

Theory of General Competitive Equilibrium and Optimization Models of the Consumption and Production

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Abstract— the work deals with the comparative and critical analysis of macroeconomic aspects of general equilibrium theory. It is about the problem of general equilibrium in production, general equilibrium in exchange and the problems of simultaneous equilibrium in production and exchange. The research of cited problems contributes to a better understanding the complete economic mechanisms. Besides, general equilibrium model represents the basis for considering welfare economics and the optimization theory of contemporary market economies.

Key words: partial equilibrium, general equilibrium, contract curves, Pareto optimum, transformation curve in production

I. INTRODUCTION

Most microeconomic models analyze equilibrium states of individual partial markets. Such market in these analyses is considered as an independent system, isolated and independent of the whole economy. This partial analysis enables perception of optimization in firm behavior and creating partial equilibria. A partial analysis, however, has necessarily its limits. It does not give satisfying answers to numerous fundamental questions. The basic shortage of partial analysis models is the same; it does not explain functions of the connected system of partial markets, i.e. the whole economy. Therefore, connecting models of analyzing partial markets and models of general economic equilibrium represent completing and connecting contemporary microeconomic analyses into a unit system.

The theory of general economic analysis, except its complexity and difficulties in practical implementation, gives invaluable benefits in the domain of analyzing efficiency and welfare in microeconomic researches and offering great support in macroeconomic modeling. The subject of this work is just the comparative and critical analysis of microeconomic aspects of fundamental questions of the general competitive equilibrium.

Introducing problems of general equilibrium into economy is connected with physiocrats. We find it in the F. Quesnay's *Tableau Economique* in 1758, and partly in Turgot's work. The role of market mechanisms in creating equilibrium by means of competition, i.e. the "invisible hand" of market we find in A. Smith's teachings. K. Marx brilliantly explained the effects of mechanisms of the law

of value. In his works, we find the laws of stable rate of reproduction and expanded social reproduction, which, in principle, explain the possibilities of dynamic and balanced economic growth.

Nevertheless, the first developed general equilibrium theory we find with the representatives of the Lausanne School of the Marginalist theory in the works by L. Walras and V. Pareto in the second part of the 19th century. Further contributions to development of this theory were the input-output analysis by Wassily Leontief, whose theory, according to many authors, is based on Marx's schemes of social reproduction. Kenneth Arrow, F. Hahn [8] and G. Debreu [6] founded contemporary neoclassical theory of general equilibrium on their works. To understand better general equilibrium theory, Schumpeter's teaching are important, who, beside partial and general equilibrium, differentiates the so-called aggregate equilibrium. To his opinion, partial equilibrium is the equilibrium of economic spheres. Aggregate equilibrium is the equilibrium between selected aggregate quantities (selected in relation to the analysis, which should be done), while general equilibrium represents the equilibrium of national economy. On the other side, with A. Pigou, we find differentiated stable, neutral and unstable equilibriums.

We should get down to the analysis of general equilibrium as any other analysis in economic theory, from the row of simplified suppositions. We take the supposition of competitive market, pure exchange model, production equilibrium, and then simultaneous equilibrium is considered.

II. GENERAL EXCHANGE EQUILIBRIUM

In the analysis of general equilibrium, theoreticians Pareto, Edgeworth, Walras and others start from researching the so-called general exchange equilibrium, i.e. in the this analysis, they firstly abstract money, i.e. commodity prices. Equilibrium conditions are explained starting from the so-called "Edgeworth box" or Edgeworth diagram. At the beginning, they usually take two people who exchange two goods. Take A and B, and goods to be exchanged are X and Y.

In Figure 1, in relation to the ordinate OA several curves of indifference (line of equal utility) of participants in exchange are taken. All the points of the curve of indifference represents such alternative combinations of goods X and Y for an individual A giving him the same level of utility. The slope of determined indifference curve expresses the so-called MRS (Marginal Rate of Substitution), i.e. marginal rate of goods exchange in a determined point of indifference curve, where the utility level of actor A remains unchanged. If some curve of indifference is further from the origo, the utility level representing mutual indifferent combinations, presented on it, is bigger. For that, the relation can express utility values represented by indifference curves of the individual A: $A_3 > A_2 > A_1$.

