

# Design of a Translating Parallel Robot Based on 3-CPU Kinematics

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**Abstract:** *The paper presents the mechanical design of an innovative parallel robot for motions of pure translations, based on the 3-CPU architecture. Its simple kinematics resembles the Cartesian robots and allows for the achievement of a wide isotropic workspace. The resulting prototype shows interesting static and dynamic performances, with the only drawback of requiring precise manufacturing and mounting tolerances.*

**Keywords:** *Parallel Kinematics Machines, Translating Parallel Mechanism, Design, Optimization*

## I INTRODUCTION

Kinematic analysis and synthesis are indeed the most important phases of machine design but they must be faced with the following phases of mechanical design and prototype's realization in order to assess the real value of the initial concept. This is even more important in the challenging case of robotics, since the complexity of the envisaged architectures may fail to provide effective solutions to the problems at hand.

This is the case of the robot described in the present paper, whose concept had been previously outlined in [1-2], then its kinematics has been characterised in [3] and finally optimised in [4]: the design of the first prototype, hereby presented, has been addressed with the aim of achieving good static performances, so that the robot could be used in operations

implying a contact with the environment, as for instance in mechanical assembly.

The paper, after outlining the geometric and kinematic features of the robot, describes the main phases of the design that led to the construction of the physical prototype, whose performances had been previously assessed by computer simulation and are presently being evaluated through actual experimentation.

## II DESCRIPTION OF KINEMATICS

The 3-CPU concept shown in Fig. 1 is based on the parallel actuation of the mobile platform by means of 3 identical legs: each leg is composed by two links, joined by a prismatic pair (P), and is connected to the ground by a cylindrical joint (C) and to the mobile platform by a universal joint (U). This mechanism is characterised by 3 dof's and, if the axes of outer

pairs of each leg are parallel one to the other, it allows the mobile platform to translate in space without rotating. As a matter of fact, different settings of the joints are possible in space, still assuring the mentioned conditions, but an optimization process [4] proved that the symmetric architecture shown in Fig. 1 provides the best performances as for workspace volume, robot's dexterity and mobile platform's overall dimensions. The optimal configuration of the robot would also require a point-like platform, a proper length  $d_{max}$  of the limbs ( $d_{max} = \sqrt{2}a_{max}$ , see further on) and orthogonal ground joints' axes: of course the first condition has been only approximately satisfied, while the last one requires demanding geometric manufacturing tolerances.

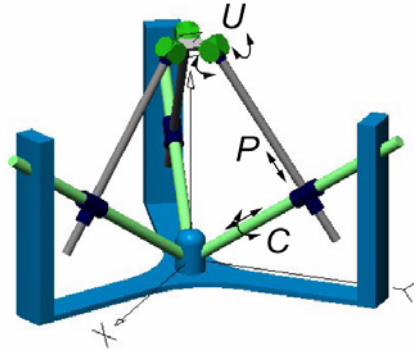


Figure 1  
3-CPU translating parallel mechanism

A simple kinematic study [3] allows to obtain the single solution for both direct (1) and inverse (2) position kinematics problems for the selected configuration:

$$\begin{cases} p_x = \frac{2a_1 - a_2 - a_3}{\sqrt{6}} \\ p_y = \frac{a_2 - a_3}{\sqrt{2}} \\ p_z = \frac{a_1 + a_2 + a_3 - 3t}{\sqrt{3}} \end{cases} \quad (1)$$

$$\begin{cases} a_1 = t + \sqrt{\frac{2}{3}}p_x + \sqrt{\frac{1}{3}}p_z \\ a_2 = t - \sqrt{\frac{1}{6}}p_x + \sqrt{\frac{1}{2}}p_y + \sqrt{\frac{1}{3}}p_z \\ a_3 = t - \sqrt{\frac{1}{6}}p_x - \sqrt{\frac{1}{2}}p_y + \sqrt{\frac{1}{3}}p_z \end{cases} \quad (2)$$

where  $p_x, p_y, p_z$  represent platform position,  $a_i, i=1,2,3$  are sliders' strokes along base guideways, and  $t$  is the radius of the circle inscribed inside the triangular mobile platform.

