Fuzzy Reliability for a System with N Components

Cornelia Victoria ANGHEL

S.l.dr.eng. - Department of Computer Science, „EFTIMIE MURGU" University of Resita, P-ta Tr.Vuia, No.1-4, 1700 Resita, Romania, Phone: 0745130272, Fax:+40-255-210230, E-mail: canghel@uem.ro

Abstract The paper presents an eloquent example considering a system with an n component that is degraded until he becomes completely defect. We study the fuzzy system reliability for K components in linear and parallel case. In conclusion we can say that the variation are very small or nonsemnificativ.

1. Fuzzy reliability.

As the nuance sets allow the gradual assignment of the appurtenance functions, they offer a natural description mode of the systems where both the performance and the own elements degrade. These notions have different "work" degrees from the "full functioning to the complete non-functioning" state.

We consider S = {S_1, S_2, … , S_n} the possible state set. We define a fuzzy event W = "the system got out of order", through the following fuzzy nuance sets:

$$\tilde{W} = \{(S_i, \mu_W(S_i));\ \ i = \overline{1,n}\};$$

$$\tilde{F} = \{(S_i, \mu_F(S_i));\ \ i = \overline{1,n}\} \qquad (1)$$

where μ_W and μ_F are the appurtenance degrees for each state in $\tilde{W}$, respectively in $\tilde{F}$.

We observe that every state appurtenance to both finding sets and fault conditions, but through different appurtenance degrees, depending on the fact if it is as close as possible in findings or as faulty as possible. These being gradually the transition state "in findings" to the "fault system" as complementary events for system. Generally $\mu_W(S_i) = 1 - \mu_F(S_i)$ making "system in operation" and "fault system" as complementary events $\tilde{W} = \tilde{F}$, but this is not obligatory.

As in the conventional reliability theory, we are interested in the finding to the fault condition transition. If the event T constitutes such a transition, the reliability of a system from the moment t_0 to the moment t_0 + t are defined by :

R(t_0, t_0+ t) = P(T doesn't appear in the [t_0, t_0 + t] interval) (2)

Because $\tilde{W}$ and $\tilde{F}$ are both nuance states, the transitions between them are obviously fuzzy, from $\tilde{T}$, the transition between $\tilde{W}$ and $\tilde{F}$ is a fuzzy event. $\tilde{T}$ appear only when there are certain transition between those n state systems {S_1, S_2, … , S_n}, so that T is defined in the field {r_{ij} ; i, j = 1, 2, … , n}, where r_{ij} defined the transitions from the state S_i to the state S_j with the appurtenance function $\left\{\mu_{\tilde{T}}\left(r_{i,j}\right),\ i, j=\overline{1,n}\right\}$; $\tilde{T}=\left\{\left(x_{ij}, \mu_{\tilde{T}}\left(x_{ij}, i=\overline{1,n}\right)\right)\right\}$. We consider:

$$\beta_{\tilde{W},\tilde{E}}\left(S_i\right)=\frac{\mu_{\tilde{F}}\left(S_i\right)}{\mu_{\tilde{F}}\left(S_i\right)+\mu_{\tilde{W}}\left(S_i\right)};\ \ i=\overline{1,n} \qquad (3)$$

where $\beta_{\tilde{W}\tilde{F}}\left(S_i\right)$ is the relative fault degree for the S_i state.

If $\mu_{\tilde{W}}\left(S_i\right)=1-\mu_{\tilde{F}}\left(S_i\right)$, then $\mu_{\tilde{W}\tilde{F}}\left(S_i\right)$is reduced to $\mu_{\tilde{F}}\left(S_i\right)$.

In the transition phenomena from the state S_i to the state S_j it is reasonable to say that the system damaged on a certain extension, if only if $\mu_{\tilde{W}\tilde{F}}\left(S_i\right)<\mu_{\tilde{W}\tilde{F}}\left(S_j\right)$.