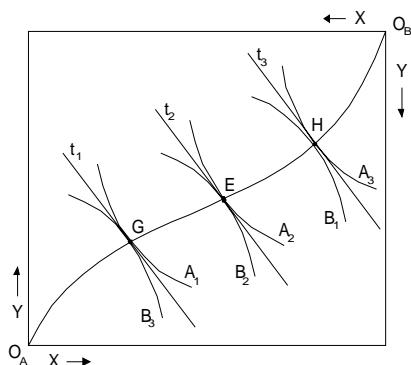


Figure 1.

The origo B is in the opposite northeast angle of the Edgeworth closed diagram, and in relation to it, the indifference curve of Participant B in exchange, i. e. the curves $B_3 > B_2 > B_1$. Every of them represent numerous mutually equivalent or indifferent combinations of goods X and Y for the individual B. The slope of indifference curve is now MRS but for the trader B.

The exchange of goods for individuals is useful, until they are not on indifference curves, which have not points of contact. The marginal rates of goods substitution X and Y for exchangers become equal in points of contact of their indifference curves. As MRS of consumption goods are appropriate to the relations of their indifference curves, it results that exchange is done until marginal utilities of these goods X and Y become equal for A and B. The row of these points represents the general equilibrium of exchange for A and B in the observed model. The geometric set of equilibrium points gives the so-called Edgeworth contract curve, which connects OA with OB. When participants of exchange are in this curve, they reach the so-called Pareto-optimality in exchange. It is said that the “distribution is optimal in the Pareto sense if it is such that every improvement of the situation causes the aggravation of other situations.” In other words, some distribution is Pareto optimal only if there is no possibility of such change, which could improve the situation of one not damaging others. Therefore, every point in the contract curve represents the Pareto optimality, and this curve is the geometric place in Pareto optimality. The Pareto optimality was dome in 1896 in the work Cours d’économie politique. When the ordinary understanding of utility was defined (i.e. not the absolute but the relative

value or the level of utility providing comparative combinations of properties) his work *Manuale d’economica*, the policy in 1906, reached its real importance. In this work, namely, for the first time, the attitude was defined that maximization of aggregate social utility can be reached with the relations of exchange, when no individual utility can be increased without decreasing the of somebody else’s utility.

In Figure 1, along the Edgeworth contract curve, there are numerous alternative equilibrium combinations of exchange. To make the choice between them, it is necessary to define the so-called Social Welfare Function (SWF). To define this function is a very complex task. We should start from the evaluation of values of some situations, evaluation of preferences of social subjects, especially those who create economic policy, and so on. However, this task can be solved with some exactness. Knowing social welfare functions, the Pareto criterion enables eliminating non-optimal combination in exchange.

Now, look at Figure 2. Suppose that the initial distribution X and Y between A and B is at point N, where the curves of utility and indifference A2 and B cut, it is easy to understand that all points for participants in exchange in the shaded surface represent better combinations than the ratio of exchange expressed by point N. The shaded area in Diagram is called the “region of mutual advantages”, and the interval of Edgeworth contract curve between points G and H is called the “core or pith of economy” [Stojanovic, 1944].

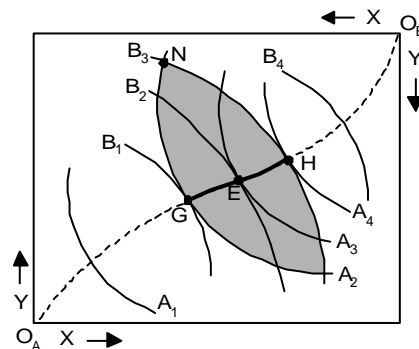


Figure 2.

Correct definition of the position of SWF enables the choice between optimal exchange combinations in the line between points G and H.

Completing general equilibrium in exchange requires the introduction of relative prices of properties and incomes of consumers. Namely, it is generally accepted that consumers or households as traders try to optimize their economic position, or, in other words, to maximize their consumption utility, starting from the following factors:

1. Preference consumers’ system expressed by indifference functions;
2. Amount of money income of consumers;
3. Prices of individual goods and services, P (price), as the indicator of the level of social utility of goods and services.