Velocity kinematics is expressed by a constant Jacobian matrix  $J$ , as follows:

$$\begin{bmatrix} \dot{a}_1 \\ \dot{a}_2 \\ \dot{a}_3 \end{bmatrix} = \frac{\sqrt{3}}{3} \begin{bmatrix} \sqrt{2} & 0 & 1 \\ -\sqrt{2}/2 & 1 & 1 \\ -\sqrt{2}/2 & -1 & 1 \end{bmatrix} \begin{bmatrix} \dot{p}_x \\ \dot{p}_y \\ \dot{p}_z \end{bmatrix} \quad (3)$$

### III FUNCTIONAL DESIGN

#### Design Requirements

The functional requirements driving the design had been aimed at the realisation of a research prototype able to perform assembly tasks or other operations constrained by the contact with the environment: therefore static performances have been advantaged while still trying to achieve an acceptable dynamic behaviour. In the end, by taking also into account economy of realisation, the following requirements have been imposed:

- Nominal vertical thrust:  $300 N$
- Maximum vertical velocity:  $1 m/s$
- Maximum moment at the end-effector:  $30 Nm$
- Workspace:  $> 0.2 m^3$
- Overall dimensions:  $< 2 \times 2 \times 2 m^3$

#### Geometrical Dimensions

The specified requirements allow to assume the following tentative dimensions:

$$\begin{aligned} a_{max} &= 750 \text{ mm} \\ d_{max} &= 1060 \text{ mm} \\ t &= 100 \text{ mm} \end{aligned} \quad (4)$$

The minimum stroke  $a_{min}$  of ground sliders is bound by the geometric mounting condition  $a > t$  and by the hindrance of physical joints, therefore it is assumed the limit value:  $a_{min} = 150 \text{ mm}$ . The other dimensions have been worked out by means of computer simulation:

$$\begin{aligned} 150 \text{ mm} &\leq a_i \leq 750 \text{ mm} \\ 71 \text{ mm} &\leq d_i \leq 1060 \text{ mm} \\ -40.6^\circ &\leq \theta_i \leq 40.6^\circ \end{aligned} \quad (5)$$

where  $\vartheta_i$  is the tilt angle of the generic limb around the ground pairs. The resulting cubic workspace has a volume of  $V = 0.216 \text{ m}^3$  and is shown in Fig. 2.

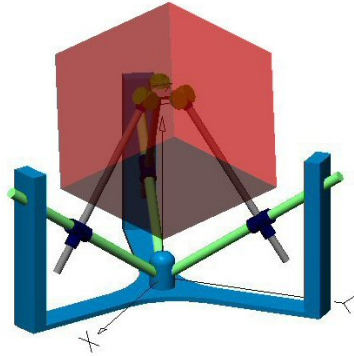


Figure 2  
Cubic workspace of the robot

### Analysis of Static Loads

Due to the simple kinematics of the machine, it is straightforward to compute the effect on the joints and on the links of a wrench applied at the centre  $P$  of the mobile platform. For instance, Fig. 3, it can be seen that the legs are usually loaded by both torque and bending moments:

$$\frac{\mathbf{M}_t}{\sqrt{3}/6} = \begin{bmatrix} c\theta_1 & 0 & 0 \\ 0 & c\theta_2 & 0 \\ 0 & 0 & c\theta_3 \end{bmatrix} \begin{bmatrix} -2 & 0 & 2\sqrt{2} \\ 1 & -\sqrt{3} & 2\sqrt{2} \\ 1 & \sqrt{3} & 2\sqrt{2} \end{bmatrix} \mathbf{M}_{ext} \quad (6)$$

$$\begin{aligned} \frac{\mathbf{M}_b}{\sqrt{3}/6} &= \begin{bmatrix} s\theta_1 & 0 & 0 \\ 0 & s\theta_2 & 0 \\ 0 & 0 & s\theta_3 \end{bmatrix} \begin{bmatrix} -2 & 0 & 2\sqrt{2} \\ 1 & -\sqrt{3} & 2\sqrt{2} \\ 1 & \sqrt{3} & 2\sqrt{2} \end{bmatrix} \mathbf{M}_{ext} \\ &+ \begin{bmatrix} d_1 & 0 & 0 \\ 0 & d_2 & 0 \\ 0 & 0 & d_3 \end{bmatrix} \begin{bmatrix} 2\sqrt{2} & 0 & 2 \\ -\sqrt{2} & \sqrt{6} & 2 \\ -\sqrt{2} & -\sqrt{6} & 2 \end{bmatrix} \mathbf{F}_{ext} \end{aligned} \quad (7)$$

where  $\mathbf{F}_{ext} = [F_x \ F_y \ F_z]^T$  and  $\mathbf{M}_{ext} = [M_x \ M_y \ M_z]^T$  are the external forces and moments applied in  $P$ ,  $\mathbf{M}_t = [M_{t1} \ M_{t2} \ M_{t3}]^T$  and  $\mathbf{M}_b = [M_{b1} \ M_{b2} \ M_{b3}]^T$  are the torque and bending moments on the three legs respectively. It results that the application of a pure force  $\mathbf{F}_{ext}$  does not yield any torque on the legs while the arising bending moment is highly dependant upon their stroke.

