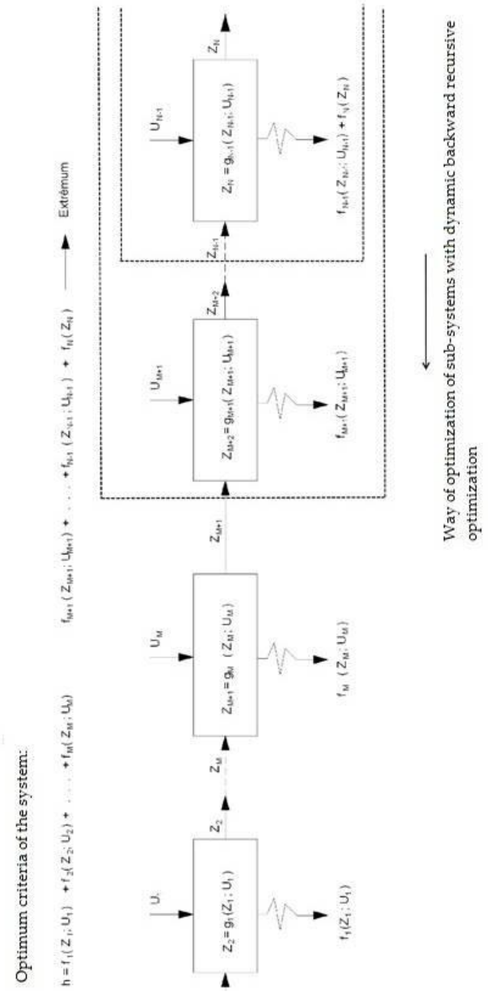

Figure 1. White box model of serial decision system [7]

With the appropriate choice of U_M decision variable in the function of Z_M decision variable we have the optimum of the partial system containing M, M+1,…, N-1 stages, that is the optimal cost of $U_{M,opt}$, $U_{M+1,opt}$, …$U_{N-1,opt}$ optimal decisions. This optimization is called dynamic, backward recursive optimization.

In the first phase of optimization we define the $O_{N-1}((U_{N-1}, Z_{N-1}), Z_N)$ function as follows

$$O_{N-1}(Z_{N-1}) = \min_{U_{N-1}}\left\{ f_{N-1}\left(Z_{N-1}, Z_N, U_{N-1}\right)\right\}. \quad (4)$$

As a second step

$$O_{N-2}(Z_{N-2}) = \min_{U_{N-2}}\left\{ f_{N-2}\left(Z_{N-2}, U_{N-2}\right) + O_{N-1}(Z_{N-1})\right\}, \quad (5)$$

but it is observable that

$$Z_{N-1} = g_{N-1}(U_{N-2}, Z_{N-1}), \quad (6)$$

therefore

$$O_{N-2}(Z_{N-2}) = \min_{U_{N-2}}\left\{ \begin{array}{l} f_{N-2}\left(Z_{N-2}, U_{N-2}\right) + \\ + O_{N-1}(g_{N-1}(U_{N-2}, Z_{N-1})) \end{array}\right\}. \quad (7)$$

By this optimization of the decision variable $O_{N-2}(Z_{N-2})$ for U_{N-2} we can define optimal value of U_{N-2}, which then we substitute to the function $O_{N-2}(Z_{M-2}, U_{N-2})$. Hence, we get to know the optimal (maximal or minimal) costs of the studied stages in the function of Z_{N-2} inputs.

Most frequently the inquiry of the optimal value of the decision variable is done numerically, taking into consideration the discrete character of cost functions and variables.

We discretize the possible value aggregation of the actual status variable – for example Z_M – , and for each exact value we search for the $U_M(Z_M)$ value of decision variable, resulting in optimal minimal value $O(Z_M)$, which we store.

III. Decision System Theory Model of Operating Heat Pump Systems with U-Tube Constallation

When describing to the optimization of an operating system, we refer to the search of those operating parameters of an installed and operating system – considering heat demand of the consumer - , by which the operating costs of the system are minimal. For this we have to know precisely the type and size of the elements in the system and the demand of the consumer. We demonstrate the system theory scheme of an operating system in “Fig. 2”.

Decision variables determining the operation during optimization of an operating system:

- By U-tube heat source: mass flow of primer liquid ($\dot{m}_p$), temperature of primer liquid (forward flow) upcoming from U-tube (T_{pe});
- By evaporator: mass flow ($\dot{m}_h$) of refrigerant applied at cycle;
- By compressor: condensation temperature (T_c) and evaporation temperature (T_o);
- By consumer: mass flow of heating water ($\dot{m}_s$), temperature of the forward heating water (T_{se});

Objective function of the decision system of heat pumps, demonstrated in “Fig 2” is as follows:

$$\begin{array}{l} K(\dot{Q}_{consumer}) = \min \Sigma K_{ü} = \min(K_{consumer} + K_{condensator} + \\ + K_{compressor} + K_{evaporator} + + K_{U-tube}) \end{array}. \quad (8)$$