In the two-dimensional model of consumers' choice limits or the so-called budget consumption limitation are expressed by the so-called line or consumption limit. In case that the variables of money income of traders A and B are equal in exchange, i.e. $I_A = I_B$, and prices of properties X and Y which come in exchange between them P_x and P_y , the budget limitation of consumption for both exchangers can be expressed by the relation:

$$I_{A,B} = X \cdot P_x + Y \cdot P_y \text{ whence} \quad (1)$$

$$X = \frac{I}{P_x} - \frac{P_x}{P_x} \cdot Y \quad (2)$$

$$Y = \frac{I}{P_y} - \frac{P_x}{P_y} \cdot X \quad (3)$$

Under the above supposition, the identical budget line can present the limits of consumers' choice for traders A and B. With unchanged prices of goods and amount of money income, the position of budget line remains unchanged, and its slope expresses the relation of property prices, i.e. P_x and P_y . Look now at Figure 3, the possibilities of general equilibrium in exchange, taking into consideration the amount of money income and prices of exchanged goods

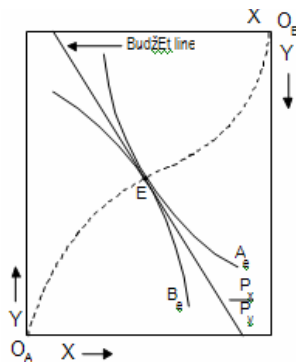


Figure 3.

In Figure 3, general exchange equilibrium between A and B is at point E on the Edgeworth contract curve. At this point, namely, we have matching up the slopes of indifference curves of trader A, on the curve A_e and the slope of indifference curve B, the other trader in exchange. In this point (as in all other points of the contract curve), we have matching up of marginal substitution rates of X and Y for traders A and B. However, contrary to other points on the contract curve, in E, the slope of their marginal substitution slope, i. e. MRS (it is in fact the slope of tangent in some point) is matching up with the slope of their budget line. The slope of budget line expresses relative prices, in other words, the relation of prices of goods in exchange, i.e. X and Y. At the equilibrium point E, the following equalities are valid:

$$MRS_{A(X,Y)} = MRS_{B(X,Y)} = \frac{P_x}{P_y} = \frac{MU_x}{MU_y} \quad (4)$$

(MU – Marginal Utility)

At point E, equilibrium goods prices have the same mutual relations in marginal rate of goods substitution, i.e. MRS is equal for both traders in exchange, and at the

same time, it is appropriate to relations of marginal utility of exchanged goods. It means that the equilibrium point E is suitable for relation of the proportion where supply and demand of exchanged goods become equal.

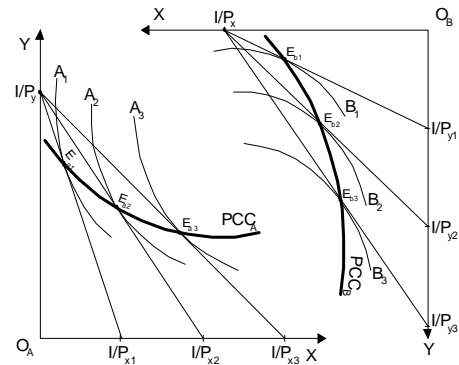


Figure 4.

By the combination of indifference curves and budget lines we obtain the so-called PCC (Price Consumption Curve), which show the structure of optimal consumer baskets in case of price changes of some goods (supposing that prices of other goods, the level of money incomes of consumers and their preference system are steady). In Figure 4, we presented the curves of relations of prices and consumption for individuals A and B, supposing that the product price X changes for trader A; for trader B, the product price Y is changeable. All other relations and conditions remain unchangeable.

The curve of relations of price and consumption of trader A, i.e. curve PCCA shows that, starting from the combination of goods in point E_{a1} , which is located along with the starting position of budget line, the trader A is ready to offer increasing quantity of product X for increasing less quantity of product Y. It is done by gradually price reduction of product X presented by budget lines I/P_{x2} and I/P_{x3} . This trader tries to optimize his consumer utility in the conditions of changed prices. Contrary to trader A, in northeast angle of Diagram, considering PCCB, i.e. the curve of relations of price and consumptions of trader B, we see that the latter trader is ready to exchange more product Y whose price reduces for increasingly less quantity of product X, in view of maximization of his consumer utility.