In this case, we define:

$$\tilde{R}\left(t_0, t_0+t\right)=1-\sum_{i=1}^{n}\sum_{j=1}^{n}\mu_{\tilde{T}}\left(r_{i,j}\cdot P\left(r_{i,j}\right)appearce\ in\ \left[t_0, t_0+t\right]\text{int}\, erval\right) \qquad (4)$$

If the transitions are restraint at the state S_i, we can go only on the immediately inferior state $\mu_{\tilde{F}}\left(S_{i-1}\right)>\mu_{\tilde{F}}\left(S_i\right)$, we obtain:

$$\tilde{R}\left(t_0, t_0+t\right)=1-\sum_{i=1}^{n-1}\mu_{\tilde{T}}\left(r_{i+1,i}\right)\cdot P\left(r_{i+1,i},\ \ can\ hapened\ \ in\ \left[t_0, t_0+t\right]\text{int}\, erval\right)$$

. (5)

If the system is in S_n state at the t_0 moment, and in S_i state at the t_0 + t moment, then the transition $r_{n,n-1}$, … , $r_{j+1,j}$ can appear in [t_0, t_0+t] interval. As the transition can appear only at the anterior states, if $r_{j+1,j}$ appears, the system can exist only in one state, less S_{j+1}, at the moment t_0 + t.

That is why $P(r_{j+1,j})$ appears in interval $[t_0, t_0 + t] = \sum_{i=1}^{j} \mu_{\tilde{T}}(r_{i+1,i}) \cdot \sum_{k=1}^{i} P$ (the system is in S_k state at t_0+t moment).

Changing the summation order, we have:

$$\tilde{R}(t_0, t_0 + t) = 1 - \sum_{i=1}^{n-1} P \text{ (the system is in } S_k \text{ state at } (t_0+t) \cdot \sum_{i=k}^{n-1} \mu_T \cdot (r_{i+1,i})) \quad (6)$$

and taking into account that $\mu_{\tilde{T}}(r_{i,j}) = \mu_{\tilde{T}}(r_{i,k}) + \mu_{\tilde{T}}(r_{k,j})$, we have

$$\tilde{R}(t_0, t_0 + t) = 1 - \sum_{k=1}^{n-1} \mu_{\tilde{T}}(r_{n,\ k}) \cdot P \text{ (the system is in } S_k \text{ state at } t_0 + t) \quad (7)$$

in which case $\tilde{R}(0,\ 0 + t) = \tilde{R}(t)$.

The equation (7) expresses the fuzzy reliability of the system at the t moment. It can be interpreted as the probability that the system can damage with a certain degree between (0, t).

2. Application.

We consider a system with n components that degrades in time, until it becomes completely faulty. Many redundant systems behave in this manner. In the conventional reliability, this type of system is considered outworn if it is degrades until a minimum limit**.** If K is minimum as a number of components necessary for the system to be in operation, its reliability will be in the general case, with the fault rate λ = const., and P(t) the probability that the component operates until t moment:

$$R_k(t) = \sum_{k=K}^{n} [P(t)]^k \cdot [1 - P(t)]^{n-k} \quad (8)$$

and

$$R_k(t) = \sum_{k=K}^{n} \left[e^{-\lambda \cdot t}\right]^k \left[1 - e^{-\lambda t}\right]^{n-k} \quad (9)$$

In the extreme case, $K = n$ and $K = 1$, reduce the equations (8) and (9) are reducing at series systems, when no fault is tolerated, and at the parallel one, when n components are damaged. This type of system can be shaped having a series of states S_j where the system is in operation, from S_i i = 1,2, …, n, where the components are in operation, with S_n - state of complete functioning and S_0 of non-functioning. For the states I there are possible linear or parabolic degradations.

