Features of non-linear robot position-controllers by simulation of control model

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Abstract
The developed methods of control are in general based of laws of linear controls, however the operation of realized controllers go over to non-linear domain because of motor-voltage limit.
This paper investigates the features of non-linearity and uses some model-process which improve adaptiv properties of this type non-linear control. This process are based on the analysis of function relationship of parameters investigated by simulation of control model.

Keywords: robot, position control, adaptiv control, non-linear control, numeric simulation

Introduction

At position control of robots is desirable the approach aperiodic of target, without overshooting.
Control can shift from overdamped to underdamped mode while following a single trajectory. Underdamped control is undesirable in an industrial robot, as overshot can cause damage to the objects the robot is manipulating, hence, controllers are tuned to give near critically damped response at normal operating speed. At high speed the inertial loads change rapidly, and at low speed some robots move with noticeable vibration [1].
All these dynamic effects generate disturbances which cause errors in following of trajectory, hence robot designers try to keep the loop-gains as high as possible. Adaptive control systems adjust the gain of the control loops in order to maintain critical damping over a range of manipulator configuration.

It is known that many controllers of robot are non-linear. The gain of control for a critical damped case is calculable with classic methods for linear control loops, however we can only give an approach for non-linear control loops. With advanced simulation methods (e.g. Matlab Simulink) we can model the real system nearly in the desired level of the operation, consequently we can derive such results from the simulation which are utilised in the design.
The aim of this thesis to create a model for adaptiv non-linear position-control, in which an experimental function takes the influences of the

changing inertia into consideration for the gain, which is desirable while the robot is moving.
This functions are based on running many simulations. The Fig.1 shows the model.

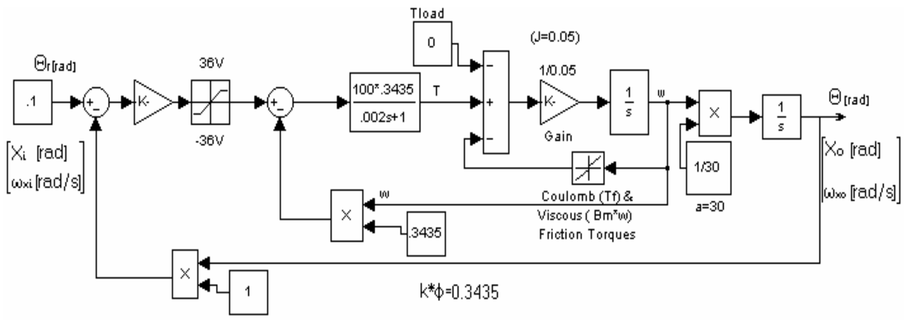

Figure 1: Model of position controlled robot link drived by permanent magnet DC motor.

There is no current limitation in this model. The gain is P-type, the loop is 1-type because of the integration. $\boldsymbol{\Theta_r}$ is a rotation reference signal .
If we take an axis 1st of an R6-type robot into consideration when the 2nd and 3rd axis are moving, the value of inertia can be changed by the variation of configuration maybe in ratio 1:100.
We can preset the value of Coulomb and viscous friction in the model. With regarding to the effect of friction-reduction of PWM controlled motor, we
set the value of Coulomb friction low. Practically there is not backlash in modern robot-constructions.
The value of the load torque is zero in steady-state, there is not contact force during the movement of the robot TCP, since the movement of the robot is unrestricted. The gear-ratio is 1:30.

1. The features of non-linearity

In this case the non-linearity of control-loop derives from voltage-limit which is 36V in our case.
This value of voltage permits a speed of the 1st link to 3.5 rad/s. This speed is sufficient to follow relatively low frequency or amplitude of input signals (Fig.2,3).