In fact, the curve of relations of prices and consumption of the trader A, i. e. PCCA, represents the curve of offer A, i.e. its readiness to change goods X for Y in exchange. On the other side PCCB, i.e. the curve of relations of prices and consumption of trader B, presents the curve of offer B, i.e. acceptable relation of goods exchange X and Y for Individual B, depending on price change of product Y. Traders of exchange A and B along their offer curves, starting from points E_{a1} and E_{b1} to points E_{a3} and E_{b3} get to indifference curves which become more distant from the origos, i.e. which express an increasing level of utility for them.

Present, finally, in Figure 5, the curve of relations of prices and consumption of both traders, i.e. PCCA and PCCB, inside of the so-called "region of mutual advantages", limited by their starting indifference curves A_e and B_e .

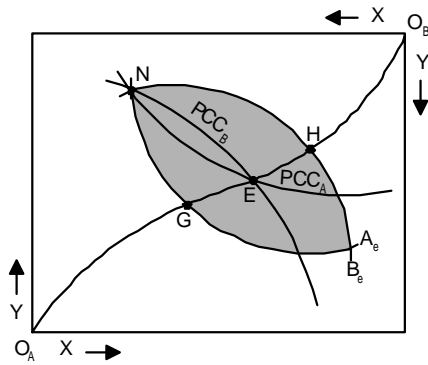


Figure 5.

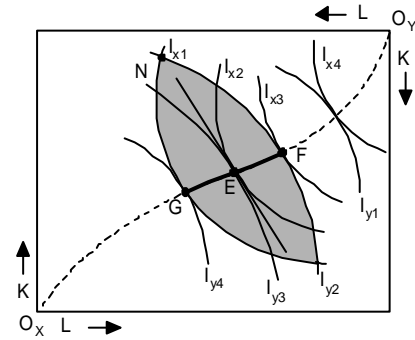


Figure 6.

We can see the point of section of supply curves of observed traders of exchange is on Edgeworth contract curve and inside of the core of exchange in the interval of points G and H in point E. It expresses the relations of exchange where the product offer of the individual A (i.e. his demand for product Y) and product offer by trader B (i.e. his demand for product X). The general exchange equilibrium is established in point E – of course, in the simplified model with two traders and two products. However, this model enables to define the general law on equilibrium of exchange in the following sense: general exchange equilibrium is established with equality of marginal substitution rates of traders along with equality of supply and demand that they present.

The analysis of general equilibrium mechanism is needed to continue by researching the mechanism of general production equilibrium.

III. GENERAL PRODUCTION EQUILIBRIUM

In analyzing of general production equilibrium, we can take the simplified production model analog to the model used in the analysis of general exchange equilibrium. In the model, we suppose that a producer makes two products X and Y, with combination of only two inputs, labor and capital, i.e. by means of L (Labor) and C (Capital). The general production equilibrium is established when marginal technical rate of factor substitution equalize, i.e. MRTS (Marginal Rate of Technical Substitution) for both products. Equilibrium can be established on the so-called Edgeworth closed production box [Kopanyi, 2003], i.e. in Figure 6. The curves IX1, IX2, IX3 and IX4 represent isoquants or the so-called curves of equal product of production of product X. Therefore, isoquants IY1, IY2, IY3 and IY4 show the curves of equal products for product Y. If the starting point is point N in the section of isoquants IX1 and IY3, it is visible that production maximization X and Y is not realized here, therefore, nor general production equilibrium. The producer can increase both production X and production Y, i.e. reaches isoquants at the higher position (i.e. further than the origo) reducing capital consumption for production X on behalf of production Y and conversely, increasing labor consumption in production X, on behalf of consumed labor in production Y.