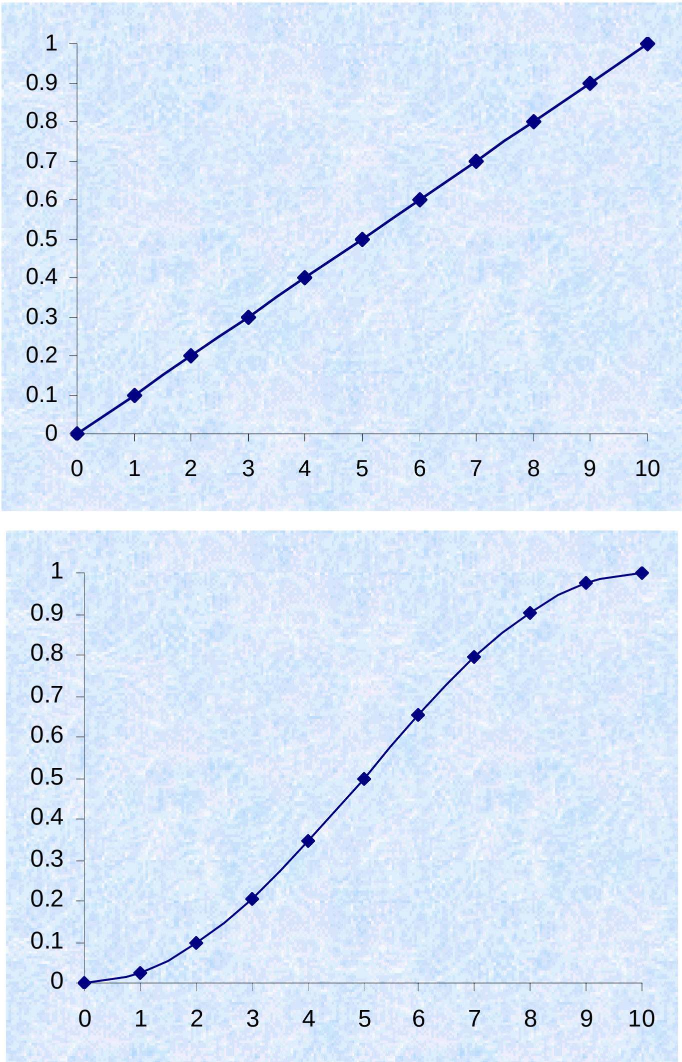

Fig.1.
Linear and quadric appurtenance functions for degradation states using a fuzzy system.

T [hours]	$\tilde{R}(t)$		$R_k(t)$		
	Linear	Quadric	k=0 Parallel	k=5	k=10 Series
	series 1	series 2	series 3	series 4	series 5
0	1	1	1	1	1
100	0.9048	0.9646	0.9999	0.9998	0.3678
200	0.8187	0.9048	0.9999	0.9961	0.1353
300	0.7408	0.8288	0.9999	0.9764	0.0497
400	0.6703	0.7442	0.9999	0.9271	0.0183
500	0.6065	0.6574	0.9999	0.8444	0.0067
600	0.5488	0.5732	0.9996	0.7359	0.0024
700	0.4965	0.4948	0.9989	0.6146	0.0009
800	0.4493	0.4239	0.9974	0.4938	0.0003
900	0.4065	0.3650	0.9945	0.3834	0.0001
1500	0.2310	0.1335	0.9199	0.0506	-
2000	0.1353	0.0599	0.7663	0.0063	-