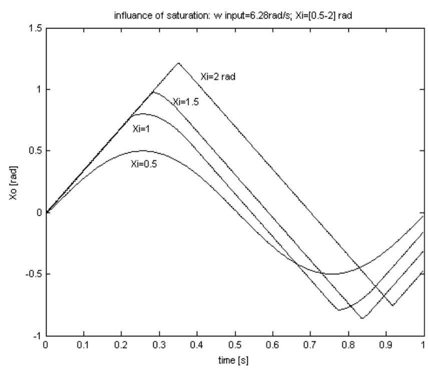

Figure 2: Influence of saturation-type non-linearity in case of sinusoidal input, ω_{in} =const

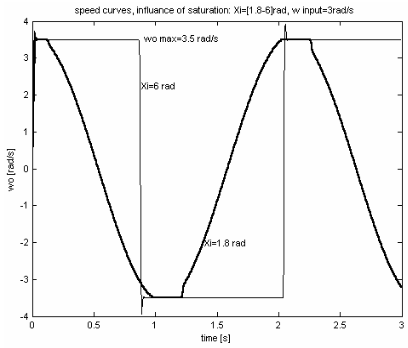

Figure 3: Speed curves in case of saturation: $\boldsymbol{\omega_{in}}$=const, ***Xi***=1.8 and 6 rad

The well-known features are as follows [4]:

- the value of output signal depends on also magnitude of input signal,
- the method of the descriptive-function is based on the following conditions :
  - the values of the parameters of non-linear elements are stable in time,
  - there are not constant and subharmonic components in the output signal,
  - the output signal is periodic and their frequency is equal to the frequency of the input signal,
  - there is only one non-linearity in the loop.

The disadvantages are detailed as follows:

- verification of accuracy is difficult in this way;
- the process can be used for only quantitative estimation;
- the behavior in time-domain is not can be estimated.

The features of the non-linearity, which is applied in our model is shown in Fig . 4.

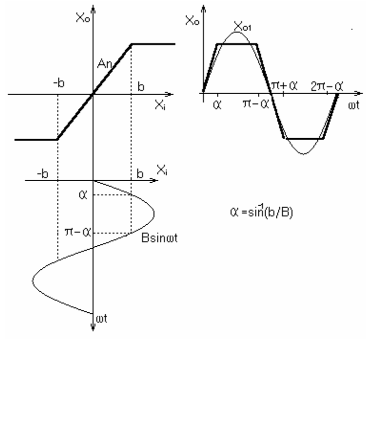

Figure 4: The describing-function of saturation-type non-linearity

In the method of describing-function we take only into account the 1st harmonic [4]:
$X_{o1}(t)=B_1 sin\omega t+A_1 cos\omega t$. Be it:

$C_1=(B_1^2+A_1^2)^{1/2}$, $sin\varphi_1=A_1/C_1$, $cos\varphi_1=B_1/C_1$. With this is describable: $X_i(j\omega)=B$, $X_{o1}(j\omega)=C_1 e^{j\varphi 1}$.

According to the method of describing-function, the quotient of signal amplitudes of output X_o and input X_i can be given by the following expression :
$N(B,j\omega)=C_1(B,j\omega)e^{j\varphi 1(Bj\omega)}/B$, where

$X_i(j\omega)=B$, $X_{o1}(j\omega)= C_1 e^{j\varphi 1}$. It is shown in the figure that in linear section the transfer constant is A_n. If the amplitude of input is sinusoidal, if $X_i=B>b$ then output X_o is shaped cut away.
$x_o(t)=A_n x_i (t)= A_n B sin\omega t, \quad 0 \le \omega t \le \alpha,$
$x_o(t)= A_n b, \quad \alpha \le \omega t \le \pi-\alpha,$
$x_o(t)= A_n x_i (t)= A_n B sin\omega t, \quad \pi-\alpha \le \omega t = \pi,$
where b is an input-value produce the saturation,

and $\alpha=sin^{-1} (b/B)$. $A_1=0$,

$$B_1=(2/\pi) \int_0^{\alpha} A_n B sin^2\omega t d\omega t+2/\pi \int_{\alpha}^{\pi-\alpha} A_n b\ sin\omega\ d\omega t+ +2/\pi \int_{\pi-\alpha}^{\pi} A_n B\ sin^2\omega t d\omega t$$

After integration

$B_1=(2/\pi) A_n B[\alpha+ sin2\alpha/2]$.

Here $C_1=B_1$ and $\varphi_1=0$, $N(B,j\omega)=(2/\pi)A_n[\alpha+sin2\alpha/2]=N(B)$, $B\ge b$.

If $B>b$, the value of the function is less then A_n, but if $B\le b$, the behavior of the function is the same as the behavior of the linear element with value A_n .

The above-mentioned features are shown in Fig 5.