With these relations and limitations, production maximization of both products is reached in the tangential point of production isoquants X and Y, i.e. at point E. In point E, the curve slopes of equal products X and Y, i.e. I_{x2} and I_{y3} are equal, i.e. the marginal technical rates of substitution in their production are equalized. Thence, these relations are valid:

$$MRTS_{L,K(X)} = MRTS_{L,K(Y)} \quad \text{Then} \quad (5)$$

$$MRTS_{L,K} = \frac{MP_L}{MP_K}; \quad (6)$$

It means that production equilibrium criterion is realized with the condition

$$\left(\frac{MP_L}{MP_K} \right)_X = \left(\frac{MP_L}{MP_K} \right)_Y \quad (7)$$

The point of equilibrium is on the so-called Edgeworth contract production curve, which connects origo O_X and origo O_Y . When production of goods is on this curve, it is not possible any more to increase production of one material product without decreasing production of the other product. On the contract production curve, there are such combinations of production of goods, which realize the so-called Pareto production equilibrium [Pareto, 1971]. In the above analysis of production equilibrium, two marginal rates of technical substitution and marginal products of observed are taken into consideration, but not using the price factor.

In further analysis, suppose that the sum of engaged resources (or TC – total costs) is the constant for production of goods X and Y. Then, suppose that, with ceteris paribus, labor price, i.e. $PL1 < PL2 < PL3$ in production of product X decreases gradually. So, we obtain isocost lines (lines of equal costs) in different positions for production of product X. Connecting tangential points of appropriate isoquants and isocost lines with different labor prices, we obtain the production curve of product X, which expresses the optimal combination of input under cited conditions, i.e. the curve T_x .

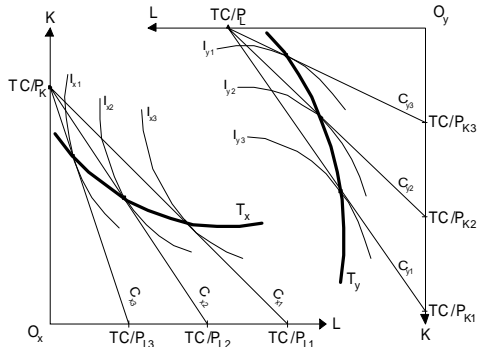


Figure 7.

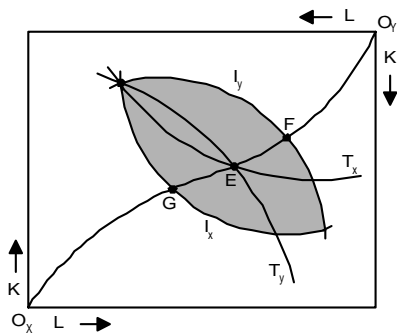


Figure 8.

In production of product Y, we gradually reduce the price of capital use ($PK1 < PK2 < PK3$) and analog to logic in production optimization X, we obtain the curve of production T whose points show the optimal combination of input with different capital prices, i.e. the function T_y . production function T_x and T_y are presented in Figure 7. Curves T_x and T_y cut inside the so-called “region of mutual production advantages” limited by isoquants I_x and I_y . the curve T_x , on the one side, represents the factor demand curve (L, K) used for production of product X, and at the same time, it is the product supply curve X, on the other side. Therefore, the curve T_y represents the factor demand curve for production of goods Y, but also the supply curve Y.

The slopes of isoquants, i.e. the curve of equal product MRTS, express the technical substitution possibilities of input factors in the given isoquant point (in fact, the slope of tangent along with this point) and with the condition that the volume of production remains the same. The slopes of isocost lines (line of equal costs) express the relations of prices used and combined factors in production, i.e. the relation P_k/P_l . That means that isoquants and isocost lines of marginal substitution rates of production factors in the points of tangents are appropriate to the relations of current prices of these factors. It is, together, the criterion of optimal factor combination. Thus, this relation is valid:

$$MRTS_{L,K} = \frac{MP_L}{MP_K} = \frac{P_L}{P_K} \quad (8)$$

Hence, it gives the following equality on the contract production curve:

$$\left(\frac{MP_L}{MP_K} \right)_X = \left(\frac{MP_L}{MP_K} \right)_Y = \frac{P_L}{P_K} \quad (9)$$

These equalities “must exist even when more goods are made and more production factors are used – more than two” [Stojanovic, 1994].

After all these considerations, we can start to define general production equilibrium in the sense of the Pareto optimum. In Figure 9, we again draw the amounts of products X and Y.