A. Optimal function by the consumer’s stage

The consumer’s heat demand is given, which is the known output of the decision stage. The input of the stage is the circulated heating mass flow, which we consider as parameter. We link the optimal function of the stage to this parameter, which is the electric power cost of the heating water’s circulation. Mass flow $\dot{m}_s$ of the circulated heating water is parameter and at the same time decision variable.

$$O_1(\dot{m}_s) = k_e \cdot \left[R_s \left(\frac{\dot{m}_s}{\rho_s} \right)^3 \right] \cdot \frac{1}{\eta_e} \cdot \frac{1}{\eta_m}, \quad (9)$$

Whereby R_s is the coefficient of hydraulic resistance of the known pipe system with given geometric parameters, k_e is the unit cost of electric power, η_e is the pump efficiency and η_m is the electric motor efficiency.

The function expresses the utilized electric power efficiency for satisfying consumer’s demand with a given parameter of pump in the function of the parameter, like

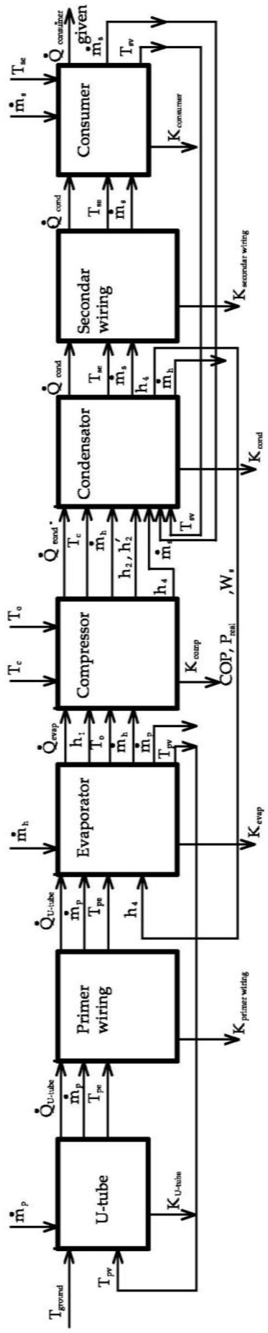

Figure 2. Decision system theory model of an operating heat pump system

the mass flow of the circulated heating water on the secondary side. Provided that we set particular exact values of $\dot{m}_s$ parameter, then correspondingly, we can calculate the secondary forward going and returning water temperature with the help of formulas (10) and (11).

$$T_{sv} = T_{se} - \frac{\dot{Q}_{consumer}}{\dot{m}_s \cdot c_s} = T_b + \frac{\dot{Q}_{consumer}}{k_{rad} \cdot A_{rad}} - \\ - \frac{\dot{Q}_{consumer}}{2 \cdot \dot{m}_s \cdot c_s}, \tag{10}$$

$$T_{se} = T_{sv} + \frac{\dot{Q}_{consumer}}{\dot{m}_s \cdot c_s} = T_b + \frac{\dot{Q}_{consumer}}{k_{rad} \cdot A_{rad}} + \\ + \frac{\dot{Q}_{consumer}}{2 \cdot \dot{m}_s \cdot c_s}. \tag{11}$$

B. The optimal function for the partial system with a condensator

While in operation, new operating costs do not emerge by the decision stage of the condensator. Therefore,

$$O_{21}(T_c) = O_1(\dot{m}_s). \tag{12}$$

In case of an operating system mass flow of the operating system on the secondary side by the stage of the condensator remains as parameter. Optimum of the stage equals with the optimum of the previous level. We do not perform optimization. Condensation temperature is determined. It is known from former stages that

$$T_c = T_b + \left(\frac{\dot{Q}_{consumer}}{k_{rad} \cdot A_{rad}} + \frac{\dot{Q}_{cond}}{k_{cond} \cdot A_{cond}} \right), \tag{13}$$

by which

$$O_{21}(T_c(\dot{Q}_{consumer})) = O_1(\dot{m}_s). \tag{14}$$

For the production of $O_1(\dot{m}_s)$ we search for the lowest allowed (lower than nominal) circulate able heating mass flow $\dot{m}_s$.

C. The optimal function for the partial system supplemented by compressor stage

A new cost element enters, namely the electric power utilization of the compressor. In the optimization function we place this electric power utilization of the compressor next to the $O_{21}(\dot{m}_s)$ taken from the previous decision stage. To the optimal function of the newly supplemented 321 system we involve the evaporation temperature as parameter, given that electric power use and COP value of the compressor is determined by condensation and evaporation temperature.

$$O_{321}(T_o) = \left\{ \begin{matrix} E(T_o, T_c(\dot{Q}_{consumer})) + \\ + O_{21}(T_c(\dot{Q}_{consumer})) \end{matrix} \right\}. \tag{15}$$

Hereby $E[T_0, T_c(\dot{Q}_{consumer})]$ is the electric power utilization of the compressor, and its cost. The $T_c(\dot{Q}_{consumer})$ and T_o determines the value of COP and the value of $\dot{Q}_{evap}$ as well, whereby $COP = \frac{\dot{Q}_{evap} + P_{real}}{P_{real}}$. The values of $\dot{Q}_{evap}$ are calculated by equations (16) and (17), while P_{real} can be calculated by equation (18). $\dot{Q}_{evap}$ equals with heat quantity extractable by U-tube.