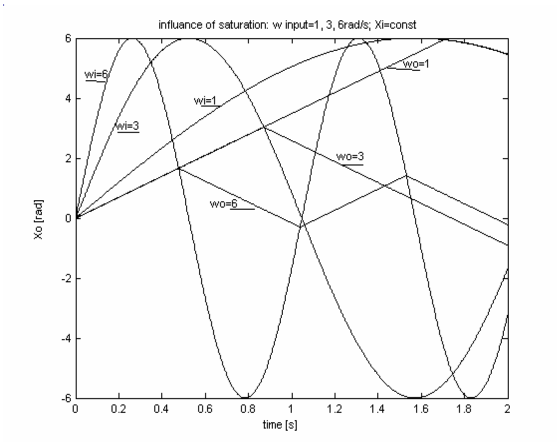

Figure 5: Influance of saturation ω_{in}=1,3,6 rad/s, X_i=const

Here the maximum speed of angle is ω=3.5 rad/s, and this is not able to follow a bigger speed input signal. The maximum gradient of output X_o (see in Fig. 5) means the limit of speed, which is derived from the voltage limit of the motor.

The saturation-type non-linearity have an effect on accuracy of following which is an another important feature of control.
If input of the control demands a higher speed at output then as possible due to the saturation, this can cause a very significant distortion (see Fig. 6).
Parameters of employed sinusoid signals for investigation and the computed maximum values of speeds from this are shown in title of figures.

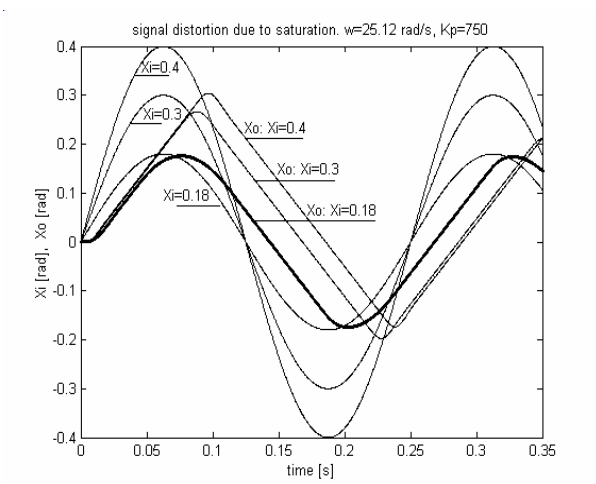

Figure 6: Distortion of output signal due to the saturation. ω=25.12 rad/s, Xi variable

Because of the saturation the possible value of speed is ω=3.5 rad/s (on 1st axis) as mentioned in introduction. The Fig. 7 shows the curve of speed on peaks which of looks the distortion if the speed is in the domain of non-linearity.

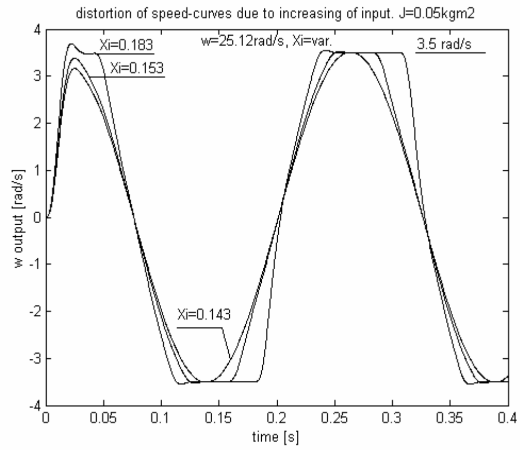

Figure 7: Distortion of speed-curves due to non-linearity

The features of following are of minor importance in usual tasks of robotics when the robot approaches their target but in case of following of the trajectory it’s very important to follow punctually the input of the control, mostly in the movement rectilinear or circular, otherwise this link of the robot will no in the position required in time prescribed. Hence maybe no realized the
coordinated movement among in axis of robot, consequently the movement will no the shape prescribed.

In the figures below the computed speed of input is ω=3.5 rad/s, in this way is attained the condition of linear operation. The visible alterations are derived from the changing the values of the gain, the inertia and the friction. The Fig. 8 shows the output signal of the position control regarding to changing the gain K_p (here the inertia and friction are constant).
is in the domain of non-linearity.