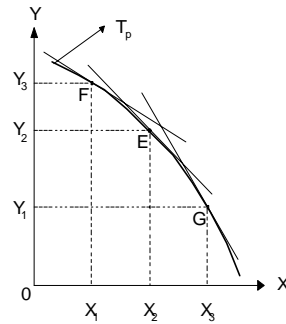


Figure 9.

The contract curves of production transferred from previous diagrams (Figure 6 and 8), change into the function of production possibilities or the curve of production transformation. The slope of transformation curve expresses the marginal rates of product transformation X and Y, i.e. MRT (Marginal Rate of Transformation). Marginal transformation curves with coordinate axes express extreme cases, i.e. when only one or only other kind of product is produced. Individual points of production transformation curves express different alternatives combinations of final products X and Y, which can be maximally produced by full employment of available inputs R and K with available technology. It means that all the combinations along the transformation production curve satisfy the criteria of Pareto optimum, i.e. general production equilibrium. In the area under the curve of alternative production possibilities there are differently realizable possibilities X and Y with suboptimal use of resource use, and combinations in the area of alternative production possibilities out or above the curve of production transformation are unrealizable based on available possibilities.

After the differentiated analysis of conditions and suppositions of general equilibrium of production and exchange, we can analyze the general economic equilibrium in the economy, i.e. the equilibrium of production and exchange.

IV. GENERAL EQUILIBRIUM OF PRODUCTION AND EXCHANGE

The cited conditions of general exchange equilibrium and general production equilibrium must unite in order to make the simultaneous equilibrium of production and

exchange, as in reality, economies where only production or exchange of goods do not exist [Stojanovic, 1994].

In Figure 10, the curve of transformation TP(X,Y) represents the combinations of goods X and Y. The curve OAOB represents the so-called contract consumption curve – exchange. The simultaneous or general equilibrium of production and exchange, or the so-called Pareto optimal equilibrium is realized with the condition:

$$MRT_{XY} = (MRS_{X,Y})_A = (MRS_{X,Y})_B \quad (10)$$

where exchangers of goods X and Y are participants A and B.

The general production equilibrium is realized at point M (or OB) which provides production per 10 units X and Y. The curve tangent of production possibilities Tp(x,y) at point M expresses the marginal transformation rate X in Y, i.e. MRTX,Y. The tangent along the curve Tr(X,Y) expresses the marginal rate of substitution of cited goods in production, i.e. MRSX,Y. At points M and E there is parallelism of the cited tangents. They provide general production equilibrium at point M and general exchange production at point E. Point E shows also the distribution of made equilibrium amount of goods X and Y between A and B in the sense: A gets six pieces of X and five units of Y, while B will obtain four pieces X and five pieces Y.

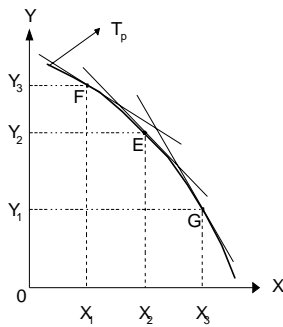


Figure 10.

The criterion of simultaneous equilibrium of production and exchange in the sense of equalizing marginal transformation rates and marginal substitution rates can be generalized, of course, in case of the existence of more kinds of goods, i.e. more producers and consumers. It makes the economic analysis more complete and near its reality.

MRT, i.e. marginal rate of transformation of goods in equilibrium reflect the relations of their marginal production costs, and MRS, i.e. the marginal rate of substitution of goods at the equilibrium level equalize marginal utility goods (and these reflect the relations of their equilibrium prices). Thus, the criterion of simultaneous general equilibrium in production and exchange can be expressed in the following way:

$$\frac{MC_X}{MC_Y} = \left(\frac{MU_X}{MU_Y} \right)_A = \left(\frac{MU_X}{MU_Y} \right)_B \quad (11)$$

The above relation is valid, of course, also in the model with a large number of goods and services in case of many producers and consumers.

The presented (and simplified) analysis of general equilibrium of exchange and production, besides completing the knowledge of functioning connected and complex system of market mechanisms has a broader importance. The conditions and criteria of general competitive equilibrium are applied in the analysis of a very important and current subject matter of economic theory to the so-called welfare economics and the theory of optimum (efficiency) of contemporary market economy. However, this will be the subject of researching in the next work.

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