A proper selection of the motors can be easily done by observing that their holding force  $\mathbf{f}$  is given, as usual, by:

$$\mathbf{f} = \mathbf{J}^{-T} \cdot \mathbf{F}_{ext} \quad (8)$$

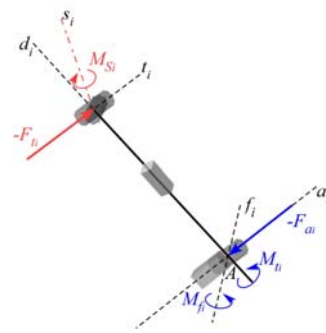


Figure 3  
Loads acting on the limbs

On the other hand, the application of a pure moment at the platform is not reflected on the actuators. It is noted

that the internal actions are highly dependant upon the configuration.

### Robot Actuation

The most convenient way of driving the robot is to actuate the translation of the ground cylindrical pairs: the alternative solution of directly controlling legs' variable lengths would have the advantage of charging the limbs by normal loads only but would need to bring about the motors during limbs motion with higher inertias and a more complex design.

Therefore the ground cylindrical joint of each leg is practically realized by splitting it into the elemental revolute and prismatic pairs with parallel joint axes: a slider carries the revolute joint that connects the limb and runs along the fixed railways actuated by means of rotary motors coupled with ball screws to obtain a linear motion.

The motors must be selected together with the ball screws, in order that both requirements on nominal thrust and task-space velocity are met. The relations (3) and (8) can be usefully implemented in a Matlab program to test the satisfaction of such requirements. In the end, 3 brushless motors with nominal torque and speed of  $M_n=1,2 Nm$  and  $n_n=2\ 300 rev/min$  respectively are coupled with single thread ball-screws of  $16 mm$  pitch and  $16 mm$  diameter (see Fig. 4).

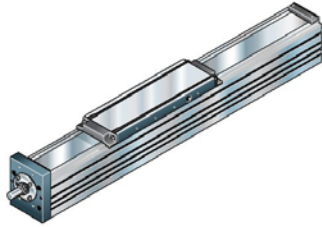


Figure 4  
Ball-screw module for the driving of the linear axes

## IV STRUCTURAL DESIGN

### Selection of Off-The-Shelf Components

Several loading scenarios have been taken into consideration trying to figure out the most severe operating conditions of the machine: in all test cases, a *Matlab* procedure determined the reaction loads on each part throughout the workspace, evidencing the highest values.

Figure 5, for instance, plots the maximum bending moment on the legs when the platform spans the horizontal plane at height  $z=1m$ .

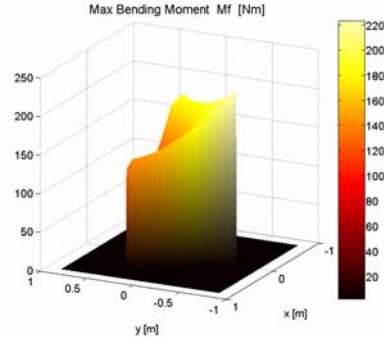


Figure 5  
Maximum bending moment on legs (plane  $z=1m$ )

The most severe case that has been tried resulted to be characterised by the following task space loads:

$$\begin{cases} F_x = 200N \\ F_y = 100N \\ F_z = 100N \\ M_z = 30Nm \end{cases} \quad (8)$$

that gave rise to the following maximum joint reactions, see Fig. 3:

$$\begin{cases} F_{ii} = 210N \\ M_{si} = 58Nm \\ F_{ai} = 210N \\ M_{ni} = 58Nm \\ M_{\beta i} = 223Nm \end{cases} \quad (9)$$

With this information, it has been possible to select the commercial



Figure 6  
CPU kinematic structure of the limbs

components used to realize the joints of each leg, see Fig. 6: due to the high number of kinematic pairs needed for the complete machine (15 joints), it was important to take into consideration both the overall stiffness and the possibility of a fine registration of axes alignments.

The first cylindrical pair has been realised by means of a linear module whose carriage holds the support for the revolute joint shown in the following Fig. 7a; the intermediate prismatic pair has been realized by a ball-bearing guide and the final universal joint by using two revolute pairs: the inner one is based on two taper-roller bearings while the outer one, connecting the limb to the mobile platform, is idle, therefore a simple journal bearing has been used.

### Design of Manufactured Parts

The design was constrained both by the admissible resulting stress in the critical parts (i.e. usually the joints) and by the cogent requirements on the maximum allowed deformations, mainly in the limbs whose deflection would cause a significant decrease in robot's stiffness and end-effector accuracy.