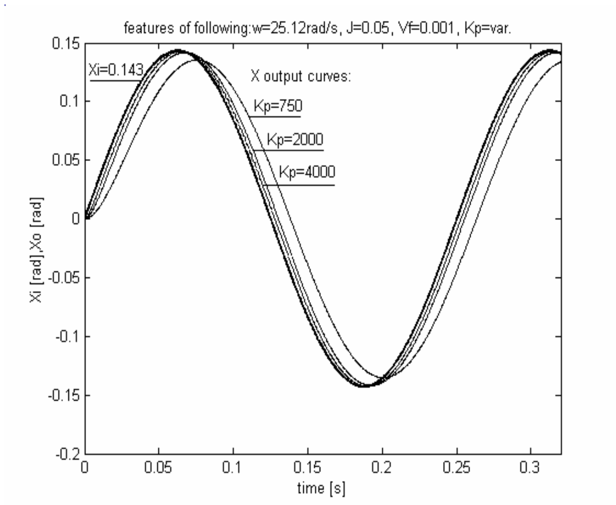

Figure 8: Features of following at changing gain, ***Kp***

The figure 9 shows the influance of changement of Viscous-friction, at values 0.09 and 0.0009. Here the gain ***Kp***=1200, ***ω***=25.12 rad/s, ***J***=0.05 kgm^2.

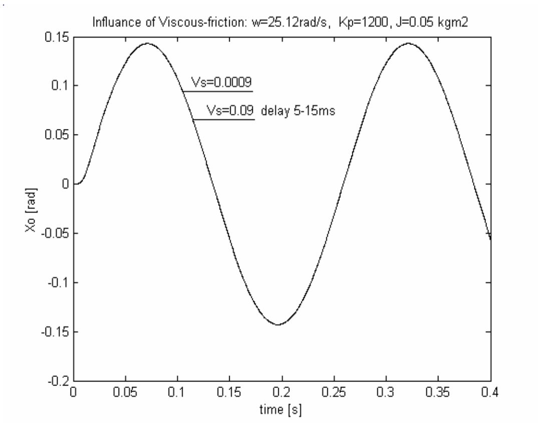

Figure 9: Influance of changement of Viscous-friction

The alterations timely or in shape indicate the features of followings. The influance of Viscous –friction is of no significance, 5-15 millisecond delay, is can be shown only by magnifying.
The influance of changing of values of the inertia and the gain are more important

in the delay however this cause no distortion.
It's known that delay decreases with application the higher value of gain and this is noticeable in that cases too. Considering that the setting time of position control is also decreasing at higher value of gain, it is expedient to set the gain to a possible highest value.

2. Improving the adaptivity of control

2.1. Finding the values of function $Kp(\Theta_r)$ and $Kp(J)$

For improve the adaptivity of non-linear control we need to know the relationship between the suitable gain and the actual values of inertia. Running the model, the gain ***(Kp)*** was variable, while the reference signal of angle of rotation (Θ_r) and inertia (J) were constant. Here J means $J_{reduced}$.
We have done numerous model-runs to find the value of gain which produces a minimal time aperiodic course. In these investigations Θ_r was changed between 0.0001 and 6 radian, and J changed between 0.0005 and 0.05 kgm^2. The was resulted curves ***Kp(J)*** at Θ_r=const are shown in Fig. 10

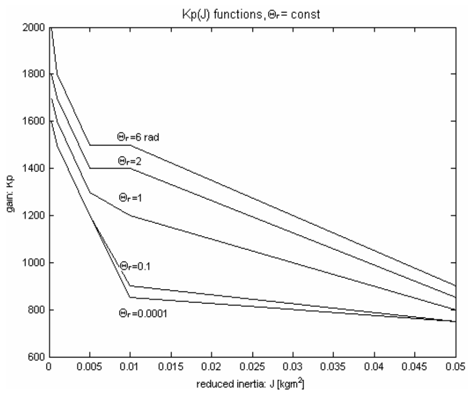

Figure 10: ***Kp(J)*** functions of control loop (see fig.1), resulted minimal time aperiodic approaches.

Note that the influence of change of inertia decreases at values higher Θ_r.

We can draw that in strongly saturated non-linear position control the function between the critical gain-values and inertia is near hyperbolic.

2.2. Approximations with 2^{nd}, 3^{rd}, 4^{th}, 5^{th} order polynoms.

The coefficients calculated by Matlab process. The ones of curves are shown in Fig. 11.