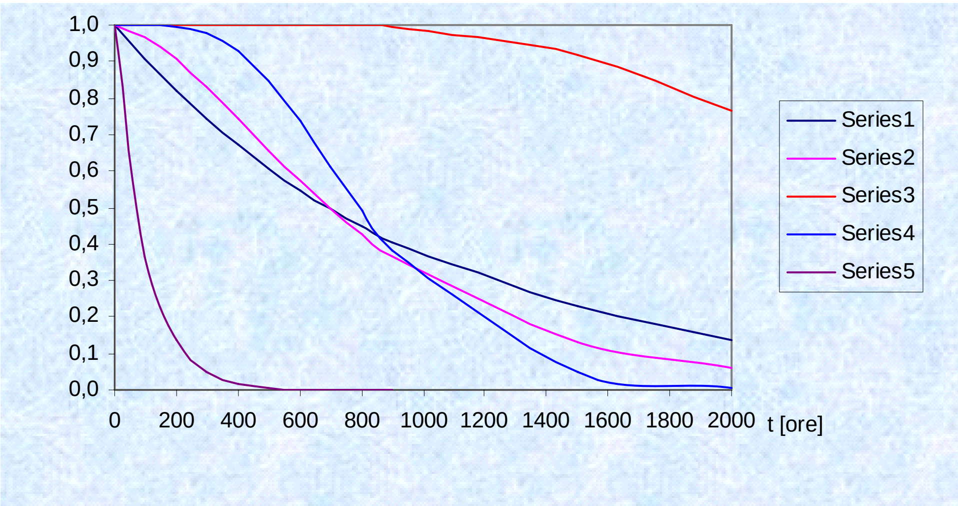

Fig.2.

Comparison between $\tilde{R}(t)$ and $R_k(t)$, n = 10, $\lambda = 0{,}001$ degradations on hours.

For the linear degradation $\mu_{\overline{w}} = \frac{i}{n}$ and

quadric $\mu_w(S_i) = \begin{cases} \frac{2i^2}{n^2} & ; \quad n \in \left(0; \frac{n}{2}\right) \\ 1 - \frac{2(n-1)^2}{n^2} & ; \quad n \in \left(\frac{n}{2}; n\right) \end{cases}$

The appurtenance function can be possibly interpreted “a system in operation being given the possibility that the component i is in operation is $\mu_w(S_i)$”. It is noticed that the curve concavity changes at $\frac{n}{2}$. The degradation function $\mu_{\overline{F}}(S_i) = 1 - \mu_w(S_i)$. If we assume that all the components operate at $t = 0$, the appurtenance function for the degrading transitions of the events from S_n la S_i are expressed by:

$$\mu_{\tilde{T}}(r_{n,i}) = \frac{n-i}{n} \quad (linear);$$

$$\mu_T(r_{n,i}) = \begin{cases} 1 - \frac{2i^2}{n^2} & ; i \in \left[0; \frac{n}{2}\right] \\ \frac{2(n-1)^2}{n^2} & ; i \in \left[\frac{n}{2}; n\right] \end{cases} \text{(quadric)}$$

From equation (7) the system reliability can be written $\tilde{R}(t) = 1 - \sum_{k=0}^{n-1} \mu_{\tilde{T}}(r_{n,k}) P$ (the system is in S_k state at the t moment). In case that $\lambda = ct$, then P(the system is in k state at the moment t $= \left[e^{-\lambda t}\right]^k \left[1 - e^{-\lambda t}\right]^{n-k}$

So that:

$$\tilde{R}(t) = 1 - \sum_{k=0}^{n-1} \mu_{\tilde{T}}(r_{n,k}) \binom{n}{k} \left[e^{-\lambda t}\right]^k \left[1 - e^{-\lambda t}\right]^{n-k} \qquad (10)$$

the values of $\tilde{R}(t)$ and $\tilde{R}(k)$ for $k = 0{,}5$ and 10 from (9) are indicated in the table and in figure 2 for $n = 10$ components and $\lambda = 0{,}001$ degradations on hour. Both curves are contained between $R_0(t)$ and $R_{10}(t)$, from which it differs drastically. They are similar to $R_5(t)$, which show that the reliability for the short values and for the long values of the time are practically plane, that is the variation of $R(t)$ are very little or even unsignificant.

References

[1]. Mendel, J. – "Introduction in Fuzzy Logic Systems", Ed. Hardcover, Univ. of Southern California, Los Angeles, dec, 2000.