Figure 7a, for example, shows the parts that compose the revolute joint connecting the carriage to the limb: a FEM analysis allowed to assess that the support's deformations were less than 0.5 mm but in this case its state of stress were well beyond the admissible thresholds, Fig. 7b, therefore the part had to be re-designed, as shown in Fig. 7c. A specific attention has been paid to the optimization of the moving parts, in order to limit their masses without reducing their resistance.

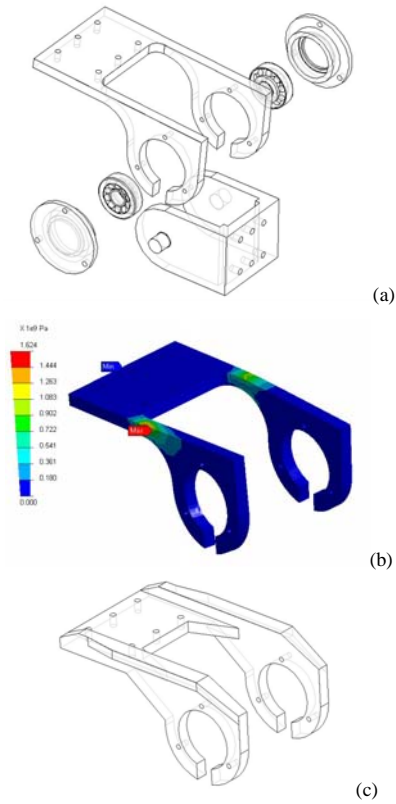


Figure 7  
First design of carriage revolute joint (a) and state of stress of limb's support under the maximum loads (b); final design of the support (c)

## V ROBOT'S PERFORMANCES

### Kineto-Static Performances

Once the machine has been completely designed, it can be characterised by computer simulation before the final prototyping stage.

Actual workspace can be computed taking into account the real stroke of base guides; it results a cube of  $0.275 \text{ m}^3$  volume, as shown in Fig. 8.

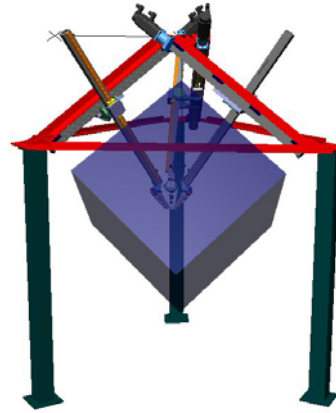


Figure 8  
CAD model of the robot (workspace shaded)

The maximum thrust of the robot is not limited by the nominal torque of the motors but by the mechanical resistance of the ball-bearing guide: therefore, when the legs are completely stretched out, that is the most stressing configuration, the maximum vertical thrust is some  $1620 \text{ N}$  or  $815 \text{ N}$  when pointing downwards or upwards respectively.

As for platform's maximum velocity, the constancy of the Jacobian matrix generates at every point of the workspace the same velocity field shown in Fig. 9, with a maximum vertical velocity of  $1 \text{ m/s}$ .

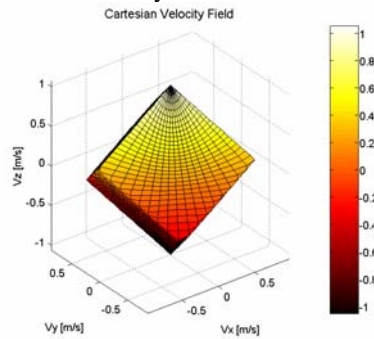


Figure 9  
Cartesian velocity field

### Dynamic Performances

By means of a simulation software, it is possible to assess also the dynamic performances: the maximum and minimum accelerations are yielded in the vertical direction with about  $24.0 \text{ m/s}^2$  or  $4.4 \text{ m/s}^2$  for downwards or upwards motions respectively, while the acceleration along different directions varies between these two values, according to robot's configuration.



Figure 10  
Prototype of the 3-CPU translating parallel machine

### Conclusions

A physical prototype of the described design has been built and is shown in Fig. 10. The robot is characterized by a large cubic workspace with no translation or rotation singularities. As a matter of fact, machine's kinematic relations are very simple and the constant Jacobian matrix grants constant kineto-static properties throughout all workspace. Moreover,

platform's overall dimensions are pretty small. On the other hand, the machine requires very strict geometric manufacturing tolerances in order to grant the satisfaction of the translation conditions. Moreover, the workspace can result difficult to access due to the particular structure of the machine, but this would result in a problem only for certain kinds of applications.

The 3-CPU robot is now available at the laboratories of the Department of Mechanics, where the Authors are presently developing the control system and performing the first experimentations.

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