$$\dot{Q}_{evap} = \dot{m}_p \cdot c_p \cdot (T_{pe} - T_o) \cdot \left(1 - e^{-\frac{k_{evap} \cdot A_{evap}}{\dot{m}_p \cdot c_p}} \right), \tag{16}$$

$$\dot{Q}_{evap} = \dot{m}_p \cdot c_p \cdot \left(T_{pe} - T_0\right) \cdot \left(\frac{2 \cdot k_{evap} \cdot A_{evap}}{k_{evap} \cdot A_{evap} + 2 \cdot \dot{m}_p \cdot c_p} \right), \quad (17)$$

$$P_{real} = \dot{m}_h \cdot W \cdot \frac{1}{\eta_i} \cdot \frac{1}{\eta_m}. \quad (18)$$

D. The optimal function of the partial system supplemented by evaporation stage

A new cost emerges.

$$O_{4321}\left(T_o, \dot{Q}_{evap}\right) = \left\{O_{321}(T_o)\right\}, \quad (19)$$

Whereby $\dot{Q}_{evap}$ is the known value added to T_0.

Heat quantity provided by evaporator

$$\dot{Q}_{evap} = \dot{Q}_{consumer} - P_{real} = COP \cdot P_{real} - \\ - P_{real} = P_{real} \cdot \left(COP - 1\right) \quad (20)$$

The stage's optimum is expressed by function $O_{4321}(T_0, \dot{Q}_{evap})$ in the function of evaporation temperature and the heat output of the evaporator.

E. Optimal function for the partial system supplemented by U-tube stage

The new cost is the electronic power use of the circulated fluid in the U-tube's hydraulic system.

$$O_{54321}\left(\dot{m}_p, T_o\right) = \min_{\dot{m}_p, T_{pv}} \left\{ \begin{array}{l} k_e \cdot \left[R_p \left(\frac{\dot{m}_p}{\rho_p} \right)^3 \right] \cdot \\ \cdot \frac{1}{\eta_e} \cdot \frac{1}{\eta_m} + O_{4321}\left(T_o\right) \end{array} \right\}, \quad (21)$$

Whereby R_p is hydraulic resistance coefficient of the U-tube's hydraulic system, k_e is the unit cost of electric power, η_e is the pump efficiency and η_m is the motor efficiency. The function equation (21) is solved numerically. We set $\dot{m}_p$ and make T_0 a known, fixed parameter value. Fixed to T_0 $\dot{Q}_{evap} = \dot{Q}_{U-tube}$ is known as well. By this we are able to calculate the value of T_{pv}, which is

$$T_{pv} = T_0 + \frac{\dot{Q}_{U-tube}}{k_{evap} \cdot A_{evap}} - \frac{\dot{Q}_{U-tube}}{2 \cdot \dot{m}_p \cdot c_p}. \quad (22)$$

Based on differential equations (23), (24) the obtained values [8], [9], [10], [11] are utilized to determine the extractable heat capacity belonging to value T_{pv} and calculated by equation (22) to the mass flow of certain primary fluids. Then, we compare this heat capacity with the heat capacity calculated during optimization, with the values of $\dot{Q}_{evap} = \dot{Q}_{U-tube}$ heat capacity utilized in equation (22). Provided that the heat capacity value of the U-tube calculated by equations (23), (24) equals to or is smaller than heat capacity values utilized in equation (22) belonging to mass flow $\dot{m}_p$, which was applied as parameter, then the given $\dot{m}_p$ is applicable. If however, the calculated heat capacity is bigger, then the values belonging to the given $\dot{m}_p$ fall out of the optimization.

$$\dot{m}_p \cdot c_p \cdot \frac{dT_{pv}(H)}{dH} = s + \frac{\left(T_{ground}(H) - T_{pv}(H)\right)}{R_{overall}} + \dot{q}', \quad (22)$$

$$\dot{m}_p \cdot c_p \cdot \frac{dT_{pe}(H)}{dH} = s + \frac{\left(T_{ground1}(H) - T_{pe}(H)\right)}{R_{overall}} + \dot{q}'. \quad (23)$$

IV. SUMMARY

With the utilization of the above described decision system theory models heating systems with U-tube installation can be optimized. By calculating transformation equations introduced at certain stages in the function of decision variables we can determine input and output values of certain stages by given consumer's heat demand.

The decision system theory scheme demonstrated in "Fig. 2" can be utilized at any operating heat pump system, besides objective functions of the system can be calculated, which is the minimization of operation costs. In case of emerging individual need the hereby described decision system theory schemes can be supplemented by further decision stages. Nevertheless, in this case supplementation has to be performed according to the laws of system theory, and the relationships between certain decision stages have to be conducted accordingly. We described the basics of this method in our earlier papers [3], [4], [5], [6].

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