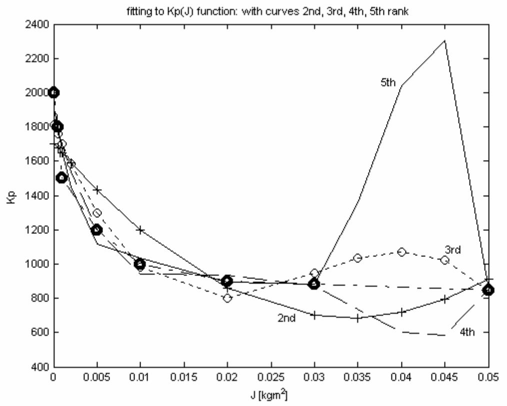

Figure 11: 2^{nd}, 3^{rd}, 4^{th} and 5^{th} order approximations of ***Kp(J)*** function.

The curve 5^{th} order are can not be used, because their inaccuracy is unacceptable at values between 0.03-0.05 of ***J***.
The application of the curve 2^{nd} and 4^{th} order is possible with compromise, because the values of ***Kp*** at values ***J***=0.025-0.045 is too less.
The approximations with polynoms result lower inaccuracy in those cases, when the reference signal is higher.

2.3. Approximations with a hyperbolic function

In the case Θ_r=const we can fit a hyperbolic function on the ***Kp(J)*** curve.
It can be fitted a function given by expression ***Kp***=$[350/J^{0.21}+130]$ on Θ_r. This function was derived from investigations (see Fig 12.) The shape of this function reflects the monoton nature of the function ***Kp(J)*** better.

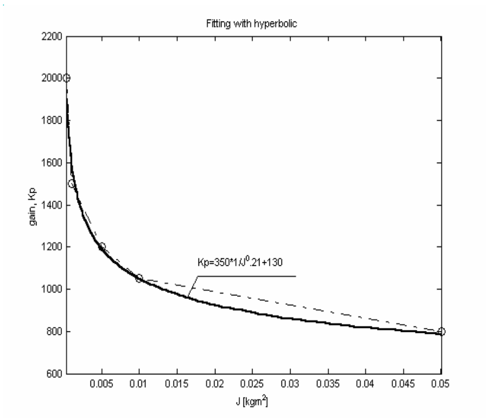

Figure 12: Approximation of the ***Kp(J)*** function with hyperbolic

The adaptive position control model which uses a hyperbolic approximation function can be shown in Fig. 13.

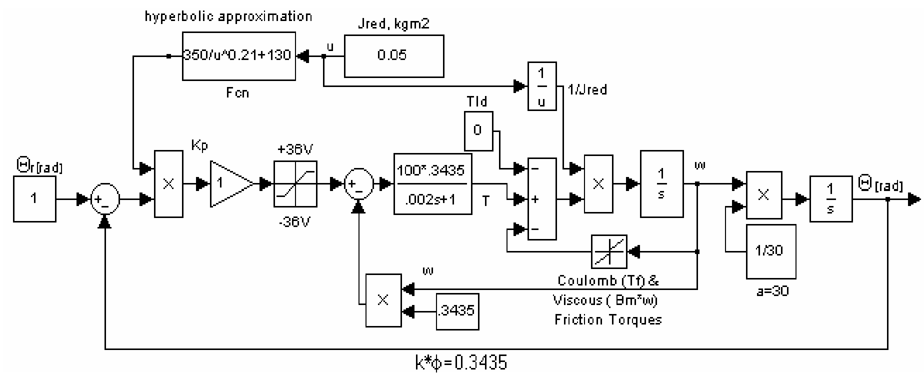

Figure 13: Inertia-adaptiv control model operating with a ***Kp(J)*** function

The control loop given by an actual value of inertia calculates the actual value of gain ***Kp***. Deficiency of that model it does not take into consideration the function ***Kp(Θ$_r$),*** so it is necessary to reduce ***Kp*** to its critical value.

Conclusions

The operation of saturation type non-linear position control can properly be modeled and investigated with simulation methods.

In the task to following a trajectory it is need to use the control in linear domain. For this it’s important to compute the speed at the trajectory beforehand and then to set the input of control as no higher then what provides the operation of control in linear domain.
It was found that the minimum time aperiodic approach of the target can be solved even with highly change parameters. These functions can be determined, and their applications realize a better behavior in the course of operation of the control loop. With a suitable fitting of these functions can be design a non-linear adaptive position controller for hardly changing conditions.

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